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1.1 The Universe as seen in different scales

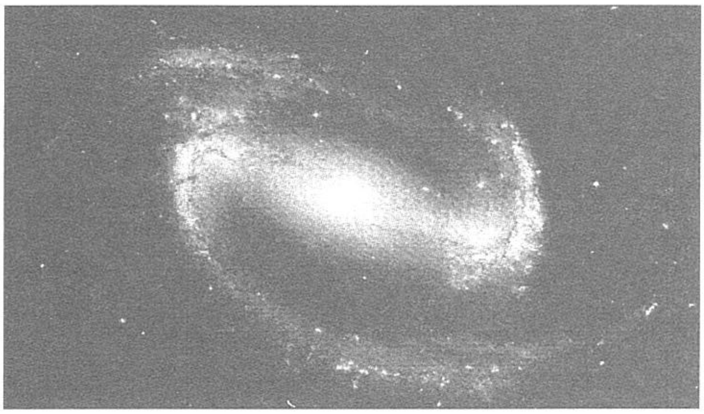
Use the following data wherever necessary :

Astronomical unit	AU = 1.50×10^{11} m
Light year	ly = 9.46×10^{15} m
Parsec	pc = 3.09×10^{16} m = 3.26 ly = 206265 AU
Speed of light in vacuum	$c = 3.00 \times 10^{8}$ m s^{-1}

Part A :

The following questions marked with { } are the past DSE examination questions.
The question marked with {SP} is the Sample Paper question.
The number inside the brackets represents the year of the DSE examination.

M1. Which of the following is NOT contained in the astronomical object shown in the figure ?
{SP}



A. Cluster of galaxies

B. Nebula

C. Star

D. Star cluster

M2. Arrange the following celestial bodies in ascending order of distance from the Earth :
{13}
(1) Sun
(2) Sirius, which is 8.6 ly from Earth
(3) Uranus, which is 19 AU from Earth

A. (1) (2) (3)
B. (1) (3) (2)
C. (3) (1) (2)
D. (3) (2) (1)

M3. Mercury is 0.39 AU from the Sun. Which of the following is NOT a possible distance between Mercury and the Earth ?
{14} Assume that the orbits of Mercury and the Earth are circular and co-planar.

A. 1.20 AU

B. 1.00 AU

C. 0.78 AU

D. 0.50 AU

M4. Given that a typical galaxy in the form of a circular disc is of diameter 10^5 ly and thickness 10^3 ly containing about 10^{11} stars, {14} estimate the average separation between two neighbouring stars within the galaxy assuming that the stars are uniformly distributed.

A. 4.3 ly

B. 6.8 ly

C. 8.9 ly

D. 43 ly

Part B :

The following questions are designed to give supplemental exercise for this chapter.

M5. An asteroid is at 3.84 AU from the Sun. What is the maximum possible distance between the asteroid and the Earth, expressed in light minutes ?

A. 24 light minute

B. 32 light minute

C. 40 light minute

D. 48 light minute

M6. The Milky Way Galaxy extends over 75000 ly. Calculate the distance in km that spans over.

A. 1.1×10^{13} m

B. 1.1×10^{16} m

C. 7.1×10^{17} km

D. 7.1×10^{20} km

M7. Which of the following celestial bodies is NOT revolving around the Sun ?

A. Dwarf planet

B. Moon

C. Comet

D. Meteoroid

M8. Which of the following statements concerning the Milky Way is/are correct ?

(1) The Milky Way is a spiral galaxy.

(2) The Solar system is at the centre of the Milky Way.

(3) The Milky Way contains supercluster of galaxies.

A. (1) only

B. (1) & (2) only

C. (2) & (3) only

D. (1), (2) & (3)

M9. If light takes 8 minutes 20 seconds to reach the Earth, what is the time taken for light to reach Neptune from the Sun, given that Neptune is at 30 AU away from the Sun ?

A. 150 minutes

B. 200 minutes

C. 250 minutes

D. 300 minutes

M10. Which of the following shapes of galaxies is not found in the universe ?

A. spiral galaxy

B. elliptical galaxy

C. irregular galaxy

D. cubic galaxy

M11. Which of the following concerning the Venus are correct ?

(1) Venus has its own moon.

(2) Venus cannot be seen at the mid-night.

(3) Venus is the brightest planet that can be seen from the Earth.

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

M12. Arrange the following celestial objects according to their sizes in descending order.

(1) galaxy

(2) star cluster

(3) supercluster

A. (1), (2), (3)
B. (2), (1), (3)
C. (3), (1), (2)
D. (3), (2), (1)

M13. Which of the following statements can be used to explain why stars in the sky rise in the east and set in the west ?

A. The Earth rotates from west to east with a period of a day.

B. The Earth rotates from east to west with a period of a day.

C. The Earth rotates from west to east with a period of a year.

D. The Earth rotates from east to west with a period of a year.

Answers

1.	A	6.	C	11.	C
2.	B	7.	B	12.	C
3.	D	8.	A	13.	A
4.	A	9.	C		
5.	C	10.	D		

Solution

1. A

The object shown is a spiral galaxy.

It may contain stars, star cluster or nebula, but not cluster of galaxies.

Galaxy should be contained inside a cluster of galaxies.

2. B

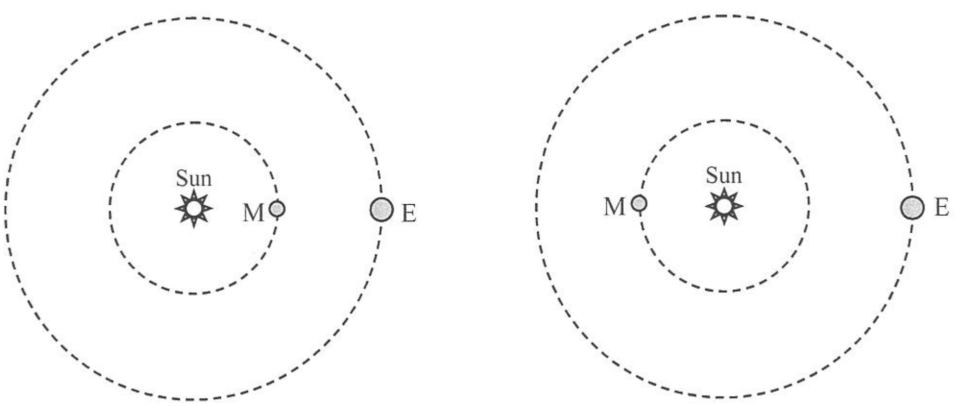
(1) The distance of the Sun from the Earth is 1 AU $= 1.50 \times 10^{11}$ m

(2) The distance of Sirius from the Earth is 8.6 ly $= 8.6 \times 9.46 \times 10^{15}$ m $= 8.14 \times 10^{16}$ m

(3) The distance of Uranus from the Earth is 19 AU $= 19 \times 1.50 \times 10^{11}$ m $= 2.85 \times 10^{12}$ m

Thus, the distances in ascending order are (1), (3), (2)

3 D



The distance of the Earth from the Sun is 1 AU by definition of AU.

When Mercury (M) and Earth (E) are at the same side of the Sun, the separation is the smallest.

Smallest separation = 1 AU − 0.39 AU = 0.61 AU

When Mercury (M) and Earth (E) are at the opposite sides of the Sun, the separation is the greatest.

Greatest separation = 1 AU + 0.39 AU = 1.39 AU

The possible distances between Mercury and Earth are between 0.61 AU to 1.39 AU.

Thus, the distance of 0.50 AU is impossible.

4 A

Total volume of the galaxy $= \pi r^2 \times t = \pi \times (10^5 \times \frac{1}{2})^2 \times (10^3) = 7.854 \times 10^{12}\ \text{ly}^3$

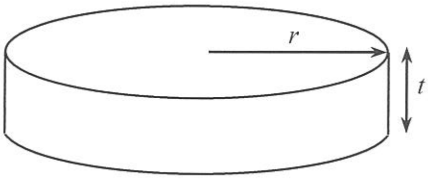
Let the average separation between two neighbouring stars be s.

Average volume occupied by each star $= s^3$

As the number of stars in the galaxy is 10^{11},

$\therefore\ 10^{11} \times s^3 = 7.854 \times 10^{12}$

$\therefore\ s = 4.3$ ly



5. C

The distance of the Earth from the Sun is 1 AU, by definition.

If the asteroid and the Earth are at the two extreme ends of the orbit, then their maximum separation is 3.84 + 1 = 4.84 AU

Maximum possible distance = 4.84 AU

$$= 4.84 \times 1.50 \times 10^{11}\ \text{m} = 7.26 \times 10^{11}\ \text{m}$$

$$= \frac{7.26 \times 10^{11}}{3 \times 10^8}\ \text{light second}$$

$$= 2420\ \text{light second}$$

$$= 40.3\ \text{light minute} \approx 40\ \text{light minute}$$

6. C

Distance $= 75000$ ly

$= 75000 \times 9.46 \times 10^{15}\ \text{m} = 7.1 \times 10^{20}\ \text{m} = 7.1 \times 10^{17}\ \text{km}$

7. B

✓ A. Dwarf planet, similar to the eight large planets, are revolving around the Sun.

× B. Moon is satellite, must be revolving around a planet.

✓ C. Comet revolves around the Sun in elliptical orbits.

✓ D. Meteoroid, comes from comets or asteroids, are orbiting around the Sun.

8. A

✓ (1) The Milky Way is the galaxy that contains our Solar system.

× (2) The Solar system is at the spiral arm, not the centre of the Milky Way galaxy.

× (3) The Milky Way galaxy belongs to the Local Group, which in turns belongs to a supercluster of galaxies.

9. C

Distance of the Earth from the Sun is 1 AU, by definition.

Since the time taken is proportional to the distance, thus, time taken = 8 min 20 sec × 30 = 250 minutes

10. D

Cubic galaxy has not been observed.

11. C

×	(1)	Venus does not have its own moon (satellite).
✓	(2)	At mid-night, Venus is at the other side of the observer on the Earth, thus it cannot be seen.
✓	(3)	Other than the Sun and the Moon, Venus is the brightest object seen in the night sky.

12. C

Supercluster contains galaxy,
galaxy contains star cluster.

13. A

As the Earth rotates from west to east with a period of a day,
all the stars in the sky seem to rotate from east to west with also a period of a day,
thus every star rises at the east and sets at the west.

1.2 Astronomy through history

Use the following data wherever necessary :

Astronomical unit	AU = 1.50×10^{11} m
Speed of light in vacuum	$c = 3.00 \times 10^{8}$ m s^{-1}
Light year	ly = 9.46×10^{15} m

Part A :

The following questions marked with { } are the past DSE examination questions.

The questions marked with {SP} are the Sample Paper questions.

The question marked with {PP} is the Sample Paper question.

The number inside the brackets represents the year of the DSE examination.

M1. According to the Ptolemy's geocentric model,
{SP} A. the orbit of Mercury is elliptical.
B. The Earth-Venus distance is always smaller than the Earth-Sun distance.
C. The Earth-Mars distance is always smaller than the Earth-Sun distance.
D. It is not possible to observe Jupiter at mid-night.

M2. Which statement is INCORRECT ?
{SP} A. Planets move around stars in elliptical orbits.
B. Comets move around the Sun in elliptical orbits.
C. The cube of the orbital period of a planet is proportional to the square of the length of its semi-major axis around the Sun.
D. For a given planet orbiting around the Sun, the speed of the planet increases as its distance from the Sun decreases.

M3. The average of the closest and farthest distances of a comet from the Sun is 18 AU. What is its orbital period ?
{SP} A. 6.9 yr
B. 18 yr
C. 76 yr
D. 200 yr

M4. Which of the following statements about retrograde motion of planets is correct ?
{PP} A. Retrograde motion can only be observed at locations near the equator.
B. Only planets closer to the Sun than the Earth exhibit retrograde motion.
C. During retrograde motion, an observer on Earth sees the planet move from east to west over the course of several weeks or months.
D. The geocentric model cannot explain the retrograde motion of planets.

M5. Which statement about the motion of the Earth around the Sun is **INCORRECT** ?
{12} A. The speed of the Earth in its orbit is not constant.
B. The Sun is at the centre of the Earth's orbit.
C. The distance from the Sun to the Earth changes periodically.
D. In general, the instantaneous velocity of the Earth is not perpendicular to Sun's gravitational force.

M6. Which of the following statements about Ptolemy's geocentric model and Copernican's heliocentric model of the universe
{13} is/are correct ?

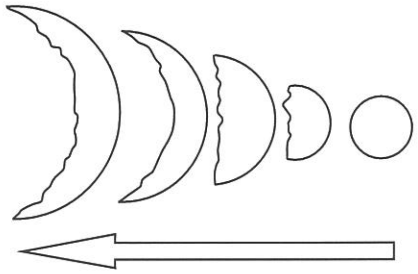
(1) Orbits are circular in both models.

(2) The Earth is at the centre of the Moon's orbit in both models.

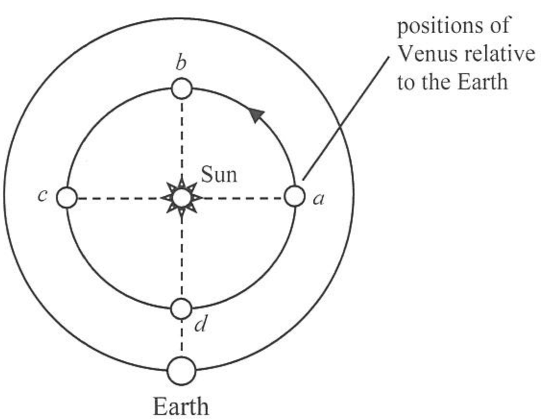
(3) Retrograde motion can be explained in both models.

A. (1) only
B. (3) only
C. (1) & (2) only
D. (1), (2) & (3)

M7. The following diagram is Galileo's drawing in 1610 showing the phases of Venus.
{14}

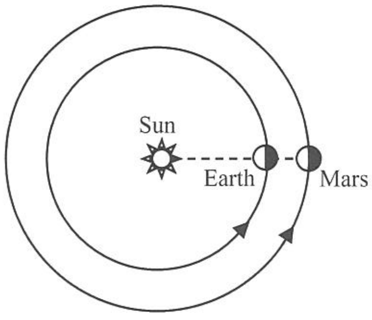


Which of the following parts of the orbits of Venus represent the above change of phases from right to left ?



A. $a \rightarrow b \rightarrow c$
B. $b \rightarrow c \rightarrow d$
C. $c \rightarrow d \rightarrow a$
D. $d \rightarrow a \rightarrow b$

M8. When the Earth lines up with the Sun and Mars as shown, how does Mars appear to move across the night sky as viewed from
{15} Earth ?



A. Mars moves from west to east against the background stars.
B. Mars moves from east to west against the background stars.
C. Mars does not move against the background stars.
D. The movement of Mars cannot be determined because the east and west directions are not known.

M9. Which of the following observations by Galileo contradict with the geocentric model of the universe ?

{15}

(1) the discovery of satellites of Jupiter

(2) the retrograde motion of Mars

(3) the changing phase of Venus

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

Part B :

The following questions are designed to give supplemental exercise for this chapter.

M10. Which of the following is/are the main features of the Copernican model of the solar system ?

(1) Planets move around the Sun, which is at the centre of the orbits.

(2) The orbits of the planets are elliptical.

(3) Planets move with uniform speed in their orbits.

A. (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

M11. The closest distance of a comet from the Sun is 32.5 AU and the farthest distance is 52.9 AU. What is its orbital period ?

A. 12.2 years
B. 185 years
C. 279 years
D. 385 years

M12. Which of the following concerning the Kepler's second law of planetary motion for a certain planet is/are correct ?

(1) The line joining the planet and the Sun sweeps equal angles in equal times.

(2) The planet must move with varying speed along the orbit.

(3) The planet moves faster when it is farther away from the Sun.

A. (1) only
B. (2) only
C. (1) & (3) only
D. (2) & (3) only

M13. Galileo observed the complete phases of Venus. Which of the following is/are the correct deductions ?

(1) Venus emits different amount of light at different times.

(2) Venus must orbit around the Sun.

(3) Venus must have self rotation.

A. (1) only
B. (2) only
C. (1) & (3) only
D. (2) & (3) only

M14. Galileo has made some important astronomical discoveries through the observation by telescope. Which of the following are the impacts of his discoveries ?

(1) Celestial bodies are not perfect sphere.

(2) The Earth is not the centre of every celestial body's orbit.

(3) The orbits of the planets are not circular.

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

M15. In 2012, a comet reaches the farthest distance from the Sun, which is 25.2 AU. If the shortest distance of the comet from the Sun is 12.6 AU, at which year will the comet reach this point ?

A. 2032

B. 2043

C. 2053

D. 2094

M16. When Galileo first observed the four celestial bodies near Jupiter through a telescope, how can he be sure that they are satellites but not stars ?

(1) They appeared to be smaller than Jupiter.

(2) Their positions change at different nights.

(3) They appeared to be carried by Jupiter and their positions were never far away from Jupiter.

A. (1) only
B. (1) & (2) only
C. (2) & (3) only
D. (1), (2) & (3)

M17. Mercury is called the 'morning star' and the 'evening star' as it always appears besides the Sun. How can Ptolemaic model explain this phenomenon ?

(1) The distance between the Earth and Mercury was always shorter than the distance between the Earth and the Sun.

(2) Mercury moved around the Sun.

(3) The centre of the epicycle of Mercury was tied to the line joining the Earth and the Sun.

A. (1) only
B. (3) only
C. (1) & (3) only
D. (2) & (3) only

M18. How can retrograde motion of Mars be explained by using Ptolemaic model ?

(1) An epicycle was added to the deferent to explain the retrograde motion.

(2) Retrograde motion occurs when Mars is close to the Sun.

(3) Mars travels with varying speed along the epicycle.

A. (1) only
B. (2) only
C. (1) & (3) only
D. (2) & (3) only

M19. How can retrograde motion of Mars be explained by using Copernican model ?

(1) The Earth travels with greater speed than Mars.

(2) Retrograde motion occurs when the Earth overtakes Mars.

(3) Epicycle was still needed to explain the retrograde motion..

A. (1) only
B. (1) & (2) only
C. (2) & (3) only
D. (1), (2) & (3)

M20. The farthest distance of a comet from the Sun is 55 AU. If the comet reaches this farthest point every 300 year, what is the shortest distance of the comet from the Sun ?

A. 24.6 AU

B. 34.6 AU

C. 44.6 AU

D. 54.6 AU

M21. Which of the following is NOT the basic concept of the ancient Greek astronomers ?

A. The Earth was static and situated at the centre of the Universe.

B. All celestial bodies move in elliptical orbit around the Earth.

C. All celestial bodies move with constant speed.

D. The Venus is sometimes a morning star and sometimes an evening star.

M22. Which of the following concerning the Ptolemy's geocentric model are correct ?

(1) The Earth-Mercury distance is always less than the Earth-Sun distance.

(2) The retrograde motion of Mars is explained by the epicycle and deferent.

(3) Venus always appears near the Sun is explained by assuming that the centre of the epicycle of Venus is tied to the line joining the Earth and the Sun.

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

M23. Galileo made many astronomical discoveries through the observation by telescope. Which of the followings is **NOT** one of his discoveries ?

A. Venus undergoes a complete cycle of phases.

B. The planet Jupiter has four satellites.

C. The Milky Way consists of numerous faint stars.

D. The seventh major planet Uranus was discovered.

M24. The closest distance of a comet from the Sun is 26 AU and the farthest distance is 42 AU. Find its orbital period.

A. 133 years

B. 136 years

C. 198 years

D. 272 years

M25. Which of the following deductions from Kepler's laws of planetary motion is/are correct ?

(1) The Sun is at the centre of the orbits of the planets.

(2) The planets move with non-uniform speed around the Sun.

(3) The orbital periods of the planets are proportional to the semi-major axes of their orbits.

A. (1) only
B. (2) only
C. (1) & (3) only
D. (2) & (3) only

M26. Which of the following astronomical discoveries is/are made by Galileo ?

(1) The Milky Way consists of numerous stars.

(2) The surface of Mars is rough and uneven.

(3) Saturn has many satellites revolving around it.

A. (1) only
B. (2) only
C. (1) & (3) only
D. (2) & (3) only

M27. Galileo observed that Jupiter had four satellites moving around it, but sometimes one satellite was missed. Which of the following may be the reason ?

(1) The missed satellite was too close to Jupiter and cannot be resolved as a separate body.

(2) The missed satellite was behind Jupiter and cannot be seen.

(3) Two satellites collided together at the moment.

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

M28. Titania is a satellite orbiting around the planet Uranus. Its orbital period is 8.71 days with semi-major axis of 436 000 km. Juliet is another satellite orbiting around Uranus, and its orbital period is 0.49 days. What is its semi-major axis ?

A. 62000 km

B. 64000 km

C. 70000 km

D. 86000 km

M29. The following shows the orbital semi-major axis and the orbital period of four dwarf planets. Which set of data may not be correct ?

	Dwarf planet	Semi-major axis	Orbital period
A.	Ceres	2.77 AU	4.60 years
B.	Haumea	43.1 AU	283 years
C.	Makemake	54.5 AU	306 years
D.	Eris	67.9 AU	560 years

M30. The semi-major axis of two planets are in the ratio of 1 : 2. What should be the ratio of their orbital periods ?

A. 1 : 1.6

B. 1 : 2.8

C. 1.6 : 1

D. 2.8 : 1

Answers

1. B	6. D	11. C	16. C	21. B	26. A
2. C	7. B	12. B	17. B	22. D	27. C
3. C	8. B	13. B	18. A	23. D	28. B
4. C	9. B	14. A	19. B	24. C	29. C
5. B	10. B	15. C	20. B	25. B	30. B

Solution

1. B

✗ A. The orbit of Mercury should be circular.

✓ B. Since the Earth is at the centre of the model, and Venus is closer to the Earth than the Sun, thus, Earth-Venus distance must always be smaller than the Earth-Sun distance.

✗ C. According to the model, Mars is farther away from the Earth at centre than the Sun, thus, Earth-Mars distance should always be greater than the Earth-Sun distance.

✗ D. According to the model, Jupiter and the Sun may be at the two opposite sides of the Earth, thus, it is possible to observe Jupiter at mid-night.

2. C

✓ A. Planets move in elliptical orbits around the Sun which is a star.

✓ B. Comets move in elliptical orbits around Sun.

✗ C. It should be the square of the orbital period of a planet is proportional to the cube of the length of its semi-major axis around the Sun.

✓ D. When a planet is closer to the Sun, its speed increases, according to Kepler's second law.

3. C

The average of the closest and farthest distances is equal to the semi-major axis a.

By Kepler's third law of planetary motion : $T^2 \propto a^3$

Compared with the Earth that has period of 1 year and semi-major axis of 1 AU.

$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$ $\quad\therefore \left(\frac{T_1}{1}\right)^2 = \left(\frac{18}{1}\right)^3$ $\quad\therefore T_1 = 76\text{ yr}$

4. C

✗ A. Retrograde motion of planets can be observed at any suitable latitude, not only at equator.

✗ B. Retrograde motion of occurs for planets both closer to and further away than the Earth.

✓ C. During retrograde motion, the planet moves from east to west on the celestial sphere over a period of weeks or months.

✗ D. Both the geocentric model and the heliocentric model can explain the retrograde motion of planets.

5. B

✓ A. By Kepler's second law, the speed of the Earth is greater when it is nearer to the Sun and is smaller when it is further away from the Sun.

× B. By Kepler's first law, the Sun is at one of the focus of the elliptical orbit of the Earth, not at the centre.

✓ C. Since the Earth orbits around the Sun in elliptical orbit, the distance from the Sun to the Earth varies.

✓ D. Since the Earth orbits around the Sun in elliptical orbit but not circular orbit, the direction of the velocity is not perpendicular to the force.

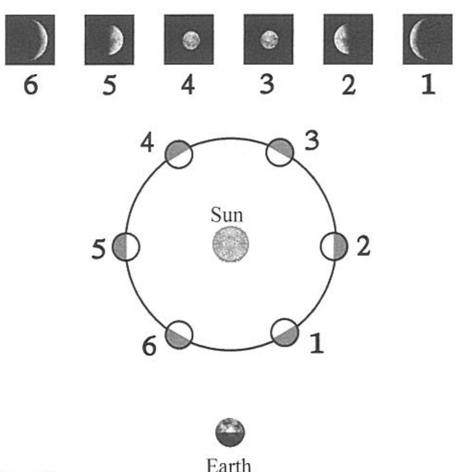
6. D

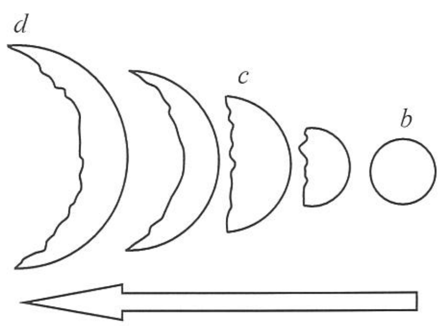
✓ (1) Both models assume that the orbits are circular. Kepler discovered that the orbits are actually elliptical.

✓ (2) In Ptolemy's model, the Earth is at the centre of all the celestial bodies, including the Moon.
In Copernican's model, the Earth revolves around the Sun, but the Moon revolves around the Earth.

✓ (3) In Ptolemy's model, retrograde motion is explained by epicycle.
In Copernican's model, retrograde motion is explained by the different speed in different cycles.

7. B





Refer to the figure,

At position between ③ and ④, that is, position b, the Venus shows full phase.

At position ⑤, that is, position c, Venus shows a half phase.

At position ⑥, that is, position near d, Venus shows a eclipse phase.

Thus, the sequence is $b \rightarrow c \rightarrow d$

8. B

When the Earth lines up with the Sun and Mars, Mars is undergoing retrograde motion.

Thus, Mars would appear to move from east to west across the sky against the background stars in a few nights.

9. B

✓ (1) The discovery of satellites of Jupiter shows that Earth is not the orbital centre of every celestial body.

× (2) The retrograde motion of Mars is not observed by Galileo.
Moreover, retrograde motion can be explained by geocentric model of the universe.

✓ (3) The changing phase of Venus cannot be explained by the geocentric model of the universe.

10. B

✓ (1) Copernican model is a heliocentric model, with the Sun at the centre.

× (2) Copernicus wrongly assumed that every planet moved in circular orbit.

✓ (3) Copernicus wrongly assumed that every planet moved with uniform speed along their orbits.

11. C

Semi-major axis : $a = (32.5 + 52.9) / 2 = 42.7$ AU

By Kepler's third law of planetary motion : $T^2 \propto a^3$

Compared with the Earth that has period of 1 year and semi-major axis of 1 AU.

$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$ $\quad \therefore \left(\frac{T_1}{1}\right)^2 = \left(\frac{42.7}{1}\right)^3$ $\quad \therefore T_1 = 279$ year

12. B

× (1) The line joining the planet and the Sun should sweep equal areas, not equal angles, in equal times.

✓ (2) In order to sweep equal areas in equal times, the planet must move with varying speed.

× (3) When the planet is farther away from the Sun, since the distance from the Sun is longer,
the distance travelled should be shorter,
thus the planet should move with smaller speed when it is farther away from the Sun.

13. B

× (1) Venus reflects light from the Sun, it would not emit light itself.

✓ (2) Venus must orbit around the Sun, so that the Sun may fully lit Venus to give full phase at some positions.

× (3) The phases of Venus have no relation with the self rotation of Venus.

14. A

✓ (1) The observation that the surface of the Moon is rough and uneven, and the observation that there are sunspots on the Sun's surface, lead to the conclusion that celestial bodies are not perfect spheres.

✓ (2) Since there are satellites moving around Jupiter, not every celestial body moves around the Earth, thus, the Earth may not be at the centre of every orbit.

× (3) Galileo has not found evidence for non-circular orbits, the discovery of elliptical orbits was by Kepler.

15. C

Semi-major axis : $a = (25.2 + 12.6) / 2 = 18.9$ AU

By Kepler's third law of planetary motion : $T^2 \propto a^3$

Compared with the Earth that has period of 1 year and semi-major axis of 1 AU.

$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$ $\quad \therefore \left(\frac{T_1}{1}\right)^2 = \left(\frac{18.9}{1}\right)^3$ $\quad \therefore T_1 = 82$ year

It takes half of the period to move from the farthest point to the nearest point, thus $t = 2012 + (82 / 2) = 2053$

$\therefore$ The year that the comet reaches the shortest point is 2053.

16. C

× (1) Many stars appear to be smaller than Jupiter, as their distances are so far away.

✓ (2) Stars would not change their positions after a night, but satellites moving around a planet would.

✓ (3) If they are satellites moving around Jupiter, they must always be around Jupiter.

17. B

× (1) In Ptolemaic model, the Earth is at the centre,
thus Earth-Mercury distance must always be shorter than the Earth-Sun distance.
However, this cannot explain why Mercury is always beside the Sun.

× (2) Ptolemaic model is a geocentric model, and in this model, Mercury moved around the Earth.

✓ (3) By assuming that the centre of the epicycle of Mercury is fixed to the line joining the Earth and the Sun, Mercury always appeared around the Sun.

18. A

✓ (1) Ptolemaic model made use of epicycle attaching to deferent to explain the retrograde motion of Mars.

× (2) Retrograde motion occurs when Mars is close to the Earth, not the Sun.

× (3) Mars travel with constant speed along the epicycle, according to Ptolemaic model.

19. B

✓ (1) As the Earth has smaller radius than Mars, Earth moves with greater speed than Mars.

✓ (2) When the Earth overtakes Mars, Mars seems to move backwards, and it is the retrograde motion.

× (3) In Copernican model, there is no need to use epicycle to explain the retrograde motion.

20. B

By Kepler's third law of planetary motion : $T^2 \propto a^3$

Compared with the Earth that has period of 1 year and semi-major axis of 1 AU.

$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3 \quad \therefore \left(\frac{300}{1}\right)^2 = \left(\frac{a_1}{1}\right)^3 \quad \therefore a_1 = 44.8 \text{ AU} \quad \therefore$ semi-major axis is 44.8 AU

By length of major axis = shortest distance + farthest distance

$\therefore 44.8 \times 2 = 55 + d \qquad \therefore d = 34.6 \text{ AU}$

21. B

✓ A. The Greek believed that the Earth was chosen by God and must be most important in the Universe, and of course the Earth must be at the centre.
On the other way, they did not feel the movement of the Earth and believed that the Earth is static.

× B. The Greek believed that all celestial bodies move in circle, which is the perfect shape.

✓ C. The Greek believed that all celestial bodies move in uniform speed, which is the perfect motion.

✓ D. The Greek found that Venus would sometimes become a morning star and sometimes an evening star.

22. D

✓ (1) Since the epicycle of Mercury is always between the Earth and the Sun, thus the Earth-Mercury distance is always less than the Earth-Sun distance.

✓ (2) Mars travels at a constant speed along the smaller epicycle. The centre of epicycle then moves at a constant speed around the larger circle called deferent. When Mars is close to the Earth, Mars is then observed to have retrograde motion.

✓ (3) The centre of the epicycles of Venus, as well as Mercury, are assumed to be tied to the line joining the Earth and the Sun, thus they always appear around the Sun.

23. D

Uranus was not discovered by Galileo,

though Uranus was discovered after the invention of telescope.

24. C

Semi-major axis : $a = \frac{1}{2}(26+42) = 34$ AU

By Kepler's third law of planetary motion : $T^2 \propto a^3$

Compared with the Earth that has period of 1 year and semi-major axis of 1 AU.

$$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

$$\therefore \left(\frac{T_1}{1}\right)^2 = \left(\frac{34}{1}\right)^3$$

$$\therefore T_1 = 198 \text{ year}$$

25. B

× (1) According to the first law, the Sun should be at one of the focus of the elliptical orbit, not at the centre.

✓ (2) According to the second law, the planets must move with varying speed along the elliptical orbit.

× (3) According to the third law, the square of the orbital periods are proportional to the cube of the semi-major axis of their orbits.

26. A

✓ (1) Milky Way, the band of light that extends over the sky actually consists of numerous faint stars.

× (2) Galileo observed the surface of the Moon, not Mars.

× (3) Galileo discovered the satellites of Jupiter, not Saturn.

27. C

✓ (1) As it is too close to Jupiter, they are seen as a single body and cannot be resolved.

✓ (2) As the satellite moves around Jupiter, it may be behind Jupiter at some time.

× (3) Two satellites have their own orbits, they would not collide together.

28. B

Kepler's third law : $T^2 \propto a^3$

$$\therefore \quad \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

$$\therefore \quad \left(\frac{8.71}{0.49}\right)^2 = \left(\frac{436000}{a_2}\right)^3 \qquad \therefore \ a_2 = 64000 \text{ km}$$

29. C

Ceres	$a^3 = 21.3$	$T^2 = 21.2$
Haumea	$a^3 = 80100$	$T^2 = 80100$
Makemake	$a^3 = 162000$	$T^2 = 93600$
Eris	$a^3 = 313000$	$T^2 = 314000$

According to Kepler's third law of planetary motion, $a^3 \propto T^2$, and the proportional constant is approximately 1. The set of the data of Makemake shows that the proportional constant is not 1, thus it should be a set of wrong data.

30. B

Kepler's third law :

$T^2 \propto a^3$

$\therefore \ (T_1 : T_2)^2 = (a_1 : a_2)^3 = (1 : 2)^3 = 1 : 8$

$\therefore \ T_1 : T_2 = 1 : 2.8$

1.3 Orbits motion under gravity

Use the following data wherever necessary :

Speed of light in vacuum	$c = 3 \times 10^8$ m s^{-1}
Astronomical unit	AU = 1.50×10^{11} m
Acceleration due to gravity	$g = 9.81$ m s^{-2} (close to the Earth)
Universal gravitational constant	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

The following list of formulae may be found useful :

Newton's law of gravitation	$F = \dfrac{Gm_1m_2}{r^2}$
Gravitational potential energy	$U = -\dfrac{GMm}{r}$

Part A :

The following questions marked with { } are the past DSE examination questions.

The questions marked with {PP} are the Sample Paper questions.

The number inside the brackets represents the year of the DSE examination.

M1. {PP} If the acceleration due to gravity on the Moon's surface is 1/6 of that on the Earth's surface, what is the gravitational potential energy with respect to the surface of the Moon for an object of mass m which is 1 m above the Moon's surface ?

Given : R = radius of the Moon (>> 1 m)

M = mass of the Moon

G = the universal gravitational constant

g = acceleration due to gravity (close to the Earth)

A. $-\dfrac{mg}{6}$

B. $\dfrac{mg}{6}$

C. $-\dfrac{GMm}{R}$

D. $\dfrac{GMm}{R}$

M2. {PP} A satellite of mass m is in a circular orbit of radius r around a planet of mass M and radius R. What is the extra kinetic energy required by the satellite to escape the gravitational attraction of the planet ? (G = the universal gravitational constant)

A. $\dfrac{GMm}{2r}$

B. $\dfrac{GMm}{r}$

C. $\dfrac{GMm}{2R}$

D. $\dfrac{GMm}{R}$

M3. Weightlessness occurs inside a spacecraft orbiting around the Earth. Which statement is correct ?

{12} A. Weightlessness only occurs for objects inside a spacecraft orbiting around the Earth.

B. The gravitational attraction of the Earth in the spacecraft's orbit is so weak that the gravitational force is practically zero.

C. The gravitational attraction of the Earth is cancelled out by that of the Moon.

D. Both the spacecraft and the objects inside it undergo free fall towards the Earth.

M4. An interplanetary spacecraft is launched from the Earth. The initial speed is $\sqrt{\frac{3GM}{R}}$, where G is the universal gravitational

{12} constant, M is the mass of the Earth and R is the radius of the Earth. What is the speed of the spacecraft when it is very far away from the Earth ?

A. $\sqrt{\frac{GM}{R}}$

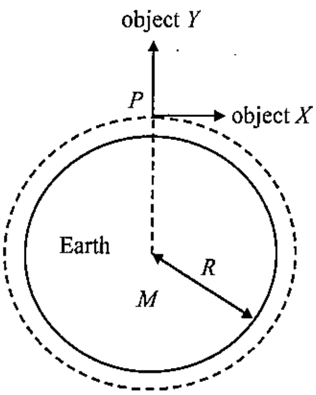
B. $\sqrt{\frac{GM}{2R}}$

C. $\sqrt{\frac{2GM}{R}}$

D. zero

M5. At a point P close to the Earth, two objects X, Y travel with the same speed v

{14} where $v = \sqrt{\frac{GM}{R}}$ with M and R being the mass and radius of the Earth respectively and G is the universal gravitational constant. X travels tangentially at P while Y travels radially outward from P. Which of the following statements about their subsequent motions is correct ? Neglect air resistance.



	Object X	**Object Y**
A.	will eventually return to the Earth.	will continue to fly in its orbit.
B.	will eventually return to the Earth.	will eventually return to the Earth.
C.	will continue to fly in its orbit.	will continue to fly in its orbit.
D.	will continue to fly in its orbit.	will eventually return to the Earth.

M6. A satellite is orbiting the Earth at a distance h from the Earth's surface. What is the gain in gravitational potential energy of

{15} the satellite in the orbit with respect to the Earth's surface ?

m = mass of the satellite

R = radius of the Earth

g = acceleration due to gravity on the Earth's surface

A. $mgh\left(\frac{R}{R+h}\right)$

B. $mgh\left(\frac{R}{R+h}\right)^2$

C. $mgh\left(\frac{R+h}{R}\right)$

D. $mgh\left(\frac{R+h}{R}\right)^2$

Part B :

The following questions marked with [ ] are the past HKAL questions.

The number inside the brackets represents the year of the examination.

M7. [81] The moon orbits the earth once every 27.3 days, with a mean orbital radius of R. What is the period of an earth satellite with an orbital radius of R / 30 ?

A. 4 hours
B. 22 hours
C. 68 hours
D. 260 days

M8. [82] A communication satellite in a circular orbit of radius R has a period of 24 hours. The period of a satellite in a circular orbit of radius $\frac{R}{4}$ is

A. 3 hours
B. 6 hours
C. 12 hours
D. 24 hours

M9. [85] If v_1 is the minimum speed for a projectile to escape from the earth and v_2 is the orbital speed of a satellite circling close to the earth, then $\frac{v_1}{v_2}$ =

A. $\frac{1}{\sqrt{2}}$
B. $\frac{1}{2}$
C. 1
D. $\sqrt{2}$

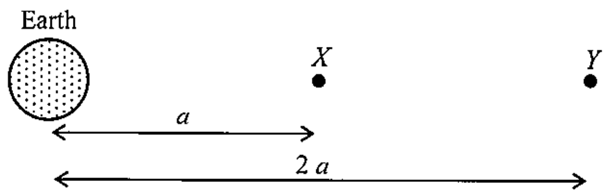
M10. [87] A space capsule is launched with speed u from the surface of the Earth to a maximum height above the ground equal to the radius of the Earth. A rocket is then fired horizontally which keeps the space capsule revolving in a circular orbit round the earth at that altitude with speed v. The ratio u : v is equal to

A. 1 : 2
B. 1 : $\sqrt{2}$
C. 1 : 1
D. $\sqrt{2}$: 1

M11. [88] A satellite moving round the Earth in a circular orbit of radius R has a period T. What would the period be if the orbit were of radius R / 4 ?

A. $\frac{T}{8}$
B. $\frac{T}{4}$
C. $\frac{T}{2}$
D. $2T$

M12.
[89]



Two points X and Y are at distances a and $2a$ from the centre of the Earth as shown in the diagram. The gravitational potential energy of a 1 kg mass at X is -8 kJ. When the 1 kg mass is taken from X to Y, the work done on the mass is

A. -4 kJ.
B. -2 kJ.
C. $+2$ kJ.
D. $+4$ kJ.

M13. Assuming the Earth to be a perfect sphere, what would its angular velocity of rotation have to be for an object at the equator
[89] to be weightless (i.e. to give a spring balance reading of zero) ? (Radius of the Earth = 6.4×10^6 m.)

A. 2.4×10^{-12} rad s^{-1}
B. 1.6×10^{-6} rad s^{-1}
C. 1.3×10^{-3} rad s^{-1}
D. 8.0×10^{2} rad s^{-1}

M14. In which of the following situations is the magnitude of the normal reaction of the supporting surface, R, equal to the weight
[91] of the body, mg ?

(1) At rest on a rough inclined plane
(2) On the floor of a spacecraft in circular orbit around the earth
(3) On the floor of a lift moving upwards with uniform velocity

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

M15. An object of mass m is released from a spacecraft at a distance $3R$ from the centre of the Earth which has radius R and
[92] mass M. On reaching the Earth's surface, the increase in kinetic energy of the object is

A. $\dfrac{GmM}{3R}$

B. $\dfrac{2GmM}{3R}$

C. $\dfrac{GmM}{2R}$

D. $\dfrac{GmM}{R}$

M16. The velocity of escape from the earth is V_o. For a planet with radius twice that of the earth and with density three times that
[93] of the earth, the velocity of escape from the planet would be

A. $\dfrac{\sqrt{3}}{2} V_o$

B. $2V_o$

C. $\sqrt{6}\, V_o$

D. $2\sqrt{3}\, V_o$

M17. For planets or satellites in circular orbits around a celestial body such as the sun or the earth, the period T is related to the
[93] radius of orbit r by Kepler's 3rd Law : $T^2 = k\,r^3$ where k is a constant.

Which of the following statements concerning the constant k is correct ?

A. It is a universal constant whose value is not affected by the choice of units.

B. It is a universal constant whose value depends on the choice of units.

C. It would have a certain value for the earth moving around the sun, but a different value for another planet moving around the sun.

D. It would have a certain value for all planets moving around the sun, but a different value for all satellites moving around the earth.

M18. Two satellites A and B of the same mass are moving in circular orbits round the earth. The radius of A's orbit is r and that of
[94] B's orbit is $2r$. Their total mechanical energies are E_A and E_B respectively. Which of the following descriptions about E_A and E_B is correct ? (Gravitational potential energy is taken to be zero at infinity.)

A. $E_A > 0$ and $E_B = \frac{1}{2}E_A$

B. $E_A > 0$ and $E_B = -2E_A$

C. $E_A < 0$ and $E_B = 2E_A$

D. $E_A < 0$ and $E_B = \frac{1}{2}E_A$

M19. On a certain planet, an object is thrown vertically upwards with an initial velocity of v_1 and it returns to the ground after
[95] time t. If the velocity of escape from the planet is v_2, find the radius of the planet.

A. $\dfrac{2v_1^2 t}{v_2}$

B. $\dfrac{4v_1^2 t}{v_2}$

C. $\dfrac{2v_2^2 t}{v_1}$

D. $\dfrac{v_2^2 t}{4v_1}$

M20. Two satellites of the same mass travel around the earth in circular orbits of different radii. The satellite in the orbit with
[96] smaller radius has

A. a greater speed.

B. a longer period.

C. a smaller acceleration towards the earth's centre.

D. a greater sum of gravitational potential energy and kinetic energy.

M21. Which of the following statements about a communication satellite in parking orbit above the earth's surface is **incorrect** ?

[99] A. It is accelerating towards the centre of the earth at all times.

B. It must be in a circular orbit above the earth's equator.

C. It must be rotating in the same sense and with the same angular speed as the earth.

D. It is at a height where its gravitational potential energy is numerically equal to its kinetic energy.

M22. In which of the following situations is the magnitude of the normal reaction of the supporting surface always equal to the
[00] weight of the body ?

(1) A ball bouncing vertically on a horizontal ground is in contact with the ground.

(2) An astronaut in a spacecraft which performs circular motion around the earth.

(3) A boy standing in a lift which is moving vertically upward with a uniform velocity.

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

M23. A satellite of mass m is launched from the earth's surface into an orbit at a height of $3R$ above the earth's surface, where R
[03] is the radius of the earth. What is the gravitational potential energy gained by the satellite during this process ?
(g is the gravitational field at the earth's surface.)

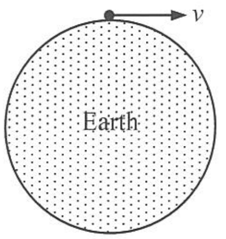
A. $\frac{1}{3}mgR$

B. $\frac{1}{4}mgR$

C. $\frac{2}{3}mgR$

D. $\frac{3}{4}mgR$

M24.
[07]



Suppose a spacecraft is launched with an initial speed v at the earth's surface and it eventually enters the close orbit of the earth. If the launching speed was increased to $2v$, the spacecraft would (Neglect air resistance.)

A. revolve uniformly in a circular orbit around the earth with a longer period.

B. revolve uniformly in a circular orbit around the earth with a shorter period.

C. revolve in an elliptical orbit around the earth with a longer period.

D. leave the earth and travel to outer space.

M25. It is known that a geostationary communication satellite is at a height of 5.6 R above the earth's surface where R is the radius
[07] of the earth. Which of the following is the best estimate for the separation between the moon and the earth ?

A. 50 R

B. 60 R

C. 150 R

D. 180 R

M26. In which of the following situations does the person concerned experience 'weightlessness' ?
[07]

(1) an astronaut in a spacecraft which is coasting in outer space with its rocket engines shut off

(2) an astronaut in a spacecraft which is decelerating to make a soft landing on the moon

(3) a parachutist descending with a constant velocity in the air

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

M27. The Earth moves around the Sun with a mean orbital radius of 1.50×10^{11} m. Which of the following is a possible planetary
[08] orbit around the Sun ?

	Orbital radius (m)	Period (year)
A.	13.4×10^{18}	11.9
B.	14.3×10^{11}	29.5
C.	2.85×10^{11}	1.9
D.	2.11×10^{11}	0.6

M28. An object is projected vertically upward with speed u from the surface of a planet. It rises to a maximum height h above the
[09] planet's surface, with h much smaller than the radius R of the planet. What is the escape velocity of the planet ? (Neglect air resistance.)

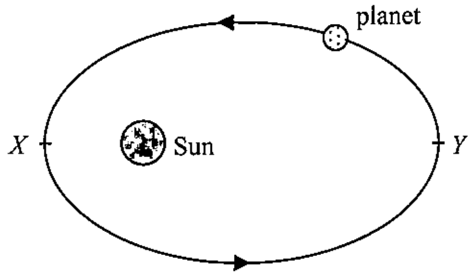
A. $u\sqrt{\dfrac{R}{2h}}$

B. $u\sqrt{\dfrac{R}{h}}$

C. $u\left(\dfrac{R}{2h}\right)$

D. $u\left(\dfrac{R}{h}\right)$

M29.
[09]



A planet is moving in an elliptical orbit around the Sun as shown. Which statement is **INCORRECT** ?

A. The gravitational force acting on the planet at X is greater than that at Y.

B. The kinetic energy of the planet at X is greater than that at Y.

C. The gravitational potential energy of the planet at X is higher than that at Y.

D. The total mechanical energy of the planet is the same at all positions in the orbit.

M30. The Moon revolves around the Earth with a period of 27.3 days and its speed is 1.0 km s^{-1}. Which combination of period
[09] and speed is possible for an artificial satellite revolving around the Earth ? Assume all orbits are circular.

(1) period = 12 hours

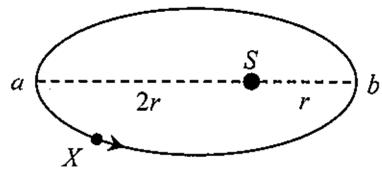
(2) period = 24 hours

(3) speed = 3.8 km s^{-1}

(4) speed = 7.6 km s^{-1}

A. (1) and (3)
B. (1) and (4)
C. (2) and (3)
D. (2) and (4)

M31.
[10]



A comet X moves around the Sun S in an elliptical orbit. Its speed at the closet point b is double that at the farthest point a. The distance of point a from the Sun is $2r$ while that of point b is r. If the kinetic energy and potential energy of the comet at point a are K and U respectively, denote the relationship between K and U.

(Take gravitational potential energy to be zero at infinity.)

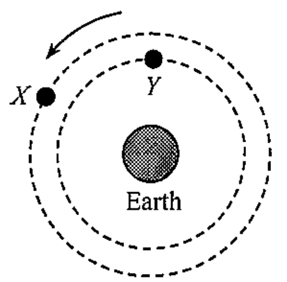
A. $U = -K$

B. $U = -2K$

C. $U = -3K$

D. $U = -4K$

M32. Two identical satellites X and Y are moving in two circular orbits around the
[11] Earth as shown. Which statement is correct ?



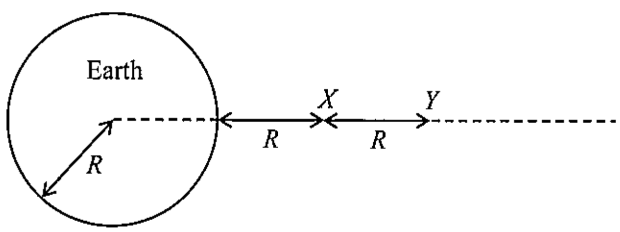
A. The period of X is smaller than that of Y.

B. The speed of X is greater than that of Y.

C. The gravitational force on X is greater than that on Y.

D. The gravitational potential energy of X is greater than that of Y.

M33.
[12]



The change in gravitational potential energy of an object is ΔU when it is brought from the Earth's surface to a point X at a distance R above the Earth's surface, where R is the Earth's radius. What is the minimum energy required to move the object a *further* distance R away from the Earth to a point Y as shown ?

A. ΔU

B. $\frac{1}{6}\Delta U$

C. $\frac{1}{3}\Delta U$

D. $\frac{1}{2}\Delta U$

M34. The acceleration due to gravity at the Earth's surface is 10 m s^{-2}. A planet X has a radius 2 times that of the Earth and a mass
[13] 10 times that of the Earth. It can be deduced that

(1) the average density of X is 1.25 times that of the Earth.

(2) the acceleration due to gravity at the surface of X is 25 m s^{-2}.

(3) the velocity of escape of both planets are the same.

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

M35. An object is projected with the velocity of escape from the Earth's surface. When the object is at a distance r from the
[13] Earth's centre, its kinetic energy and gravitational potential energy are K and U respectively. Which of the following statements about K and U is correct ?

(Take $U = 0$ at infinity.)

A. $K = -\frac{1}{2}U$

B. $K = -U$

C. $K = -2U$

D. Their relation depends on r.

Part C :

The following questions are designed to give supplemental exercise for this chapter.

M36. An astronaut in a spacecraft orbiting round the Earth experiences weightlessness. Which of the following statements is NOT correct ?

A. The resultant force acting on the astronaut is zero.

B. The gravitational attraction between the astronaut and the earth is just sufficient to provide his centripetal force.

C. There is no reaction force acting on the astronaut.

D. The centripetal acceleration of the astronaut is just equal to the acceleration due to gravity.

M37. Portia is a satellite revolving around Uranus and was discovered in 1986. Its orbital period is 0.51 days and its length of semi-major axis is 66 000 km. What is the mass of Uranus ?

A. 4.5×10^{24} kg

B. 8.8×10^{25} kg

C. 6.9×10^{26} kg

D. 2.8×10^{27} kg

M38. The planet Mars has two satellites. One is called Phobos, which has an orbital period of 0.32 days, with semi-major axis of 9000 km. The other is called Deimos, which has an orbital period of 1.26 days. What is the semi-major axis of the orbit of Deimos ?

A. 14700 km

B. 22400 km

C. 35400 km

D. 67800 km

M39. Assume that the Earth moves in circular orbit around the Sun. Estimate the mass of the Sun.

A. 1.5×10^{30} kg

B. 2.0×10^{30} kg

C. 2.5×10^{30} kg

D. Insufficient information

M40. A spacecraft is launched from the surface of the Earth with an initial speed u. It finally reaches a steady speed of 2500 m s^{-1} when it is far away from the Earth. Given that the radius of the Earth is 6400 km, find the value of u.

A. 11200 m s^{-1}

B. 11500 m s^{-1}

C. 13500 m s^{-1}

D. 13700 m s^{-1}

M41. An artificial satellite revolves around the Earth in elliptical motion. Which of the following statements is correct ?

A. The gravitational force is always perpendicular to the motion of the satellite.

B. The satellite moves with constant speed in its orbit.

C. The gravitational potential energy of the satellite is constant.

D. The kinetic energy of the satellite is the maximum when it is closest to the Earth.

M42. A satellite moves around the Earth in elliptical motion. At a point X, the satellite is closest to the Earth, and at a point Y, the satellite is farthest away from the Earth. The distance of X and Y from the centre of the Earth is 8600 km and 58000 km respectively. Calculate the time taken for the satellite to move from X to Y. Given that the mass of the Earth is 6.0×10^{24} kg.

A. 29300 s

B. 30200 s

C. 38200 s

D. 41600 s

M43. An astronaut inside a spacecraft moving in a circular orbit around the Earth is apparently weightless because

A. the astronaut is too far from the Earth to feel the Earth's gravitational force.

B. the astronaut and the spacecraft are both moving with the same acceleration towards the Earth.

C. the Earth's gravitational force on the astronaut is balanced by the reaction force of the spacecraft's floor.

D. the Earth's gravitational force on the astronaut is balanced by the centripetal force.

Answers

1.	B	11.	A	21.	D	31.	C	41.	D
2.	A	12.	D	22.	B	32.	D	42.	B
3.	D	13.	C	23.	D	33.	C	43.	B
4.	A	14.	B	24.	D	34.	C		
5.	D	15.	B	25.	B	35.	B		
6.	A	16.	D	26.	A	36.	A		
7.	A	17.	D	27.	B	37.	B		
8.	A	18.	D	28.	B	38.	B		
9.	D	19.	D	29.	C	39.	B		
10.	D	20.	A	30.	A	40.	B		

Solution

1. B

$$\Delta U = \left(-\frac{GMm}{R+1}\right) - \left(-\frac{GMm}{R}\right) = GMm\left(\frac{1}{R} - \frac{1}{R+1}\right) = \frac{GMm}{R(R+1)} \approx \frac{GMm}{R^2} \quad (R >> 1)$$

Let g_M be the acceleration due to gravity on the Moon.

$$g_M = \frac{GM}{R^2} \quad \text{and} \quad g_M = \tfrac{1}{6}g$$

$$\Delta U = \frac{GMm}{R^2} = m\,g_M = \frac{mg}{6}$$

OR

$$\Delta U = m\,g_M\,h = m\left(\tfrac{1}{6}g\right)(1) = \frac{mg}{6}$$

2. A

Total mechanical energy of the satellite in circular motion :

$$E = -\frac{GMm}{2r}$$

To escape from the gravitational attraction, the total mechanical energy of the body must be equal to zero.

Extra energy required :

$$\Delta E = 0 - \left(-\frac{GMm}{2r}\right) = \frac{GMm}{2r}$$

3. D

× A. Weightlessness occurs whenever the normal reaction is zero, that is, an object undergoes free fall, but not only for orbiting motion.

× B. If the gravitational force is zero, then there is no force to provide the centripetal force, and the spacecraft cannot perform orbiting motion.

× C. If the gravitational attraction is cancelled, then there is no force to provide the centripetal force.

✓ D. When the weight of the objects inside the spacecraft is used completely to provide the centripetal force, $m\,g = m\,a$, where a is the centripetal acceleration,
the objects are undergoing free fall since the acceleration is then equal to g.

4. A

By conservation of energy, $KE + PE = \text{constant}$ (gravitational PE is zero at infinity)

$$\frac{1}{2}m(\sqrt{\frac{3GM}{R}})^2 + (-\frac{GMm}{R}) = \frac{1}{2}mv^2 + (0)$$

$$\therefore\ v = \sqrt{\frac{GM}{R}}$$

5. D

Since P is close to the surface, the height of P above the Earth's surface is zero.

For object X:

$$\frac{GMm}{R^2} = \frac{mv^2}{R} \qquad \therefore\ v = \sqrt{\frac{GM}{R}}$$

$\therefore$ X will continue to fly in its orbit.

For object Y:

Total energy : $E = K + U = \frac{1}{2}mv^2 + (-\frac{GMm}{R}) = \frac{1}{2}m(\frac{GM}{R}) - \frac{GMm}{R} = -\frac{1}{2}\cdot\frac{GMm}{R} < 0$

$\therefore$ Y will eventually return to the Earth.

(To escape from the Earth, the minimum total mechanical energy is zero.)

6. A

$$\Delta U = (-\frac{GMm}{R+h}) - (-\frac{GMm}{R}) = GMm\,(\frac{1}{R} - \frac{1}{R+h}) = GMm\,[\frac{h}{R(R+h)}] = \frac{GM}{R^2} \times \frac{mhR}{R+h}$$

$$= mgh \times \frac{R}{R+h} \qquad (\text{ put } \frac{GM}{R^2} = g\ )$$

7. A

By Kepler's Third Law, $\frac{T^2}{R^3} = \text{constant}$

$$\therefore\ \frac{(27.3\times 24)^2}{R^3} = \frac{T^2}{(\frac{R}{30})^3} \qquad \therefore\ T = 4 \text{ hours}$$

8. A

By Kepler's Third Law, $\dfrac{T^2}{R^3}$ = constant

$\therefore \dfrac{(24)^2}{R^3} = \dfrac{T^2}{\left(\frac{R}{4}\right)^3}$ $\quad\quad \therefore T$ = 3 hours

9. D

$\dfrac{1}{2} m v_1^{\ 2} + \left(\dfrac{-G M m}{R}\right) = 0 \quad\quad \therefore v_1 = \sqrt{\dfrac{2 G M}{R}}$

$\dfrac{G M m}{R^2} = \dfrac{m v_2^{\ 2}}{R} \quad\quad \therefore v_2 = \sqrt{\dfrac{G M}{R}}$

$\therefore \dfrac{v_1}{v_2} = \sqrt{2}$

10. D

$\dfrac{1}{2} m u^2 + (\dfrac{-GMm}{R}) = (\dfrac{-GMm}{2R}) \quad\quad \therefore u = \sqrt{\dfrac{GM}{R}}$

$\dfrac{GMm}{(2R)^2} = \dfrac{m v^2}{2R} \quad\quad \therefore v = \sqrt{\dfrac{GM}{2R}}$

$\therefore \dfrac{u}{v} = \sqrt{2}$

11. A

By Kepler's Third Law, $\dfrac{T^2}{R^3}$ = constant

$\therefore \dfrac{(T)^2}{R^3} = \dfrac{(T')^2}{\left(\frac{R}{4}\right)^3} \quad\quad \therefore T' = \dfrac{T}{8}$

12. D

By $U = -\dfrac{G M m}{r} \propto -\dfrac{1}{r}$,

$\dfrac{U_Y}{U_X} = \dfrac{(a)}{(2a)} \quad\quad \therefore U_Y = \left(\dfrac{1}{2}\right)(-8) = -4$ kJ

$W = \Delta U = U_Y - U_X = (-4) - (-8) = +4$ kJ

13. C

$m g = m R \omega^2$

$(10) = (6.4 \times 10^6)\, \omega^2$

$\therefore \omega = 1.3 \times 10^{-3}$ rad s^{-1}

14. B

× (1) At inclined plane : $R = m g \cos\theta$

× (2) On spacecraft : $m g = m r \omega^2$ but $R = 0$

✓ (3) For lift moving with constant velocity : $R = W$

15. B

Gain in KE = Loss of PE = $\left(\frac{-GMm}{3R}\right) - \left(\frac{-GMm}{R}\right) = \frac{2}{3}\cdot\frac{GMm}{R}$

16. D

$$v_e = \sqrt{\frac{2GM}{R}} \propto \sqrt{\frac{\rho\cdot V}{R}} \propto \sqrt{\frac{\rho\cdot R^3}{R}} \propto \sqrt{\rho}\cdot R$$

$\frac{V_o'}{V_o} = \sqrt{3}\cdot(2)$ $\therefore V_o' = 2\sqrt{3}\,V_o$

17. D

$\frac{G M m}{r^2} = m r\left(\frac{2\pi}{T}\right)^2$ $\therefore \frac{T^2}{r^3} = \frac{4\pi^2}{G M} \propto \frac{1}{M}$ where M : mass of the central body

18. D

For circular orbital motion : $E_A = -\frac{G M m}{2 r} < 0$

By $E = -\frac{G M m}{2r} \propto -\frac{1}{r}$ $\therefore r_B = 2 r_A \Rightarrow E_B = \frac{E_A}{2}$

(Note that E_B is greater than E_A as their values are negative)

19. D

$s = u t + \frac{1}{2} a t^2$ $\therefore (0) = v_1 t + \frac{1}{2}(-g) t^2$ $\therefore g = \frac{2 v_1}{t}$

$v_2 = \sqrt{\frac{2 G M}{R}} = \sqrt{2 g R}$ $\therefore v_2 = \sqrt{2\times\frac{2v_1}{t}\times R}$ $\therefore R = \frac{v_2^{\,2}\, t}{4 v_1}$

20. A

✓ A. For circular orbit, $\frac{mv^2}{r} = \frac{GMm}{r^2}$ $\therefore v \propto \frac{1}{\sqrt{r}}$ $\therefore r\downarrow \Rightarrow v\uparrow$

× B. By Kepler's Third Law, $\frac{T^2}{r^3}$ = constant $\therefore r\downarrow \Rightarrow T\downarrow$

× C. By $g = \frac{GM}{r^2} \propto \frac{1}{r^2}$ $\therefore r\downarrow \Rightarrow g\uparrow$

× D. By $E = -\frac{GMm}{2r}$ $\therefore r\downarrow \Rightarrow E\downarrow$

21. D

✓ A. Moving in circular orbit round the Earth $\Rightarrow$ centripetal acceleration towards the centre of the Earth

✓ B. Communication satellite must orbit above the equator
so that it is always above a certain position on the Earth's surface.

✓ C. Since communication satellite has the same period as the Earth,

By $\omega = \frac{2\pi}{T}$, the satellite must have the same angular speed ω as the Earth.

× D. $U = -\frac{GMm}{r}$ but $KE = \frac{GMm}{2r}$ for circular orbit. Thus, $U \neq KE$ numerically at any position.

22. B

× (1) $R - W = \frac{mv - mu}{t}$ $\quad \therefore R = W + \frac{mv - mu}{t}$ $\quad \therefore R \neq W$

× (2) On spacecraft, $W = m r \omega^2$ but $R = 0$.

✓ (3) For lift moving with constant velocity, $R = W$.

23. D

$$\Delta U = (-\frac{GMm}{3R+R}) - (-\frac{GMm}{R}) = \frac{3GMm}{4R}$$

At the surface of the Earth, $g = \frac{GM}{R^2}$ $\quad \therefore GM = gR^2$

$$\therefore \Delta U = \frac{3(gR^2)m}{4R} = \frac{3mgR}{4}$$

24. D

The speed v is the orbital speed of a close-orbit satellite.

$$\therefore \frac{GMm}{R^2} = \frac{mv^2}{R} \qquad \therefore v = \sqrt{\frac{GM}{R}}$$

If the launching speed is $2v$, then the total mechanical energy E :

$$E = \frac{1}{2}m(2v)^2 + (-\frac{GMm}{R}) = 2m(\frac{GM}{R}) + (-\frac{GMm}{R}) = \frac{GMm}{R} > 0$$

Since the total mechanical energy is greater than zero, the spacecraft can leave the earth and travel to outer space.

25. B

The period of a geostationary communication satellite is 24 hours, i.e. one day.

The period of the moon is about 27 days.

By Kepler's third law, $T^2 \propto r^3$

$$\therefore (\frac{T_1}{T_2})^2 = (\frac{r_1}{r_2})^3 \qquad \therefore (\frac{1}{27})^2 = (\frac{6.6R}{r_2})^3 \qquad \therefore r_2 = 59.4\,R \approx 60\,R$$

26. A

✓ (1) When the rocket engines are shut off,
the weight of the astronaut is used completely to provide his acceleration.
As no normal reaction acts on him, he experiences weightlessness.

× (2) When the spacecraft decelerates downwards, normal reaction acts on the astronaut
so that $R - mg = m\,a$, the astronaut feels heavier.

× (3) The air resistance that is equal to the weight acts on the parachutist to give him the feeling of weight.

27. B

By Kepler's 3rd law, $\dfrac{T^2}{r^3}$ = constant

For the Earth, period is 1 year and radius is 1.50×10^{11} m, thus $\dfrac{T^2}{r^3} = \dfrac{(1)^2}{(1.50\times10^{11})^3} = 2.96\times10^{-34}$

× A. $\dfrac{T^2}{r^3} = \dfrac{(11.9)^2}{(13.4\times10^{18})^3} = 5.89\times10^{-56}$

✓ B. $\dfrac{T^2}{r^3} = \dfrac{(29.5)^2}{(14.3\times10^{11})^3} = 2.98\times10^{-34}$

× C. $\dfrac{T^2}{r^3} = \dfrac{(1.9)^2}{(2.85\times10^{11})^3} = 1.56\times10^{-34}$

× D. $\dfrac{T^2}{r^3} = \dfrac{(0.6)^2}{(2.11\times10^{11})^3} = 3.83\times10^{-35}$

28. B

By $v^2 = u^2 + 2as$ $\quad \therefore\ (0) = u^2 + 2(-g)h \quad \therefore\ g = \dfrac{u^2}{2h}$

As $g = \dfrac{GM}{R^2} \quad \therefore\ \dfrac{GM}{R} = gR$

Escape velocity of a planet :

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = \sqrt{2\left(\frac{u^2}{2h}\right)R} = u\sqrt{\frac{R}{h}}$$

29. C

By $F = \dfrac{GMm}{r^2}$, as the distance r at X is smaller, the gravitational force at X is greater. Thus option A is correct.

By $U = -\dfrac{GMm}{r}$, as the distance r at X is smaller, PE at X is smaller. Thus option C is **INCORRECT**.

As energy is conserved, the total mechanical energy ($K + U$) is the same at all positions. Thus option D is correct.

By $K + U$ = constant, as U at X is smaller, K at X is greater. Thus option B is correct.

Note that the equation : $K = \dfrac{GMm}{2r}$ cannot be used, as it is not a circular motion.

30. A

By Kepler's third law, $T^2 \propto r^3$

By $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$ $\quad \therefore\ r \propto \dfrac{1}{v^2}$ $\quad \therefore\ r^3 \propto \dfrac{1}{v^6}$

$\therefore\ T^2 \propto \dfrac{1}{v^6}$ $\quad \therefore\ T \propto \dfrac{1}{v^3}$ $\quad \therefore\ T v^3 = \text{constant}$

For the Moon : $T v^3 = (27.3 \text{ days})(1 \text{ km s}^{-1})^3 = 27.3$

For combination (1) and (3) : $T v^3 = (0.5 \text{ day})(3.8 \text{ km s}^{-1})^3 = 27.4 \approx 27.3$

31. C

$K = \frac{1}{2} m v^2 \propto v^2$ $\quad \therefore\ v_b = 2 v_a \Rightarrow K_b = 4 K_a = 4K$

$U = -\dfrac{GMm}{r} \propto \dfrac{1}{r}$ $\quad \therefore\ r_b = \frac{1}{2} r_a \Rightarrow U_b = 2 U_a = 2U$

By conservation of energy, $K + U = 4K + 2U$ $\quad \therefore\ U = -3K$

32. D

× A. $\dfrac{GMm}{r^2} = m r \omega^2$ $\quad \therefore\ \dfrac{GM}{r^3} = \omega^2 = (\dfrac{2\pi}{T})^2$ $\quad \therefore\ T^2 \propto r^3$ $\quad \therefore\ r_X > r_Y \Rightarrow T_X > T_Y$

× B. $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$ $\quad \therefore\ v^2 = \dfrac{GM}{r}$ $\quad \therefore\ r_X > r_Y \Rightarrow v_X < v_Y$

× C. $F = \dfrac{GMm}{r^2} \propto \dfrac{1}{r^2}$ $\quad \therefore\ r_X > r_Y \Rightarrow F_X < F_Y$

✓ D. $U = -\dfrac{GMm}{r}$ $\quad \therefore\ r_X > r_Y \Rightarrow U_X > U_Y$

33. C

From the Earth's surface to the point X :

$\Delta U = U \text{ at } X - U \text{ at surface} = (-\dfrac{GMm}{2R}) - (-\dfrac{GMm}{R}) = \dfrac{1}{2}\dfrac{GMm}{R}$

From the point X to the point Y :

$\Delta U' = U \text{ at } Y - U \text{ at } X = (-\dfrac{GMm}{3R}) - (-\dfrac{GMm}{2R}) = \dfrac{1}{6}\dfrac{GMm}{R} = \dfrac{1}{3}\Delta U$

34. C

✓ (1) $\rho = \dfrac{M}{V} \propto \dfrac{M}{R^3}$ $\quad \therefore\ \dfrac{\rho_X}{\rho_E} = \dfrac{(10)}{(2)^3} = 1.25$

✓ (2) $g = \dfrac{GM}{R^2} \propto \dfrac{M}{R^2}$ $\quad \therefore\ \dfrac{g_X}{g_E} = \dfrac{(10)}{(2)^2} = 2.5$ $\quad \therefore\ g_X = 2.5 \times 10 = 25 \text{ m s}^{-2}$

× (3) $v = \sqrt{\dfrac{2GM}{R}} \propto \sqrt{\dfrac{M}{R}}$ $\quad \therefore\ \dfrac{v_X}{v_E} = \sqrt{\dfrac{(10)}{(2)}} \neq 1$ $\quad v_X \neq v_E$

35. B

To escape, the total mechanical energy must be equal to zero.

$\therefore \quad K + U = 0$

$\therefore \quad K = -U$

36. A

For an astronaut moving in circular orbit, there must be resultant force acting on him, and this resultant force is called the centripetal force, thus the resultant force cannot be zero.

37. B

By $T^2 = \dfrac{4\pi^2 a^3}{GM}$

$\therefore \quad (0.51\times24\times3600)^2 = \dfrac{4\pi^2(66000\times10^3)^3}{(6.67\times10^{-11})M}$

$\therefore \quad M = 8.8 \times 10^{25}$ kg

38. B

By $T^2 \propto a^3$

$\therefore \quad (\dfrac{T_1}{T_2})^2 = (\dfrac{a_1}{a_2})^3 \qquad \therefore \quad (\dfrac{0.32}{1.26})^2 = (\dfrac{9000}{a_2})^3 \qquad \therefore \quad a_2 = 22400$ km

39. B

The distance r between the Earth and the Sun is 1 AU, which is given as 1.50×10^{11} m.

The orbital period T of the Earth around the Sun is 1 year, which is equal to $365 \times 24 \times 3600$ s.

By $\dfrac{GMm}{r^2} = mr\omega^2$

$\therefore \quad \dfrac{GM}{r^3} = \omega^2 = \left(\dfrac{2\pi}{T}\right)^2$

$\therefore \quad M = \dfrac{4\pi^2 r^3}{GT^2} = \dfrac{4\pi^2(1.50\times10^{11})^3}{(6.67\times10^{-11})(365\times24\times3600)^2} = 2.0\times10^{30}$ kg

40. B

By $g = \dfrac{GM}{R^2} \qquad \therefore \quad GM = gR^2 = (9.81)(6400\times10^3)^2 = 4.02\times10^{14}$

By conservation of energy, $K + U =$ constant (gravitational PE is zero at infinity)

$\dfrac{1}{2}mu^2 + (-\dfrac{GMm}{R}) = \dfrac{1}{2}mv^2 + (0)$

$\therefore \quad u^2 - \dfrac{2GM}{R} = v^2$

$\therefore \quad u^2 = \dfrac{2GM}{R} + v^2 = \dfrac{2(4.02\times10^{14})}{(6400\times10^3)} + (2500)^2 \qquad \therefore \quad u = 11500 \text{ m s}^{-1}$

41. D

× A. Gravitational force must always direct towards the centre of the Earth.
For elliptical motion, the motion may not be perpendicular to the line joining the Earth and the satellite.

× B. For elliptical motion, the speed changes as the satellite is at different distances from the Earth.

× C. The gravitational potential energy depends on the separation between the Earth and the satellite, that is not a constant.

✓ D. When the satellite is closest to the Earth, the potential energy is the minimum, thus the kinetic energy is the maximum, as the total energy of the satellite is conserved.

42. B

Semi-major axis : $a = \dfrac{8600+58000}{2} = 33300$ km

By $T^2 = \dfrac{4\pi^2 a^3}{GM}$

$\therefore \ T^2 = \dfrac{4\pi^2 (33300\times10^3)^3}{(6.67\times10^{-11})(6.0\times10^{24})}$

$\therefore \ T = 60400$ s

Time to move from X to $Y = \frac{1}{2}T = \frac{1}{2}\times 60400 = 30200$ s

43. B

× A. In circular orbit, the Earth's gravitational force must provide the centripetal force for the circular motion.
Thus, the spacecraft and astronaut cannot be so far that the gravitational force is negligible.

✓ B. Since the astronaut and the spacecraft are both moving with the same acceleration, their own weight provides their own acceleration.
Therefore, there is no normal reaction acting on the astronaut, and thus weightlessness is experienced.

× C. At the state of weightlessness, there is no reaction force acting on the astronaut.

× D. The gravitational force is the only force acting on the astronaut.
Centripetal force is the net force towards the centre.
Thus, the gravitational force provides the centripetal force, but not balanced.

Use the following data wherever necessary :

Acceleration due to gravity $g = 9.81 \text{ m s}^{-2}$ (close to the Earth)

Universal gravitational constant $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

The following list of formulae may be found useful :

Newton's law of gravitation $F = \dfrac{Gm_1m_2}{r^2}$

Gravitational potential energy $U = -\dfrac{GMm}{r}$

Part A :

The following question marked with { } is the past DSE questions. The number inside the bracket represents the year of the examination.

Q1. {13} Given : $GM = 4.0 \times 10^{14} \text{ N m}^2 \text{ kg}^{-1}$, where G is the universal gravitational constant and M is the mass of the Earth.
Mean radius of the Earth = 6400 km.
Radius of the geostationary orbit is about 42400 km, i.e. 36000 km above Earth's surface.

The following describes a way to launch a satellite into the geostationary orbit :

- The satellite is first launched by a rocket to a circular near-Earth orbit (1) at 300 km above the Earth's surface.
- At A, the satellite's engine is fired for a short period of time to give it a boost needed to enter the elliptical transfer orbit (2), with AB as the ellipse's major axis.
- At B, the satellite's engine is fired again briefly to boost it into the geostationary orbit (3).

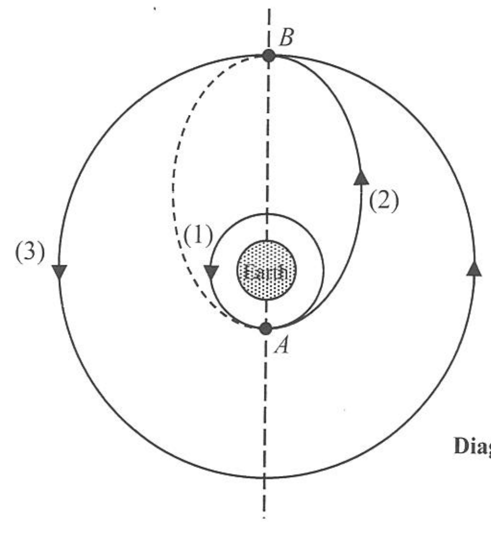


Diagram NOT drawn to scale

Assume that the three orbits are coplanar such that the elliptical orbit touches the two circular orbits at A and B respectively. During the period when the satellite travels from A to B along the transfer orbit, its engine is shut.

(a) Communications satellites are usually launched into the geostationary orbit. State and explain the advantage of such an arrangement. (2 marks)

Q1. (b) Find the speed of the satellite in the near-Earth orbit (1). (2 marks)

(c) (i) Show that for a satellite of mass m moving in a circular orbit of radius r around the Earth, its total mechanical energy is $-\dfrac{GMm}{2r}$, where M is the mass of the Earth. Take the gravitational potential energy of the satellite at infinity to be zero. (2 marks)

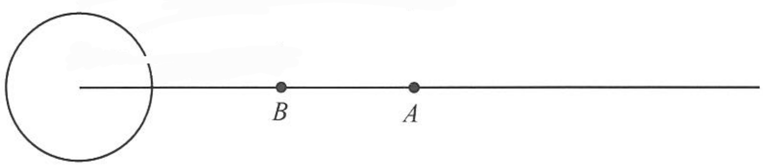
(ii) Use the result in (c) (i) to calculate the energy required to transfer a satellite of mass m = 2000 kg from the near-Earth orbit (1) through A to the geostationary orbit (3) through B. (2 marks)

(iii) How long does it take for the satellite to travel from A to B along the transfer orbit (2) ? (2 marks)

Part B :

The following questions marked with [ ] are the past HKAL questions. The number inside the brackets represents the year of the examination.

Q2. (Given : $G\,M_E = 4.0 \times 10^{14}$ N m^2 kg^{-1} where G = gravitational constant and M_E = mass of the earth.)
[91]



A spacecraft (mass m) has just finished its mission on the moon and is planning to return to the earth. At points A and B, the spacecraft is coasting through space with its motors off. If it had a velocity of 5200 m s^{-1} at point A, 28000 km from the centre of the earth, what would be its velocity at point B, 20000 km from the centre of the earth ? (3 marks)

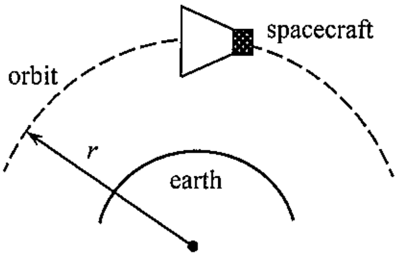
Q3. [93] (a) A rocket is fired to launch a spacecraft with an astronaut into an orbit round the earth. If the initial acceleration of the rocket is 100 m s^{-2} and the mass of the astronaut is 60 kg, find the supporting force acting on the astronaut

(i) before the rocket is fired from the earth's surface ;

(ii) during the first few seconds after firing.

Explain briefly why the astronaut should lie down in a bed-shaped seat during launching. (3 marks)

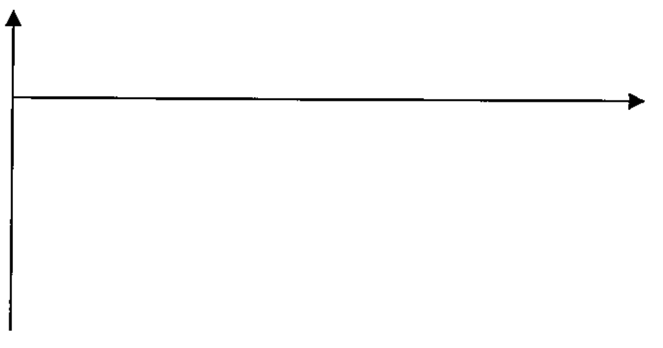
(b) Now the spacecraft with the astronaut is moving round the earth in a circular orbit of radius r with its engine turned off.



(i) Derive an expression for the total mechanical energy E of the spacecraft in terms of G, M_E, m and r,

where G = gravitational constant
M_E = mass of the earth
m = mass of the spacecraft. (3 marks)

(ii) Sketch a graph showing how E varies with r for values of r greater than the earth's radius R_E. (2 marks)



(iii) What is the extra energy required for the spacecraft to escape from the earth's gravitational field ? Express your answer in terms of G, M_E, m and r. (2 marks)

(c) The astronaut inside the orbiting spacecraft is said to be 'weightless'.

A student explains that since the orbit is at a great distance from the earth, the acceleration due to gravity and the weight of the astronaut are both zero.

Do you agree with the student ? Explain your answer. (3 marks)

Q4. The values of gravitational potential energy of a satellite of mass 2000 kg from a planet are given in the following table :
[94]

Distance from the surface of the planet / m	Gravitational potential energy / J
0	-1.25×10^{11}
800 000	-1.11×10^{11}
infinity	0

(a) If the gravitational potential energy is taken to be zero at an infinite distance from the planet, the gravitational potential energy at any point closer to the planet than infinity will be negative. What property of gravitational force ensures this ? (1 mark)

(b) The satellite is raised from the planet's surface to a height of 800 000 m above its surface.

(i) Find the change in gravitational potential energy of the satellite. (1 mark)

(ii) Calculate the kinetic energy of the satellite when it moves round the planet in a circular orbit at such a height. (3 marks)

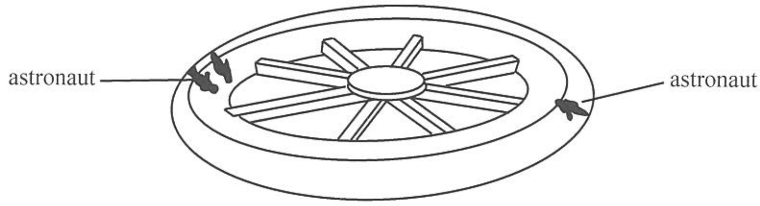
(c) Calculate the minimum speed with which a spacecraft must be fired from the planet's surface so as to escape from it. (Neglect air resistance and assume the spacecraft itself is unpowered.) (2 marks)

Q5. Given that the radius of the Earth is 6400 km.
[99]

(a) Calculate the gravitational potential energy of a 1 kg-mass at the Earth's surface. (2 marks)

(b) Find the minimum energy supplied to each kilogram of the mass of the spacecraft so as to enable it to escape from the Earth's gravitational pull. Show your working. (2 marks)

Q6. The figure shows a donut-shaped space station that is far from any planetary objects. It is designed such that the astronauts
[01] live at the periphery 1.0 km from the centre. Describe how an 'artificial gravity' of 10 N kg^{-1} can be created at the periphery.
(3 marks)



Q1. (a) The satellite will be directly above a certain location on the equator of the Earth with period of 24 hours. [1]

Thus the ground station can transmit the radio signals to the satellite in a fixed direction without tracking. [1]

(b) $\frac{GMm}{r^2} = \frac{mv^2}{r}$ [1]

$\therefore \; v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(4.0\times10^{14})}{(6400+300)\times10^3}} = 7730 \text{ m s}^{-1}$ < accept 7727 m s^{-1} > [1]

(c) (i) $U = -\frac{GMm}{r}$ [1]

$\frac{GMm}{r^2} = \frac{mv^2}{r}$ $\quad \therefore \; K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$ [1]

$\therefore \; E = U + K = (-\frac{GMm}{r}) + (\frac{GMm}{2r}) = -\frac{GMm}{2r}$

(ii) $\Delta E = (-\frac{GMm}{2r_B}) - (-\frac{GMm}{2r_A}) = \frac{GMm}{2}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$

$= \frac{(4.0\times10^{14})(2000)}{2}\left(\frac{1}{(6400+300)\times10^3} - \frac{1}{42400\times10^3}\right)$ [1]

$= 5.03 \times 10^{10}$ J [1]

(iii) Semi-major axis : $a = \frac{42400+6700}{2} = 24550$ km [1]

$T^2 = \frac{4\pi^2 a^3}{GM} = \frac{4\pi^2(24550\times10^3)^3}{(4.0\times10^{14})}$ $\quad \therefore \; T = 38200$ s

$t = \frac{1}{2}T = \frac{1}{2}\times 38200 = 19100$ s < accept 318 min. OR 5.31 hours > [1]

Q2. $\frac{1}{2}mv_A^{\,2} + (-\frac{GM_E m}{r_A}) = \frac{1}{2}mv_B^{\,2} + (-\frac{GM_E m}{r_B})$ [1]

$\frac{1}{2}v_B^2 - \frac{1}{2}v_A^{\,2} = GM_E(\frac{1}{r_B} - \frac{1}{r_A})$

$v_B^2 - (5200)^2 = 2 \times 4.0 \times 10^{14} \times (\frac{1}{20\times10^6} - \frac{1}{28\times10^6})$ [1]

$\therefore \; v_B = 6200 \text{ m s}^{-1}$ [1]

Q3. (a) (i) $S = mg = 60 \times 10 = 600$ N [1]

(ii) $S - mg = ma$

$\therefore S = mg + ma = 600 + 60 \times 100 = 6600$ N [1]

As the supporting force is very large, the astronaut should lie down in a bed-shaped seat to reduce the pressure by increasing the contact area. (**OR** to avoid the lack of blood flowing to the brain.) [1]

Q3. (b) (i) $U = -\dfrac{GM_{E}m}{r}$ [1]

Equation of circular motion :

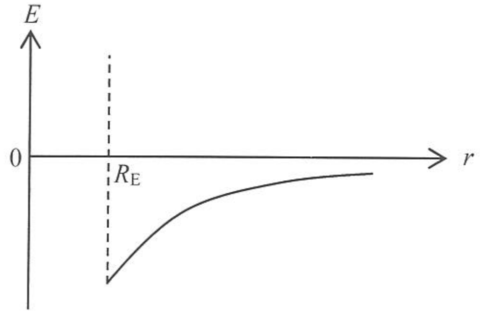
$$\frac{GM_{E}m}{r^2} = \frac{mv^2}{r}$$

$\therefore\ K = \dfrac{1}{2}mv^2 = \dfrac{GM_{E}m}{2r}$ [1]

Total Energy $= U + K$

$\therefore\ E = (-\dfrac{GM_{E}m}{r}) + (\dfrac{GM_{E}m}{2r}) = -\dfrac{GM_{E}m}{2r}$ [1]

(ii)



< shape of curve correct > [1]

< curve takes a finite negative value at $r = R_E$ > [1]

(iii) At infinity, $E_\infty = 0$ [1]

Energy required $= E_\infty - E_r = 0 - (-\dfrac{GM_{E}m}{2r}) = \dfrac{GM_{E}m}{2r}$ [1]

Q3. (c) No !

At a point in an orbit of radius r, weight of astronaut $= \dfrac{GM_{E}m}{r^2} \neq 0$ [1]

The astronaut is 'weightless'

because his weight (gravitational force) is completely used for centripetal acceleration, [1]

thus there is no normal reaction force acting on him by the floor. [1]

Q4. (a) Gravitational force is always attractive. [1]

(b) (i) $\Delta U = (-1.11 \times 10^{11}) - (-1.25 \times 10^{11}) = 1.4 \times 10^{10}$ J [1]

(ii) $U = -\dfrac{GMm}{r} = -1.11 \times 10^{11}$ [1]

$K = \dfrac{GMm}{2r} = \dfrac{1.11 \times 10^{11}}{2}$ [1]

$= 5.55 \times 10^{10}$ J [1]

Q4. (c) KE at surface + PE at surface = 0

$\therefore\ \frac{1}{2}(2000)\,v^2 + (-\,1.25\times10^{11}) = 0$ [1]

$\therefore\ v = 11200\ \text{m s}^{-1}$ [1]

Q5. (a) $U_R = -\dfrac{G\cdot M\cdot m}{R} = -\dfrac{G\cdot M}{R^2}\cdot m\cdot R = -\,m\cdot g\cdot R$ [1]

$= -\,(1)\times(9.81)\times(6400\times10^3)$

$= -\,6.28\times10^7\ \text{J}$ [1]

(b) $U_\infty = 0\ \text{J}$ [1]

Minimum energy per kg $= 0-(-6.28\times10^7)$

$= 6.28\times10^7\ \text{J}$ [1]

Q6. The space station should rotate about the axis through its centre with a constant angular speed [1]

such that the centripetal acceleration at the periphery is equal to $10\ \text{m s}^{-2}$ [1]

$\therefore\ a = r\,\omega^2$

$\therefore\ (10) = (1.0\times10^3)\,\omega^2$

$\therefore\ \omega = 0.1\ \text{rad s}^{-1}$ [1]

1.4 Stars and the Universe

Use the following data wherever necessary :

Speed of light in vacuum	$c = 3 \times 10^8 \text{ m s}^{-1}$
Astronomical unit	$\text{AU} = 1.50 \times 10^{11} \text{ m}$
Light year	$\text{ly} = 9.46 \times 10^{15} \text{ m}$
Parsec	$\text{pc} = 3.09 \times 10^{16} \text{ m} = 3.26 \text{ ly} = 206265 \text{ AU}$
Acceleration due to gravity	$g = 9.81 \text{ m s}^{-2}$ (close to the Earth)
Stefan constant	$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Universal gravitational constant	$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

The following list of formulae may be found useful :

Stefan's law	$P = \sigma A T^4$
Doppler effect	$\left\lvert\frac{\Delta f}{f_0}\right\rvert \approx \frac{v}{c} \approx \left\lvert\frac{\Delta\lambda}{\lambda_0}\right\rvert$

Part A :

The following questions marked with { } are the past DSE examination questions.

The questions marked with {SP} are the Sample Paper questions.

The questions marked with {PP} are the Sample Paper questions.

The number inside the brackets represents the year of the DSE examination.

M1. {SP} A star has a luminosity 1000 times that of the Sun and its surface temperature is 3900 K. The star is a

A. supernova.

B. white dwarf.

C. blue giant star.

D. red giant star.

M2. {SP} A star has a luminosity 1000 times that of the Sun and its surface temperature is 3900 K. What is the radius of the star ? (Assume that the surface temperature of the Sun is 5800 K and that all stars are blackbody emitters.)

A. 14 times the radius of the Sun

B. 21 times the radius of the Sun

C. 70 times the radius of the Sun

D. 4900 times the radius of the Sun

M3. {PP} In 1838, German astronomer Bessel announced that the parallax of 61 Cygni is 0.314 arcseconds. What is the distance of 61 Cygni from Earth according to Bessel's measurement ?

A. 0.98 ly

B. 1.02 ly

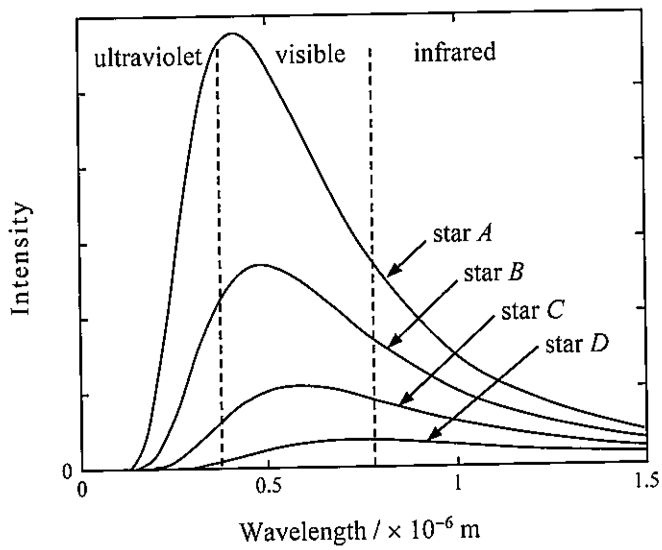
C. 3.18 ly

D. 10.38 ly

M4. American astronomer Hubble discovered that the recession velocities v of galaxies are proportional to their distance d from
{PP} the Earth, $v = Hd$, where H is the Hubble constant. Which of the following is **not** a unit of the Hubble constant ?

A. km s^{-1} Mpc^{-1}
B. m s^{-1} ly^{-1}
C. s
C. s^{-1}

(For questions 5 and 6) The figure below shows the radiation curves of four stars.



M5. Which star has the highest surface temperature ?
{PP} A. star A
B. star B
C. star C
D. star D

M6. Which of the following statements about the stars are correct ?
{PP}
(1) The area under the curve is proportional to the surface temperature of that star.
(2) The colours of the four stars are different.
(3) If stars C and D have the same luminosity, star D has a larger radius.

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

M7. Referring to the information given, which statements about stars X and Y is/are correct ?
{12}

	Absolute magnitude	**Apparent magnitude**
star *X*	2.8	4.7
star *Y*	5.4	3.2

(1) Star X is further away from the Earth than star Y.
(2) Star Y is further away from the Earth than star X.
(3) The distance of stars X and Y from the Earth can be determined from the information provided.

A. (1) only
B. (2) only
C. (1) & (3) only
D. (2) & (3) only

M8. The Sun is about 8 kpc from the centre of the Milky Way galaxy and its rotation speed about the centre is 220 km s^{-1}. How {12} long does it take to complete one rotation about the centre of the Milky Way ?

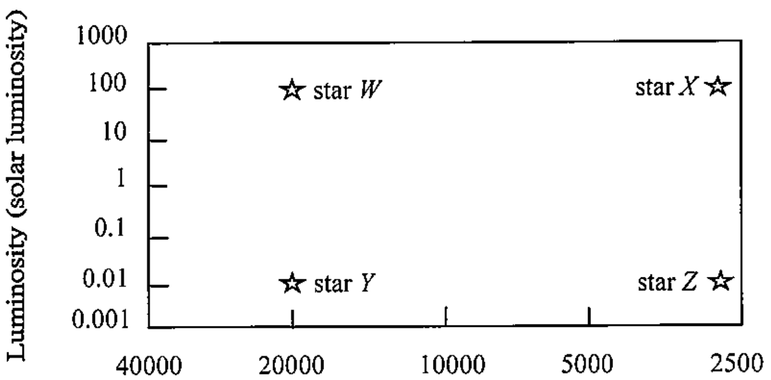
A. 2.24×10^8 years

B. 3.35×10^8 years

C. 2.24×10^{11} years

D. 3.55×10^{11} years

(For questions 9 and 10) The figure below shows some information of stars *W*, *X*, *Y* and *Z*.



M9. Which of the following statements is/are correct ?

{12}
(1) For star *X*, the intensity of red light is higher than any other colour light.
(2) For star *W*, the intensity of blue light is higher than any other colour light.
(3) Intensity ratio of red light to other colours of light is larger in star *Z* than that in star *Y*.

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

M10. The absorption spectrum of star *X* contains hydrogen absorption lines. What can be concluded from this ?

{12}
(1) Star *X* is mainly composed of hydrogen.
(2) There is hydrogen gas in the outer atmosphere of star *X*.
(3) The abundance of hydrogen in star *X* is less than that in similar star of the same temperature.

A. (1) only
B. (2) only
C. (1) & (3) only
D. (2) & (3) only

M11. Stars *P* and *Q* have the same luminosity. Star *P* is 25 times brighter than Star *Q*. We can deduce that

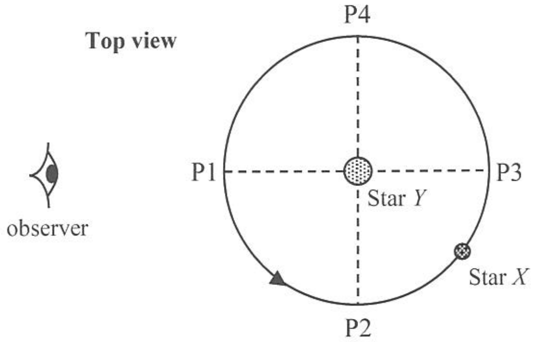
{13} A. *P*'s distance is 5 times that of *Q*.

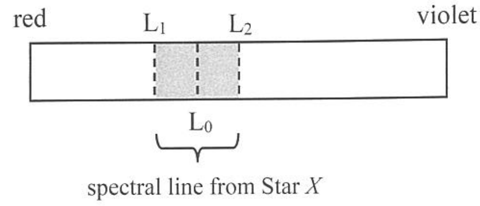
B. *Q*'s distance is 5 times that of *P*.

C. *P*'s distance is 25 times that of *Q*.

D. *Q*'s distance is 25 times that of *P*.

M12. Star X orbits around Star Y approximately in a circular orbit. An observer on the Earth viewing a spectral line from X found
{13} that its wavelength varies between the limits L_1 and L_2. L_0 is the wavelength of that line observed in the laboratory.

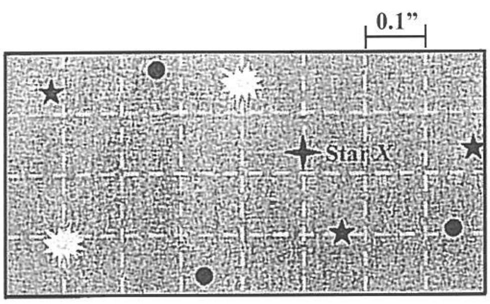




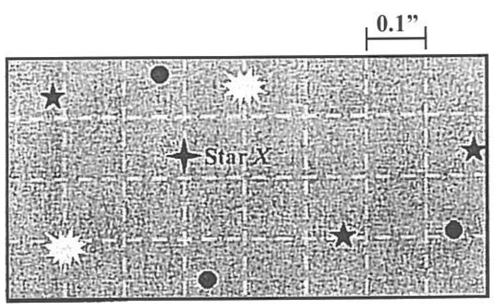
Which wavelengths correspond to positions P1, P2, P3 and P4 of Star X ?

	P1	P2	P3	P4
A.	L_0	L_1	L_0	L_2
B.	L_1	L_0	L_2	L_0
C.	L_0	L_2	L_0	L_1
D.	L_2	L_0	L_1	L_0

M13. The following are two pictures of the same region of the sky taken six months apart. Gridlines are overlaid on the pictures.
{13} Each grid square corresponds to an angular scale of 0.1 arc second. What is the distance of Star X from the Earth in unit of parsec ?



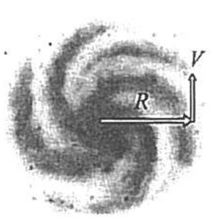
January view

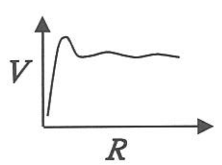


July view

A. 0.1 pc
B. 0.2 pc
C. 5 pc
D. 10 pc

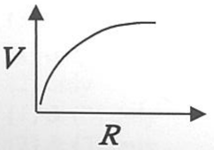
M14.
{13}



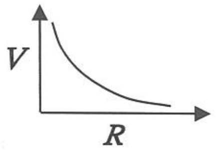


The diagram shows the top view of a galaxy and the observed variation of the rotation speed V with radius R from the galacti
centre. This curve suggests the existence of dark matters. Which of the following should be the expected rotation curv
without the existence of dark matters ?

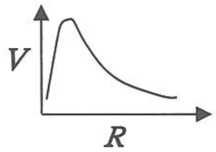
A.



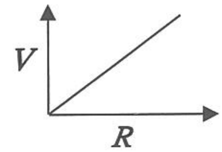
B.



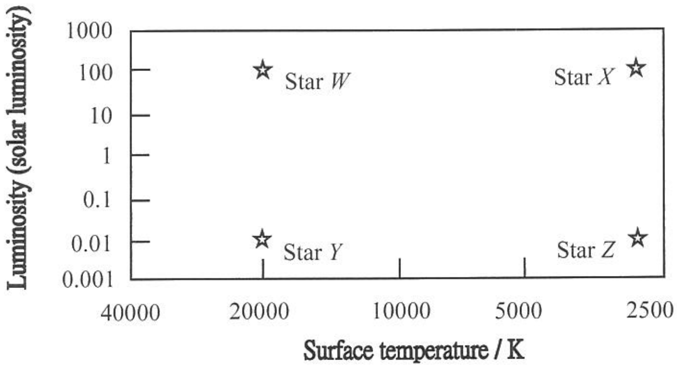
C.



D.



M15. The figure below shows some information of stars W, X, Y and Z.
{13}



Which statement(s) about the radii of stars is/are correct ?

(1) radius of X > radius of W

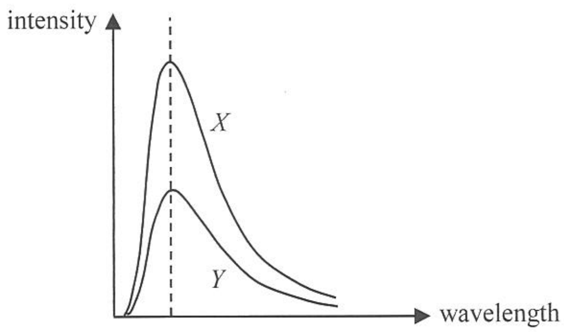
(2) radius of W > radius of Y

(3) radius of Y > radius of Z

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

M16. The violet line (410 nm) of hydrogen spectrum from a distant celestial body is blue shifted and its wavelength appears 50 nm
{14} shorter when observed. What is the observed wavelength of the red line (656 nm) from the same source ?
A. 576 nm
B. 606 nm
C. 706 nm
D. 736 nm

(For questions 17 and 18)
The diagram shows the spectra of radiation from stars X and Y with their peaks lying at the same wavelength.



M17. Which statement is correct ?
{14} A. Surface temperature of X > Surface temperature of Y
B. Surface temperature of X < Surface temperature of Y
C. Surface temperature of X = Surface temperature of Y
D. The information is not sufficient to make a comparison of the surface temperature of X and Y.

M18. Which statement is correct ?
{14} A. Star X is smaller than star Y.
B. Star X is bigger than star Y.
C. Star X and star Y are of the same size.
D. The information is not sufficient to make a comparison of the size of stars X and Y.

M19. The Earth receives solar radiation of power P_o per unit area. Estimate the power of solar radiation received per unit area on
{15} Pluto which is 40 AU from the Sun.

A. $\frac{1}{39}P_o$

B. $\frac{1}{40}P_o$

C. $\left(\frac{1}{39}\right)^2 P_o$

D. $\left(\frac{1}{40}\right)^2 P_o$

M20.
{15}

	absolute magnitude	**apparent magnitude**
star *X*	2.8	4.7
star *Y*	5.4	3.2

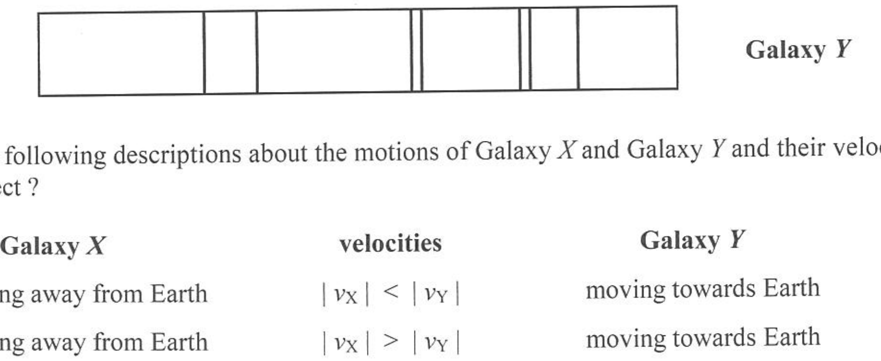
According to the information given above, which of the following about stars *X* and *Y* is/are correct ?

(1) Luminosity of star *X* is greater than that of star *Y*.

(2) A telescope collects more energy per unit area per unit time from star *X* than from star *Y*.

A. Only (1) is correct.
B. Only (2) is correct.
C. Both (1) and (2) are correct.
D. Both (1) and (2) are incorrect.

M21. The respective absorption spectra of hydrogen from Galaxy *X*, in the laboratory, and Galaxy *Y* are shown below :
{15}



Which of the following descriptions about the motions of Galaxy *X* and Galaxy *Y* and their velocities v_X and v_Y relative to th
Earth is correct ?

	Galaxy *X*	**velocities**	**Galaxy *Y***
A.	moving away from Earth	$\lvert v_X \rvert < \lvert v_Y \rvert$	moving towards Earth
B.	moving away from Earth	$\lvert v_X \rvert > \lvert v_Y \rvert$	moving towards Earth
C.	moving towards Earth	$\lvert v_X \rvert < \lvert v_Y \rvert$	moving away from Earth
D.	moving towards Earth	$\lvert v_X \rvert > \lvert v_Y \rvert$	moving away from Earth

Part B :

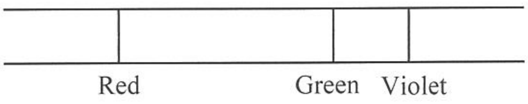
The following questions marked with [ ] are the past HKAL questions. The number inside the brackets represents the year of the examination.

M22. A hydrogen source in a laboratory emits a line spectrum with one of the lines having wavelength 656.3 nm. For a star
[93] receding from the earth at a speed of 200 km s^{-1}, what would be the wavelength of the corresponding line from the star observed on the earth ?

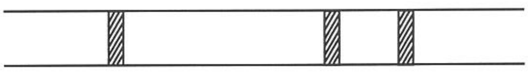
A. 655.5 nm
B. 655.9 nm
C. 656.7 nm
D. 657.1 nm

M23. The spectra produced by element X, one on earth and the other on a distant star, are compared.
[02]

Spectrum from element X on earth :



Spectrum from element X on the distant star :



What is the most probable reason for the difference in the spectra readings ?

A. The universe is expanding.
B. The star is approaching the earth.
C. The star is receding from the earth.
D. The star is spinning about its axis.

Part C :

The following questions are designed to give supplemental exercise for this chapter.

M24. A star rotates about the centre of the Milky Way galaxy with a period of 5.8×10^8 years. If its rotation speed is 840 km s^{-1}, what is the distance of this star from the centre of the Milky Way ?

A. 2.8×10^4 pc
B. 4.6×10^4 pc
C. 6.4×10^4 pc
D. 7.9×10^4 pc

M25. A star is at a distance of 15.8 ly from the Earth. What is its parallax ?

A. 0.063 arcseconds
B. 0.126 arcseconds
C. 0.206 arcseconds
D. 0.412 arcseconds

M26. What causes the energy release from a star in its main sequence ?

A. nuclear fusion
B. nuclear fission
C. radioactive decay
D. burning of fossil fuels

M27. The rotation curve of stars around galaxies shows that there is dark matter existing in the Universe. Which of the following is NOT the property of dark matter ?

A. Dark matter does not emit any type of radiation.
B. Dark matter can exert gravitational force on visible matter.
C. Dark matter has no interaction with visible matter except the gravitational force.
D. The mass of dark matter is smaller than the mass of all visible matter in the Universe.

M28. The following three units are used to express length in spatial scale.

(1) light year
(2) parsec
(3) astronomical unit

Arrange their magnitudes in ascending order.

A. (1), (3), (2)
B. (2), (3), (1)
C. (3), (1), (2)
D. (3), (2), (1)

M29. The second closest star to the Earth subtends an angle of 1.52 arc seconds when measured from the two opposite extremes of the Earth's orbit. What is its distance from the Earth in pc ?

A. 1.22 pc
B. 1.32 pc
C. 1.42 pc
D. 1.52 pc

M30. A star is at 7.85 ly from the Earth. Find the angle subtended by this star when measured from the two opposite extremes of the Earth's orbit.

A. 0.83 arc seconds
B. 0.86 arc seconds
C. 0.89 arc seconds
D. 0.92 arc seconds

M31. The apparent magnitude of star X is 3.5 and that of star Y is 4.5. Which of the following statements is/are correct ?

(1) Star X appears to be brighter than star Y.
(2) The light given out by star X is stronger than that of star Y.
(3) Star X must be closer to the Earth than star Y.

A. (1) only
B. (2) only
C. (1) & (3) only
D. (2) & (3) only

M32. The light from star X takes 200 years to reach the Earth. What is the stellar parallax of star X ?

A. 0.012"
B. 0.016"
C. 0.018"
D. 0.024"

M33. The radius of a nebula with roughly spherical shape is 10 light year. If the distance of the nebula from the Earth is 5000 pc, what is its average angular size ?

A. 3.6 arc minute
B. 4.2 arc minute
C. 4.8 arc minute
D. 5.4 arc minute

M34. Sirius is the brightest star at the night sky. It has an apparent magnitude of – 1.43. How many times is it brighter than the second brightest star, Canopus, which has an apparent magnitude of – 0.62 ?

A. 0.8
B. 1.8
C. 2.1
D. 6.5

M35. Star X has a stellar parallax of 0.12 arc second. Its apparent magnitude is 2.8. Which of the following statements concerning this star is/are correct ?

(1) Star X is brighter than another star Y with apparent magnitude of 2.6.
(2) Its absolute magnitude is greater than 2.8.
(3) Its luminosity is greater than another star Z with absolute magnitude of 2.8.

A. (1) only
B. (2) only
C. (1) & (3) only
D. (2) & (3) only

M36. A star is at a distance of 30 light years from the Earth. Its absolute magnitude is 3.5. Which of the following statements concerning the star are correct ?

(1) The star is at 9.2 pc from the Earth.
(2) The parallax of the star is 0.11 arc second.
(3) The apparent magnitude of the star is less than 3.5.

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

M37. Calculate the parallax of a star, if its apparent magnitude is exactly equal to its absolute magnitude.

A. 0.1 rad
B. 2.91×10^{-5} rad
C. 2.42×10^{-7} rad
D. 4.85×10^{-7} rad

M38. The apparent magnitude of star X is 4.5. If star Y is 30 times brighter than star X, what is the apparent magnitude of star Y ?

A. 0.8
B. 3.7
C. 6.0
D. 8.2

M39. The apparent magnitude m and the absolute magnitude M of stars X, Y, Z are listed below :

	m	M
star X	1.5	−2
star Y	3.5	4.5
star Z	4.5	2.5

Arrange the distance of the three stars from the Earth in ascending order.

A. X, Y, Z
B. Y, X, Z
C. Y, Z, X
D. Z, Y, X

M40. Assume that stars emit light as a blackbody emitter, which of the following statements is/are correct if the surface temperature of a star is increased ?

(1) The luminosity of the star increases.

(2) The peak wavelength increases.

(3) The colour shifts towards the end of ultra-violet radiation.

A. (1) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

M41. The intensity of light emitted by a star is 2×10^{8} W m^{-2}. What is the surface temperature of the star ?

A. 5500 K
B. 6600 K
C. 7700 K
D. 8800 K

M42. If a star is assumed to be a blackbody emitter, then the luminosity of the star depends on :

(1) surface temperature of the star

(2) size of the star

(3) chemical composition of the star

A. (1) only
B. (1) & (2) only
C. (2) & (3) only
D. (1), (2) & (3)

M43. If the surface temperature of a star is 6500 K and its radius is 5×10^{8} m, calculate its luminosity.

A. 1.6×10^{26} W
B. 3.2×10^{26} W
C. 4.8×10^{26} W
D. 6.4×10^{26} W

M44. The luminosity of a star is 6.5×10^{32} W. If the surface temperature of the star is 8200 K, what is its diameter ?

A. 4.5×10^{11} m
B. 6.5×10^{11} m
C. 7.0×10^{11} m
D. 9.0×10^{11} m

M45. A star gives out a total of 8.4×10^{28} J energy in one minute. If the radius of the star is 6.5×10^{8} m, what is its surface temperature ?

A. 5870 K
B. 6710 K
C. 7940 K
D. 8260 K

M46. The surface temperature of the Sun is 5800 K. A star has a radius 80% that of the Sun and its surface temperature is 4500 K. What is the ratio of the luminosity of this star to that of the Sun ?

A. 0.23
B. 0.35
C. 0.48
D. 0.68

M47. The surface temperature of the Sun is 5800 K. A star has a radius 5 times that of the Sun. If the ratio of luminosity of the star to that of the Sun is 8 : 3, what is the surface temperature of the star ?

A. 3300 K

B. 3800 K

C. 4500 K

D. 5200 K

M48. If the surface absolute temperature of a star is increased by 20%, and its radius is decreased by 40%, what would be the percentage change of its luminosity ?

A. remain unchanged

B. increase by 25%

C. decrease by 25%

D. decrease by 75%

M49. What information can be obtained by observing the spectrum emitted by a star ?

(1) The surface temperature of the star

(2) The chemical composition of the star

(3) The radius of the star

A. (1) only
B. (1) & (2) only
C. (2) & (3) only
D. (1), (2) & (3)

M50. The spectral classes of the following 4 stars are :

Star W : F3

Star X : K7

Star Y : B2

Star Z : F0

Arrange them in ascending order of their surface temperatures, i.e., from the coldest to the hottest.

A. Y, Z, W, X

B. Y, W, Z, X

C. Z, W, Y, X

D. X, W, Z, Y

M51. Stars X and Y are of the same size. Star X belongs to spectral class A while star Y belongs to spectral class B. Which of the following statements is/are correct ?

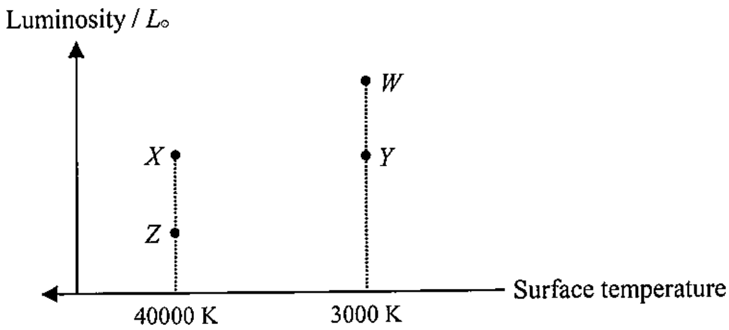
(1) The surface temperature of star X is higher than that of star Y.

(2) The luminosity of star X is greater than that of star Y.

(3) The absolute magnitude of star X is larger than that of star Y.

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

(For questions 52 and 53) The figure below shows a simplified H-R diagram. Fours stars W, X, Y, Z are marked.



M52. Arrange the sizes of the marked four stars in descending order.

A. W, X, Y, Z

B. W, Y, X, Z

C. Z, X, Y, W

D. Z, X, Y, W

M53. Which of the following statements concerning the four stars is NOT correct ?

A. Star X belongs to the spectral class O.

B. Star Y belongs to the spectral class F.

C. Star W appears red in colour.

D. Star Z appears blue in colour.

M54. The wavelength of a certain spectral line observed from the spectrum due to a celestial body is 633.72 nm. This spectra line measured in the laboratory from a stationary light source is 634.28 nm. Which of the following statements concernin the celestial body are correct ?

(1) The spectral line shows blue shift.

(2) The celestial body is approaching the Earth.

(3) The radial velocity of the celestial body is 2.65×10^5 m s^{-1}.

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

M55. A star is moving at a receding velocity of 500 km s^{-1} making an angle of 30° to the line of sight from the Earth. If the st emits a spectral line of wavelength 528.64 nm, what would be the apparent wavelength observed at the Earth ?
A. 527.76 nm
B. 527.88 nm
C. 529.40 nm
D. 529.52 nm

M56. Given that the Hubble constant is 72 km s^{-1} Mpc^{-1}. Assume that the expansion of the universe has been constant up to no estimate the age of the Universe.

A. 1.36×10^9 years
B. 1.36×10^{10} years
C. 1.36×10^{11} years
D. 1.36×10^{12} years

M57. Which of the following statements concerning dark matter is/are correct ?

(1) The rotation curve of stars around our galaxy confirms the existence of dark matter.

(2) The total mass of dark matter in the universe is approximately the same as the total mass of visible matter.

(3) Dark matter has no interaction with visible matter, including gravity.

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

M58.

A star is moving at a velocity of 560 km s^{-1} making an angle of 30° to the line of sight from the Earth as shown. If the star emits a spectral line of wavelength 624.35 nm, what would be the apparent wavelength of this spectral line observed at the Earth ?

A. 623.18 nm
B. 623.34 nm
C. 625.36 nm
D. 625.51 nm

M59. The luminosity of a star is 7.50×10^{26} W. Its brightness observed on the Earth is 9.62×10^{-9} W m^{-2}. Calculate the parallax of this star.

A. 0.28 arcsecond

B. 0.39 arcsecond

C. 0.46 arcsecond

D. 0.53 arcsecond

Answers

1. D	11. B	21. D	31. A	41. C	51. B
2. C	12. A	22. C	32. B	42. B	52. B
3. D	13. D	23. D	33. B	43. B	53. B
4. C	14. C	24. D	34. C	44. D	54. D
5. A	15. C	25. C	35. B	45. D	55. C
6. C	16. A	26. A	36. D	46. A	56. B
7. C	17. C	27. D	37. D	47. A	57. A
8. A	18. D	28. C	38. A	48. C	58. C
9. D	19. D	29. B	39. C	49. B	59. B
10. B	20. A	30. A	40. B	50. D	

Solution

1. D

Since the star is much brighter than the Sun, it is at a higher position in the H-R diagram.

Since the temperature of the star is only 3900 K, it is near the right side in the H-R diagram.

The star should be a red giant star.

2. C

By $L = 4\pi R^2 \sigma T^4$

$\therefore\ (L_s / L_\odot) = (R_s / R_\odot)^2 (T_s / T_\odot)^4$

$\therefore\ (1000) = (R_s / R_\odot)^2 (3900 / 5800)^4$ $\qquad \therefore\ R_s / R_\odot = 70$

$\therefore\ R_s = 70\, R_\odot$

3. D

$$d = \frac{1}{p} = \frac{1}{0.314} = 3.185 \text{ pc} = 3.185 \times 3.26 \text{ ly} = 10.38 \text{ ly}$$

4. C

$H = v / d$

unit of H = $\text{m s}^{-1} / \text{m} = \text{s}^{-1} \neq \text{s}$

5. A

The peak wavelength decreases with the temperature, that is, higher surface temperature gives shorter peak wavelength. As star A has the shortest peak wavelength, it has the highest surface temperature.

6. C

× (1) The area under the curve represents the total intensity of the star, which is related to the luminosity L. Luminosity L increases with the surface temperature, but not proportional. By $L = \sigma A T^4$, the area under the curve should be proportional to (temperature)^4.

✓ (2) Different peak wavelengths represent different colours.

✓ (3) By $L = \sigma A T^4 = 4\pi R^2 \sigma T^4$, as the temperature T of star D is lower, the radius R of D is larger.

7. C

✓ (1) For star X, the absolute magnitude is smaller than the apparent magnitude, thus the true distance of X is greater than 10 pc. For star Y, the absolute magnitude is greater than the apparent magnitude, thus the true distance of X is smaller than 10 pc. Therefore, the star X is further away from the Earth than star Y.

× (2) It is the opposite of statement (1), thus it is NOT correct.

✓ (3) The value of absolute magnitude M is the brightness of the star if it is placed at 10 pc. The value of apparent magnitude m compared with M can give the estimation of the distance of the star.

8. A

$$T = \frac{2\pi r}{v} = \frac{2\pi(8\times10^3\times3.09\times10^{16})}{(220\times10^3)} = 7.06\times10^{15}\text{ s} = 2.24\times10^8\text{ years}$$

9. D

✓ (1) Star X has low surface temperature, thus it emits mainly red light. It belongs to a red giant.

✓ (2) Star W has high surface temperature, thus it emits mainly blue light.

✓ (3) As star Z has surface temperature corresponding to red colour, the intensity of red light is larger.

10. B

× (1) Hydrogen is one of the components of the star, but may not be the main component.

✓ (2) The hydrogen gas in the outer atmosphere of star X absorbs photons of the hydrogen spectrum to give the absorption lines.

× (3) The abundance of hydrogen cannot be known due to the absorption lines.

11. B

By $b = \dfrac{L}{4\pi d^2}$ $\quad\therefore b \propto \dfrac{1}{d^2}$

$$\therefore \frac{b_P}{b_Q} = \left(\frac{d_Q}{d_P}\right)^2$$

$$\therefore (25) = \left(\frac{d_Q}{d_P}\right)^2$$

$\therefore d_Q = 5\, d_P$

12. A

At P_1, star X moves perpendicular to the observer, thus there is no Doppler effect, spectral line L_o is observed.

At P_2, star X moves away from the observer, thus there is red shift, spectral line L_1 is observed.

At P_3, star X moves perpendicular to the observer, thus there is no Doppler effect, spectral line L_o is observed.

At P_4, star X moves towards the observer, thus there is blue shift, spectral line L_2 is observed.

13. D

From the figure, the angular position of star X in the interval of 6 months is 0.2"

Parallax : $p = 0.2" \times \dfrac{1}{2} = 0.1"$

Distance : $d = \dfrac{1}{p} = \dfrac{1}{0.1} = 10\text{ pc}$

14. C

If there is no dark matter, the rotation speed V should decrease with r after reaching the peak, that is, the graph in C.

15. C

By using the equation for luminosity : $L = 4\pi R^2 \sigma T^4$

✓ (1) Star X and W have the same L, but W has higher temperature T, thus W has smaller radius R.

✓ (2) Star Y and W has the same temperature T, but W has greater L, thus W has greater radius R.

× (3) Star Y and Z has the same L, but Y has higher temperature T, thus Y should have smaller radius R.

16. A

By $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$

With the same source, the speed v is the same.

$\therefore \dfrac{(50)}{(410)} = \dfrac{v}{c} = \dfrac{\Delta\lambda}{(656)}$

$\therefore \Delta\lambda = 80\text{ nm}$

To have blue shifted, the wavelength is shorter.

$\therefore \lambda_{red} = 656 - 80 = 576\text{ nm}$

17. C

The peak wavelength is related to the surface temperature of a black body radiator.

Since both star X and star Y have the same peak wavelength,

thus, their surface temperatures must be the same.

18. D

The area under the curve represents the total intensity I emitted by the star.

By Stefan's Law, $I \propto T^4$.

Thus, for a perfect black body radiator, stars with the same temperature should have the same total intensity.

The difference is due to the fact that the two stars are not perfect black body radiators

but they have different degree of emission radiation.

Star X is more approaching a black body radiator.

To know the size of the star, we have to know the luminosity L but L is not given.

19. D

The solar radiation of power P_o per unit area is the brightness b or intensity I.

By $b = \dfrac{L}{4\pi d^2}$ $\quad\quad \therefore\ b \propto \dfrac{1}{d^2}$

The distance of the Earth from the Sun is 1 AU and the brightness on the Earth is P_o.

By $\dfrac{b_2}{b_1} = \left(\dfrac{d_1}{d_2}\right)^2$ $\quad\quad \therefore\ b_2 = \left(\dfrac{1}{40}\right)^2 P_o$

20. A

✓ (1) The absolute magnitude M represents the luminosity L.
Since the absolute magnitude of X is smaller than Y, the luminosity of X is greater than that of Y.

× (2) The apparent magnitude m represents the brightness b received on the Earth.
Since the apparent magnitude of X is greater than Y, the brightness of X is smaller than that of Y.
Thus, a telescope should collect less energy per unit area per unit time from X than that from Y.

21. D

Relative to the spectra in the laboratory, spectra from Galaxy X shows blue shift, thus X moves towards Earth.

Relative to the spectra in the laboratory, spectra from Galaxy Y shows red shift, thus X moves away from Earth.

By $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$ $\quad\quad \therefore\ v \propto \Delta\lambda$

As the shift of wavelength $\Delta\lambda$ for X is greater than that of Y, thus $|v_X| > |v_Y|$

22. C

$\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$ $\quad\quad \therefore\ \dfrac{\Delta\lambda}{(656.3)} = \dfrac{200\times10^3}{3\times10^8}$ $\quad\quad \therefore\ \Delta\lambda = 0.44$ nm

As the star is receding from the earth, the wavelength must be longer.

$\therefore\ \lambda' = 656.3 + 0.44 = 656.7$ nm

23. D

The spectra is neither under red shift nor under blue shift, each of the line spectrum is broadened

thus the star is spinning so that the light from one end is approaching and the light from the other end is receding

24. D

By $v = \dfrac{2\pi r}{T}$

$\therefore$ $(840 \times 10^3) = \dfrac{2\pi r}{(5.8\times10^8\times365\times24\times3600)}$

$\therefore$ $r = 2.45 \times 10^{21}$ m $= \dfrac{2.45\times10^{21}}{3.09\times10^{16}} = 7.9 \times 10^4$ pc

25. C

$d = 15.8$ ly $= \dfrac{15.8}{3.26}$ pc $= 4.85$ pc

$p = \dfrac{1}{d} = \dfrac{1}{4.85} = 0.206''$

26. A

Stars make use of nuclear fusion to produce large amount of energy for emission.

27. D

✓ A. Dark matter does not emit any type of radiation, thus it is not visible.

✓ B. Dark matter has mass, thus it exerts gravitational force on visible matter.

✓ C. The only interaction of dark matter with visible matter is the gravitational force.

✗ D. The total mass of dark matter is the universe is estimated to be much greater than the visible matter.

28. C

(1) 1 ly $= 9.46 \times 10^{15}$ m

(2) 1 pc $= 3.08 \times 10^{16}$ m

(3) 1 AU $= 1.50 \times 10^{11}$ m

$\therefore$ AU < ly < pc

29. B

Stellar parallax : $p = \dfrac{1.52}{2} = 0.76''$

Distance : $d = \dfrac{1}{p} = \dfrac{1}{0.76} = 1.32$ pc

30. A

Distance : $d = 7.85$ ly $= \dfrac{7.85}{3.26}$ pc $= 2.4$ pc

Parallax : $p = \dfrac{1}{d} = \dfrac{1}{2.4} = 0.417''$

Angle subtended at the two extremes of the Earth's orbit $= 2p = 2 \times 0.417'' = 0.83''$

31. A

✓ (1) Since the apparent magnitude of X is smaller, X is observed to be brighter.

× (2) Since the absolute magnitudes of the two stars are not known, their luminosity cannot be determined.

× (3) The distance of star cannot be determined by using the apparent magnitude only.

32. B

$$d = 200 \text{ ly} = \frac{200}{3.26} \text{ pc} = 61.35 \text{ pc}$$

$$p = \frac{1}{d} = \frac{1}{61.35} = 0.016''$$

33. B

Diameter of the nebula : $D = 2r = 20$ ly

Distance of the nebula : $d = 5000 \text{ pc} = 5000 \times 3.26 \text{ ly} = 16300 \text{ ly}$

$$\theta = \frac{D}{d} = \frac{20}{16300} = 1.23 \times 10^{-3} \text{ rad} = 1.23 \times 10^{-3} \times \frac{180^\circ}{\pi} = 0.07047^\circ = 4.2 \text{ arc minute}$$

34. C

Difference of magnitude : $n = (-0.62) - (-1.43) = 0.81$

Times of brightness $= 2.512^{0.81} = 2.1$

35. B

× (1) The smaller the apparent magnitude, the brighter is the star. Thus, star Y is brighter.

✓ (2) Distance of star $X = 1 / p = 1 / 0.12 = 8.3$ pc.
If it is placed at a distance of 10 pc, it would be dimmer. Thus, its absolute magnitude is greater.

× (3) As the absolute magnitude of star X is greater than 2.8, its actual brightness is less than that of Z, thus, the luminosity of X should be less than that of Z.

36. D

✓ (1) Distance : $d = 30 \text{ ly} = \frac{30}{3.26} \text{ pc} = 9.2 \text{ pc}$

✓ (2) Parallax : $p = \frac{1}{d} = \frac{1}{9.2} = 0.11''$

✓ (3) At a distance of 10 pc, its brightness is 3.5.
At its actual distance of 9.2 pc, it should be brighter, thus its apparent magnitude is less than 3.5.

37. D

If $m = M$, then $d = 10$ pc.

Parallax : $p = \frac{1}{d} = \frac{1}{(10)} = 0.1'' = 0.1 \times \frac{1}{60} \times \frac{1}{60} \times \frac{\pi}{180} = 4.85 \times 10^{-7}$ rad

38. A

Let the difference of their apparent magnitude be n.

$\therefore\ 2.512^n = 30$

$\therefore\ \log(2.512^n) = \log 30$

$\therefore\ n \times \log 2.512 = \log 30 \qquad \therefore\ n = 3.7$

Apparent magnitude of star $Y = 4.5 - 3.7 = 0.8$ (Note that brighter star has smaller apparent magnitude.)

39. C

The greater the value of $m - M$, the farther away is the star.

For star X: $m - M = 3.5$

For star Y: $m - M = -1$

For star X: $m - M = 2$

Thus, star Y is the nearest and star X is the farthest.

40. B

✓	(1)	If surface temperature increases, total intensity of radiation emitted by the star increases, thus luminosity increases.
×	(2)	If surface temperature increases, the peak wavelength should decrease.
✓	(3)	As the colour of the star is mainly determined by the peak wavelength, decrease of wavelength means a shift of colour towards the ultra-violet end.

41. C

By $I = \sigma T^4$

$\therefore\ (2 \times 10^8) = (5.67 \times 10^{-8})\, T^4$

$\therefore\ T = 7700\ \text{K}$

42. B

✓	(1)	The higher the surface temperature, the greater is the luminosity of the star.
✓	(2)	The larger the size of a star, the greater is the luminosity of the star.
×	(3)	Blackbody emission does not depend on the chemical composition of the emitter.

43. B

$L = 4\pi R^2 \sigma T^4 = 4\pi (5 \times 10^8)^2 (5.67 \times 10^{-8}) (6500)^4 = 3.2 \times 10^{26}\ \text{W}$

44. D

By $L = 4\pi R^2 \sigma T^4$

$\therefore\ (6.5 \times 10^{32}) = 4\pi R^2 (5.67 \times 10^{-8}) (8200)^4 \qquad \therefore\ R = 4.5 \times 10^{11}\ \text{m}$

Diameter $= 2R = 9.0 \times 10^{11}$ m

45. D

Luminosity $L = 8.4 \times 10^{28} / 60 = 1.4 \times 10^{27}$ W

By $L = 4\pi R^2 \sigma T^4$

$\therefore$ $(1.4 \times 10^{27}) = 4\pi (6.5 \times 10^8)^2 (5.67 \times 10^{-8}) T^4$

$\therefore$ $T = 8260$ K

46. A

By $L = 4\pi R^2 \sigma T^4$

$\therefore$ $(L_s / L_\odot) = (R_s / R_\odot)^2 (T_s / T_\odot)^4$

$$= (80\%)^2 \times \left(\frac{4500}{5800}\right)^4$$

$$= 0.23$$

47. A

By $L = 4\pi R^2 \sigma T^4$

$\therefore$ $(L_s / L_\odot) = (R_s / R_\odot)^2 (T_s / T_\odot)^4$

$\therefore$ $(8 / 3) = (5)^2 (T_s / 5800)^4$

$\therefore$ $T_s = 3300$ K

48. C

By $L = 4\pi R^2 \sigma T^4$

$\therefore$ $(L_2 / L_1) = (R_2 / R_1)^2 (T_2 / T_1)^4 = (1 - 40\%)^2 (1 + 20\%)^4 = 0.75$

The luminosity would decrease by 25%.

49. B

✓ (1) The surface temperature of a star can be determined by observing the peak wavelength of the spectrum.

✓ (2) The chemical composition of a star can be determined by observing the absorption line spectra.

× (3) The size of a star cannot be known by observing the spectrum only.

50. D

The surface temperatures of stars in descending order is O B A F G K M, with sub-classes from 0 to 9.

Thus, B2 is hotter than F0, F0 is hotter than F3, F3 is hotter than K7.

In ascending order of surface temperature : K7, F3, F0, B2, which is the order of X, W, Z, Y

51. B

× (1) As spectral class B is higher than class A, star Y has a higher surface temperature than star X.

× (2) By $L = 4\pi R^2 \sigma T^4$, if both stars have the same R, then $L \propto T^4$, thus the luminosity of star Y is greater than star X.

✓ (3) Since the luminosity of star Y is greater, it is brighter, thus its absolute magnitude is smaller.

52. B

By $L = 4\pi R^2 \sigma T^4$

As W and Y have the same T, and W has greater luminosity L, thus W has larger size than Y. $\therefore\ W > Y$

As X and Y have the same L, and X has higher surface temperature T, thus X has smaller radius R. $\therefore\ X < Y$

As X and Z have the same T, and X has greater luminosity L, thus X has larger size than Z. $\therefore\ X > Z$

$\therefore\ W > Y > X > Z$

53. B

✓ A. Star X is the spectral class with the highest surface temperature, it belongs to class O.

× B. Star Y should be spectral class with the lowest surface temperature, thus it should belong to class M.

✓ C. Star W has lowest surface temperature, thus appears to be red in colour.

✓ D. Star Z has the highest surface temperature, thus appear to be blue in colour.

54. D

✓ (1) As the apparent wavelength from the moving celestial body is shorter, it shifts towards the blue end.

✓ (2) As the apparent wavelength is shorter, the celestial body must be moving towards the Earth.

✓ (3) By $\frac{v_r}{c} = \frac{\Delta\lambda}{\lambda_o}$ $\therefore\ v_r = \frac{634.28 - 633.72}{634.28} \times (3 \times 10^8) = 2.65 \times 10^5 \text{ m s}^{-1}$

55. C

By $\frac{v_r}{c} = \frac{\Delta\lambda}{\lambda_o}$

$\therefore\ \frac{500 \times 10^3 \cos 30^\circ}{3 \times 10^8} = \frac{\Delta\lambda}{528.64}$

$\therefore\ \Delta\lambda = 0.76 \text{ nm}$

Since the star is receding, the spectral line has red shift, thus the apparent wavelength would be longer.

$\therefore\ \lambda' = 528.64 + 0.76 = 529.40 \text{ nm}$

56. B

SI unit of the Hubble constant : $H = 72 \times 10^3 \times (10^6 \times 3.09 \times 10^{16})^{-1} = 2.33 \times 10^{-18} \text{ s}^{-1}$

Age of the Universe $= \frac{1}{H} = \frac{1}{2.33 \times 10^{-18}} = 4.29 \times 10^{17} \text{ s} = 1.36 \times 10^{10}$ years

57. A

✓ (1) From the rotation curve of star around our galaxy,
the rotational speed of the stars does not decrease with the distance from the centre,
thus the existence of dark matter is confirmed.

× (2) The total mass of dark matter is the universe is estimated to be much greater than the visible matter.

× (3) Dark matter exerts gravitational force on visible matter.

58. C

Radial velocity : $v_r = v\cos\theta = 560 \times \cos 30° = 485 \text{ km s}^{-1}$

By Doppler effect,

$$\frac{v_r}{c} = \frac{\Delta\lambda}{\lambda_o} \qquad \therefore \frac{485\times10^3}{3\times10^8} = \frac{\Delta\lambda}{624.35} \qquad \therefore \Delta\lambda = 1.01 \text{ nm}$$

Since the star is receding away from the Earth, the wavelength must be longer.

Apparent wavelength : $\lambda' = 624.35 + 1.01 = 625.36$ nm

59. B

Brightness : $b = \dfrac{L}{4\pi d^2} \qquad \therefore (9.62\times10^{-9}) = \dfrac{(7.50\times10^{26})}{4\pi d^2} \qquad \therefore d = 7.88 \times 10^{16} \text{ m} = 2.55 \text{ pc}$

Parallax : $p = \dfrac{1}{d} = \dfrac{1}{2.55} = 0.39$ arcsecond

Use the following data wherever necessary :

Speed of light in vacuum	$c = 3 \times 10^8$ m s^{-1}
Astronomical unit	AU $= 1.50 \times 10^{11}$ m
Light year	ly $= 9.46 \times 10^{15}$ m
Parsec	pc $= 3.09 \times 10^{16}$ m $= 3.26$ ly $= 206265$ AU
Acceleration due to gravity	$g = 9.81$ m s^{-2} (close to the Earth)
Stefan constant	$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Universal gravitational constant	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

The following list of formulae may be found useful :

Stefan's law	$P = \sigma A T^4$
Doppler effect	$\left\lvert\frac{\Delta f}{f_0}\right\rvert \approx \frac{v}{c} \approx \left\lvert\frac{\Delta \lambda}{\lambda_0}\right\rvert$

Part A :

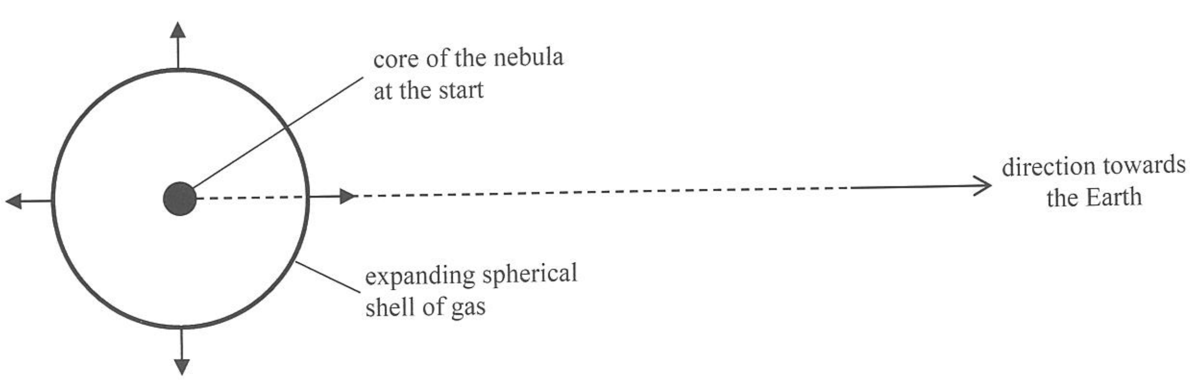
The following questions marked with { } are the past DSE examination questions.

The question marked with {SP} is the Sample Paper question.

The question marked with {PP} is the Sample Paper question.

The number inside the brackets represents the year of the DSE examination.

Q1. {SP} The Crab Nebula is an expanding, roughly spherical shell of gas in the constellation Taurus. According to a recent study, its average apparent angular size is 5.8 arc minute. The whole nebula has negligible velocity relative to the Earth, and the nebula is at a distance of 2000 pc from the Earth. The wavelength of an O III spectral line found in the spectrum of the light emitted by the gas moving towards the earth from around the middle part of the Crab Nebula is 374.13 nm along the line of sight of an observer on the Earth. The wavelength of the same spectral line observed in the laboratory is 375.99 nm.



(a) What is the radius of the Crab Nebula ? Give your answer to two significant figures in parsecs. (2 marks)

Q1. (b) Calculate the speed of that gas which is moving towards the Earth. Give your answer in km s^{-1} to two significant figures. (3 marks)

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(c) (i) The Crab Nebula was formed by the explosion of a star whose size was negligible compared with the present size of the nebula. Estimate the age of the Crab Nebula. Give your answer to two significant figures in years. State the assumption made in your calculation. (3 marks)

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(ii) Actually, the Chinese observed the stellar explosion which created the Crab Nebula in 1054 A.D. and so we know that its age is about 950 years. Give a possible reason to explain why the Crab Nebula's age estimated in (c) (i) is longer than 950 years. (2 marks)

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Q2. {PP} (a) We observe a galaxy X as shown in Figure (a). X has negligible velocity relative to the Earth. Points A and B are both 10 kpc from the centre. The wavelengths of the H-alpha lines from the hydrogen gas at points A and B are 656.83 nm and 655.73 nm respectively. The wavelength of the H-alpha line measured in the laboratory is 656.28 nm.

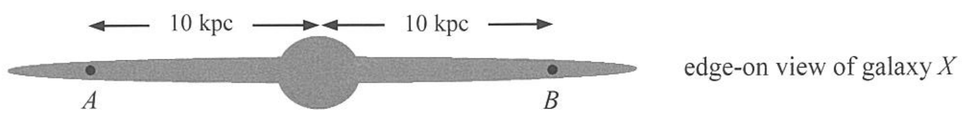

Figure (a)

(i) Determine the speed of the hydrogen gas at point A along the line of sight of an observer on the Earth. (1 mark)

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Q2. (a) (ii) Briefly explain at which point, A or B, the hydrogen gas is moving towards the Earth. (2 marks)

(iii) Assuming that the hydrogen gas at points A and B are moving in a circular path around the centre of X, and that the mass of X is concentrated at its centre, estimate the mass X. (2 marks)

(b) Observations were made on another galaxy Y, as shown in Figure (b).

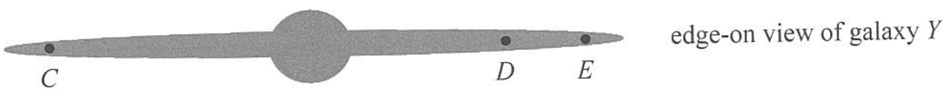


Figure (b)

(i) The angular separation between points C and E is 1.6 °. Given that Y is 950 kpc from the Earth, express the separation between C and E in kpc. (2 marks)

(ii) Further observations show that the velocities of hydrogen gas at points D and E along the line of sight of an observer on the Earth are about the same. What could be inferred about the mass distribution of Y ? Assume that the hydrogen gas at points D and E are moving in circular paths around the centre of Y. (1 mark)

(c) Briefly explain how we can estimate the surface temperature of a star by analyzing its radiation. (2 marks)

Q3. {12}

(a) Let R_S, T_S and L_S be the radius, surface temperature and luminosity of the Sun and R, T and L be the radius, surface temperature and luminosity of another star.

(i) Show that $R = \left(\frac{T_S}{T}\right)^2\left(\frac{L}{L_S}\right)^{\frac{1}{2}} R_S$. (2 marks)

(ii) Betelgeuse is a star with surface temperature 3650 K and luminosity 126000 times that of the Sun. Find the radius of Betelgeuse in terms of R_S. Take the surface temperature of the Sun to be 5780 K. (2 marks)

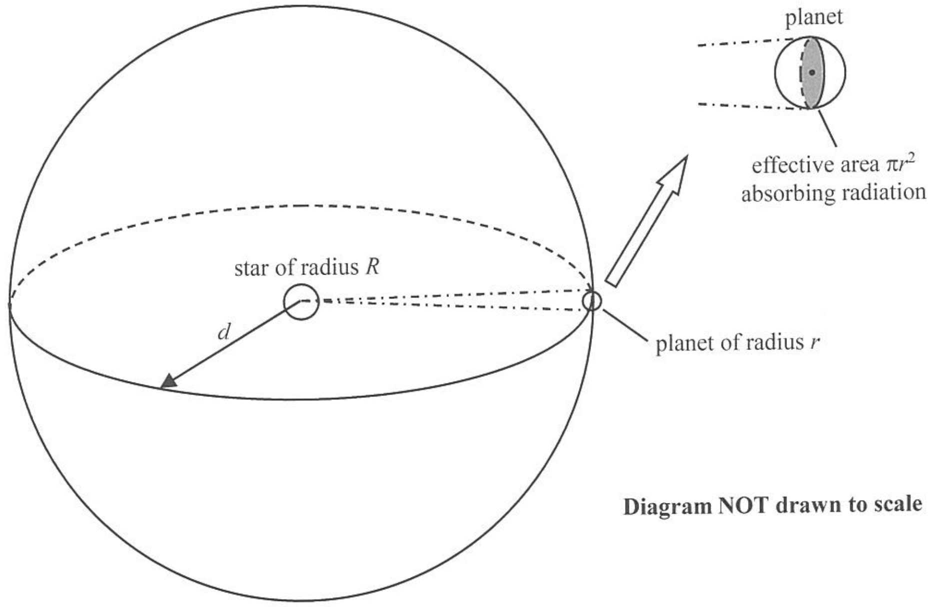
(b) (i) An estimate of the distance to Betelgeuse is 197 pc which corresponds to the luminosity given in (a) (ii). A measurement of this distance made in 2008 was 197 ± 45 pc. Without calculating the actual value, explain how the radius of Betelgeuse found in (a) (ii) would change if the upper limit of this distance measurement were used. Betelgeuse at this distance can be treated as a point source emitting light evenly in all directions. (2 marks)

(ii) Suggest a reason why it is difficult to measure accurately the distance to Betelgeuse by the method of parallax. (1 mark)

(c) In 2011, some media reports suggested that when Betelgeuse undergoes a supernova explosion (i.e. the death of a star), it will appear as the "second sun" in the sky for a few weeks. Referring to the information given below, explain whether this is true or not by comparing the brightness of Betelgeuse in supernova explosion with that of the Sun. (3 marks)

A star of similar mass as that of Betelgeuse gives off a luminosity of about 10^9 times that of the Sun for a certain period of time when the star undergoes a supernova explosion. About 1% of the power of explosion appears in the form of visible light. Take the distance of Betelgeuse to be 200 pc.

Q4.
{14}



(a) A star of radius R and surface temperature T_S (in K) emits radiation in all directions. A planet of radius r orbits the star at a distance d, which is much larger than both R and r. Assume that both the star and the planet behave like black bodies.

(i) Taking the effective area that the planet absorbs radiation emitted from the star as πr^2, show that the power absorbed by the planet is $\pi \sigma (\frac{rR}{d})^2 T_S^4$ where σ is the Stefan constant. Assume that the planet is a perfect absorber of radiation. (2 marks)

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(ii) If the planet only absorbed energy, its temperature would rise indefinitely. However, this would not happen because the planet also radiates energy as it absorbs energy so that an equilibrium state is maintained. Show that the equilibrium surface temperature of the planet is given by $T_P = \sqrt{\frac{R}{2d}} T_S$. (2 marks)

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(b) A planet called Kepler-22b was discovered orbiting a Sun-like star with orbital radius 0.84 AU (1 AU = 1.50×10^{11} m). The star has a radius of 6.82×10^8 m and its surface temperature is 5518 K.

(i) Estimate the equilibrium surface temperature of Kepler-22b using the results of (a). (2 marks)

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(ii) Liquid water is believed to be essential for life to exist on a planet. Based on the information found in (b) (i), explain whether Kepler-22b would be a favourable planet for life to exist or not. (2 marks)

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(iii) If Kepler-22b orbits a class K star instead of a Sun-like star (which is a class G star) with the same orbital radius, would its equilibrium surface temperature increases, decreases or remain unchanged ? State your reason.

Given : the sequence of spectral classes is O B A F G K M. (2 marks)

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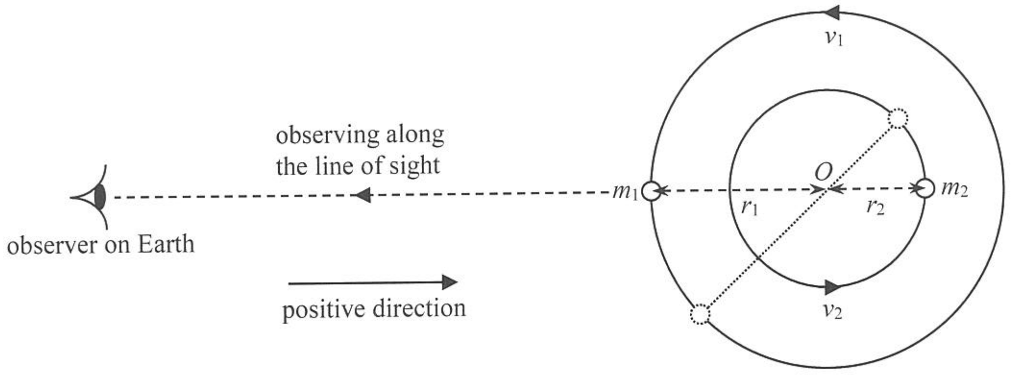
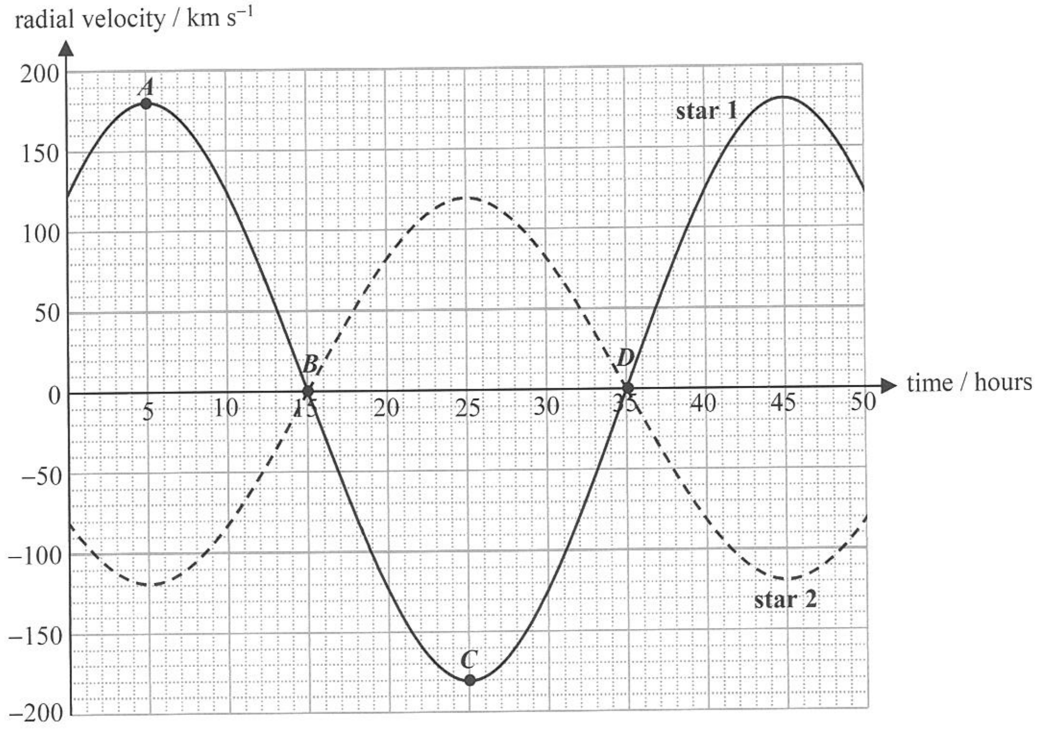
Q5. {15} Figure 1 shows a distant binary star system viewed by an observer on Earth who is also on the orbital plane of the two stars. The system consists of stars 1 and 2 with masses m_1 and m_2 respectively orbiting in uniform circular motion about their centre of mass O under their mutual gravitational force. They move with the same period in two orbits of radii r_1 and r_2 with orbital speeds v_1 and v_2 respectively.

Figure 1



By finding the radial velocity v_r of the two stars inferred from the Doppler shift ($\Delta\lambda$) of the hydrogen-alpha line (H_α) observed on Earth, astronomers are able to deduce the masses of the stars. Assume that the centre of mass O of the binary system is stationary with respect to the observer. Figure 2 shows the radial velocity curves for the two stars. The direction moving away from the observer is taken to be positive velocity.

Figure 2



(a) (i) What does it mean by radial velocity v_r of a star observed on Earth ? (1 mark)

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(ii) Identify which point, A, B, C or D, marked on the radial velocity curve corresponds to the orbital position of star 1 (in solid line) at the instant shown in Figure 1. (1 mark)

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Q5. (b) Find, from Figure 2, the orbital speed v_1 of star 1 and calculates its orbital radius r_1. Using a similar method, or otherwise, find the orbital radius r_2 of star 2. (4 marks)

(c) By considering the circular motion of star 1, calculate the mass m_2 of star 2. (2 marks)

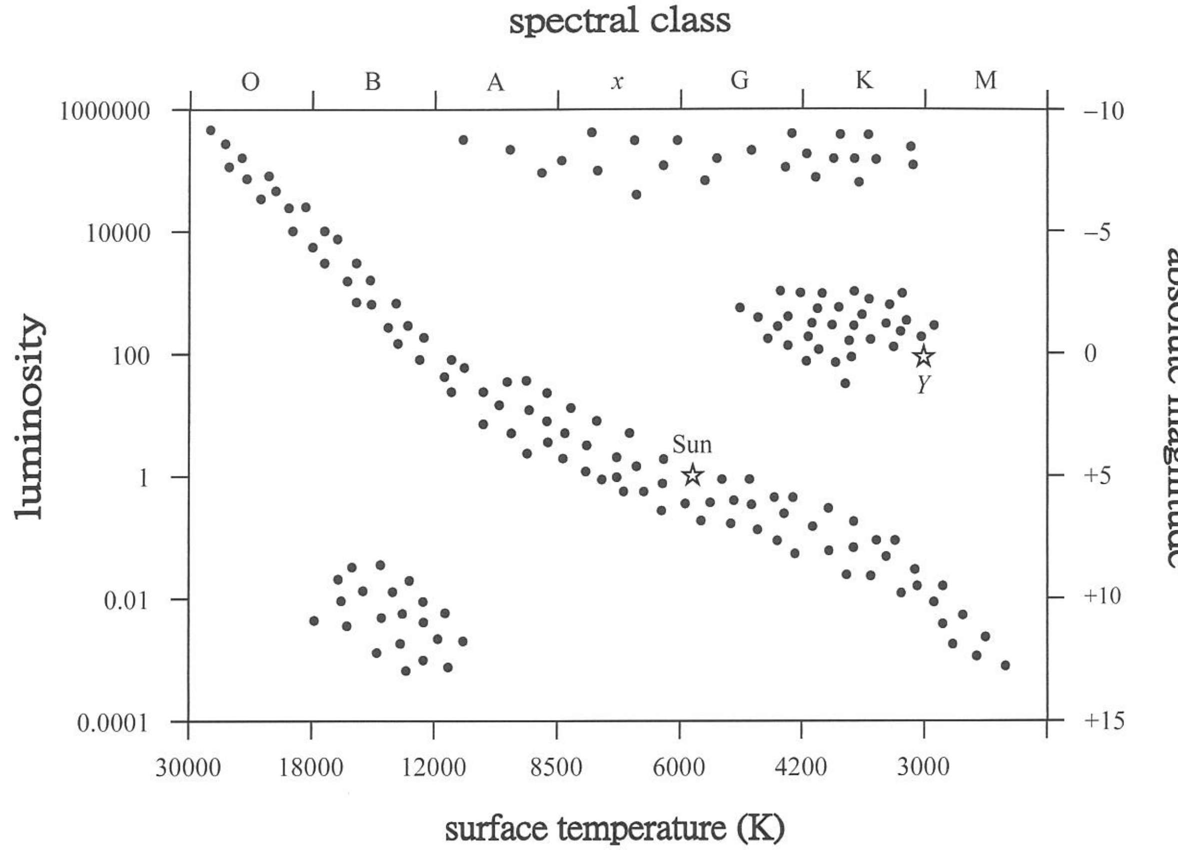
(d) A spectrometer can only measure change of wavelength larger than 0.5 nm. Explain whether this instrument is suitable to measure the Doppler shift $\Delta\lambda$ of the hydrogen-alpha line ($\lambda_o = 656.28$ nm) of the two stars. (2 marks)

Part B :

The following questions are designed to give supplemental exercise for this chapter.

Q6. State and explain one piece of evidence which suggests that the universe is expanding. (3 marks)

Q7.



The above figure shows the typical Hertzsprung and Russell (H-R) diagram. The luminosity of the Sun with the value of 3.9×10^{26} W is given the unit of 1.

(a) There are seven major spectral classes of stars corresponding to different colours, which depends on their surface temperature. One of the classes is labelled as x as shown in the diagram. What does x stand for ? (1 mark)

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(b) The Sun belongs to the main sequence stars as shown in the diagram. The surface temperature of the Sun is 5800 K. Determine the radius of the Sun. (2 marks)

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(c) The spectra obtained from stars are usually present with many faint dark lines, called absorption spectral lines. What information can be obtained by analyzing these absorption lines of a star ? (1 mark)

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(d) The vertical axis can be labelled as the absolute magnitude. State the definition of the absolute magnitude of a star. (1 mark)

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Q7. (e) The star Y marked in the above diagram belongs to the group of stars that have low surface temperature but high luminosities.

(i) Name the group of stars that Y belongs to. (1 mark)

(ii) The luminosity of star Y is 100 and its surface temperature is 3000 K. Determine the radius of the star Y relative to the Sun. (2 marks)

(iii) The absolute magnitude of star Y is 0 as shown in the diagram. Its parallax is 1.25×10^{-6} rad. By calculating the distance of star Y from the Earth, determine whether the apparent magnitude is greater or smaller than 0. (2 marks)

Q8. The brightness of a star W is observed to be 8.45×10^{-10} W m^{-2}. This brightness corresponds to the apparent magnitude of 3.69. The parallax of this star is 0.572 arcsecond.

(a) Find the distance of star W from the Earth. (1 mark)

(b) Assume star W is a point source that emits light evenly in all directions, calculate its luminosity. (2 marks)

(c) (i) Define the absolute magnitude M. (1 mark)

(ii) Explain whether star W has absolute magnitude greater or smaller than its apparent magnitude. (2 marks)

(d) Given that the surface temperature of star W is 12500 K, estimate its diameter. (3 marks)

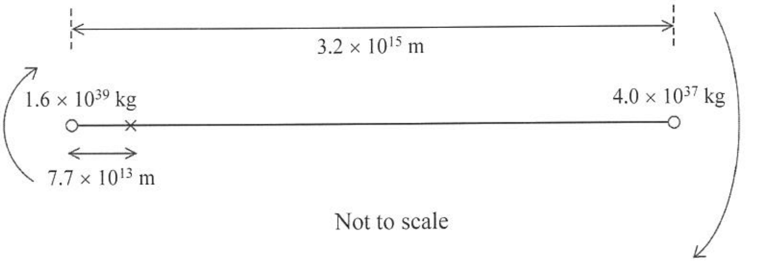
Q9. Current theory predicts that there is a massive black hole at the centre of every galaxy. It is suggested that if galaxies approach, then their central black holes begin to orbit each other until the galaxies merge.

In 2009, astronomers found convincing evidence of two such black holes orbiting as a binary system. From data collected, they estimated that the separation of the black holes was 3.2×10^{15} m and their masses were 1.6×10^{39} kg and 4.0×10^{37} kg.

(a) (i) State the origin of the force that maintains the black holes in an orbit. (1 mark)

(ii) Calculate the magnitude of this force. (2 marks)

(iii) The black holes orbit about a point 7.7×10^{13} m from the larger mass black hole.



Calculate the orbital time of the binary system, express the answer in year. (3 marks)

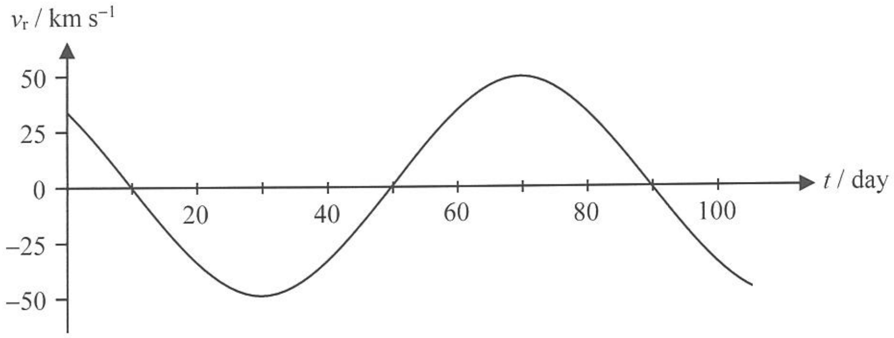
(b) As the black holes swallow up matter, radiation is emitted. To observers on Earth this radiation appears to be red shifted.

(i) State what red shift means and discuss the conclusions that can be drawn from the observation that radiation from all distant galaxies is red shifted. (3 marks)

(ii) Suggest why the light from both black holes is red shifted, even though the black holes are orbiting each other and hence moving in opposite directions. (2 marks)

(iii) The observed red shift for the two black holes was 0.38. Calculate the distance of the merging galaxies from the Earth. Given that the Hubble constant $H = 1.6 \times 10^{-18}$ s^{-1}. (3 marks)

Q10. The radial velocity curve of a star X orbiting around another more massive star Y is shown.



(a) From the curve, write down the orbital speed v and the orbital period T of the star X. (2 marks)

(b) Find the orbital radius r of the star X. (3 marks)

(c) What is the mass of the massive star Y? (3 marks)

(d) Find the maximum fractional Doppler shift of a certain wavelength observed along the line of sight from the Earth. (2 marks)

Q1. (a) Let R be the radius of Crab Nebula

By small angle approximation, $\dfrac{2R}{d} = \theta$ where d is the distance of the Crab Nebula from the Earth [1]

$\therefore \quad \dfrac{2R}{2000} = \dfrac{5.8}{60} \times \dfrac{\pi}{180}$

$\therefore \quad R = 1.7$ pc [1]

(b) Let v_r be the radial velocity of the gas.

By the Doppler shift formula,

$\dfrac{v_r}{c} = \dfrac{\Delta\lambda}{\lambda_0}$ [1]

$\therefore \quad \dfrac{v_r}{(3\times10^8)} = \dfrac{(374.13 - 375.99)}{375.99}$ [1]

$\therefore \quad v_r = -1500 \text{ km s}^{-1}$ [1]

$\therefore$ The speed of the gas moving towards the Earth is 1500 km s^{-1}.

(c) (i) Let T be the age of the Crab Nebula.
Since the nebula has negligible velocity relative to the Earth and assuming
that the expansion speed v of the nebula found in (b) is constant throughout the history of the nebula, [1]

$R = vT$

$T = \dfrac{1.7\times3.26\times9.46\times10^{15}}{1500\times10^3}$ [1]

$= 3.50 \times 10^{10} \text{ s} = 1100 \text{ yr}$ [1]

(ii) The expanding gas of the Crab Nebula is subject to the nebula's own gravitational force,
the energy required to resist such a gravitational pull reduces the kinetic energy (OR speed)
of the expanding gas. [1]

Thus, the expansion speed of the Crab Nebula was faster in the past (> 1500 km s^{-1})
and the constant expansion speed assumed in (c) (i) is incorrect.
Hence, the actual age of the Crab Nebula is less than 1100 years deduced in (c) (i). [1]

Q2. (a) (i) $\dfrac{v_r}{c} = \dfrac{\Delta\lambda}{\lambda_0}$

$\therefore \quad \dfrac{v_r}{(3\times10^8)} = \dfrac{(656.83 - 656.28)}{656.28} \qquad \therefore \quad v_r = 2.51 \times 10^5 \text{ m s}^{-1}$ [1]

(ii) The gas at point B is moving towards the Earth, [1]
since the apparent wavelength is shorter than the actual wavelength, it is a blue shift. [1]

(iii) $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$ [1]

$M = \dfrac{v^2 r}{G} = \dfrac{(2.51\times10^5)^2\,(10\times10^3\times3.09\times10^{16})}{(6.67\times10^{-11})} = 2.92\times10^{41}$ kg [1]

Q2. (b) (i) By small angle approximation, and let s be the separation.

$$\theta = \frac{s}{d}$$ [1]

$$(1.6^\circ \times \frac{\pi}{180^\circ}) = \frac{s}{(950)}$$

$$\therefore\ s = 26.5 \text{ kpc}$$ [1]

(ii) The mass distribution of Y is not concentrated at the centre. [1]

(c) The radiation from a star can be approximated by a black body radiation curve. [1]

The radiation spectrum of a black body is related to its temperature. [1]

OR

Assume the star is a black body radiator. [1]

From its radiation, the peak wavelength is related to its surface temperature. [1]

Q3. (a) (i) By Stefan's Law : $L = \sigma A T^4$ $\quad \therefore\ L = 4\pi R^2 \sigma T^4$ [1]

$$\therefore\ R^2 \propto \frac{L}{T^4} \qquad \therefore\ R \propto \frac{\sqrt{L}}{T^2}$$ [1]

$$\therefore\ \frac{R}{R_S} = \sqrt{\frac{L}{L_S}}\left(\frac{T_S}{T}\right)^2 \qquad \therefore\ R = \left(\frac{T_S}{T}\right)^2\left(\frac{L}{L_S}\right)^{\frac{1}{2}} R_S$$

(ii) $R = \left(\frac{T_S}{T}\right)^2\left(\frac{L}{L_S}\right)^{\frac{1}{2}} R_S$

$$R = \left(\frac{5780}{3650}\right)^2 (126000)^{\frac{1}{2}} R_S$$ [1]

$$R = 890\ R_S$$ [1]

(b) (i) If the upper limit is used, then the distance measured is greater.

Brightness observed : $b = \dfrac{L}{4\pi d^2}$; for the same brightness b, luminosity L increases with distance d. [1]

If the luminosity is greater, by Stefan's law, $L \propto R^2$, thus, the radius R of Betelgeuse would be greater. [1]

(ii) By $p = \dfrac{1}{d} = \dfrac{1}{197 \text{ pc}} = 5 \times 10^{-3}$ arcsecond

This parallax is too small to be measured accurately. [1]

(c) Brightness : $b = \dfrac{L}{4\pi d^2} \propto \dfrac{L}{d^2}$ [1]

$$\frac{b}{b_S} = \frac{L}{L_S} \times \left(\frac{d_S}{d}\right)^2 = (10^9 \times 1\%) \times \left(\frac{1.50 \times 10^{11}}{200 \times 3.09 \times 10^{16}}\right)^2 = 5.89 \times 10^{-9}$$ < accept 5.88×10^{-9} > [1]

The brightness of Betelgeuse is much smaller than that of the Sun, thus the report is not correct. [1]

Q4. (a) (i) Luminosity (power) given out by the star : $L = 4\pi R^2 \sigma T_S^4$ [1]

Total surface area of the sphere with radius d is $4\pi d^2$.

$$P = 4\pi R^2 \sigma T_S^4 \times \frac{\pi r^2}{4\pi d^2} = \pi\sigma(\frac{rR}{d})^2 T_S^4$$ [1]

OR

Luminosity (power) given out by the star : $L = 4\pi R^2 \sigma T_S^4$ [1]

Brightness on the planet : $b = \frac{L}{4\pi d^2} = \frac{4\pi R^2 \sigma T_S^4}{4\pi d^2}$ [1]

$$P = bA = \frac{4\pi R^2 \sigma T_S^4}{4\pi d^2} \times \pi r^2 = \pi\sigma(\frac{rR}{d})^2 T_S^4$$

(ii) Luminosity (power) given out by the planet :

$L = 4\pi r^2 \sigma T_P^2$

At equilibrium, power absorbed by the planet equals power emitted by the planet.

$$\therefore\ 4\pi r^2 \sigma T_P^2 = \pi\sigma(\frac{rR}{d})^2 T_S^4$$ [1]

$$\therefore\ 4T_P^4 = \frac{R^2}{d^2}\cdot T_S^4$$ [1]

$$\therefore\ T_P = \sqrt{\frac{R}{2d}}\, T_S$$

(b) (i) $T_P = \sqrt{\frac{R}{2d}}\, T_S = \sqrt{\frac{(6.82\times10^8)}{2\times(0.84\times1.5\times10^{11})}} \times (5518)$ [1]

$= 287$ K [1]

(ii) Liquid water exists in the range of 0°C to 100°C, that is, 273 K to 373 K. [1]

As the surface temperature of the planet is 287 K, liquid water can exist, [1]

thus it is a favourable planet for life to exist.

(iii) For a class K star, its surface temperature T_S is lower. [1]

By $T_P = \sqrt{\frac{R}{2d}}\, T_S$, $T_P \propto T_S$ for same d,

thus, the surface temperature T_P of the planet would decrease. [1]

Q5. (a) (i) Radial velocity of a star is the component of its velocity directly towards or away from the Earth. [1]

OR

Radial velocity of a star is the component of its velocity along the line of sight of the observer on Earth. [1]

(ii) Point D [1]

[At the instant shown, the radial velocity is zero.]

[After then, star 1 moves away and its radial velocity is positive.]

Q5. (b) $v_1 = 180 \text{ km s}^{-1}$ [1]

Period : $T = 40$ hours

By $T = \frac{2\pi r_1}{v_1}$

$\therefore (40 \times 3600) = \frac{2\pi r_1}{(180\times10^3)}$ [1]

$\therefore r_1 = 4.13 \times 10^9$ m [1]

$v_2 = 120 \text{ km s}^{-1}$

By $T = \frac{2\pi r_2}{v_2}$

$\therefore (40 \times 3600) = \frac{2\pi r_2}{(120\times10^3)}$

$\therefore r_2 = 2.75 \times 10^9$ m [1]

(c) $F = \frac{G m_1 m_2}{(r_1+r_2)^2} = m_1 r_1 \omega^2$ [1]

$\therefore \frac{G m_2}{(r_1+r_2)^2} = r_1 (\frac{2\pi}{T})^2$

$\therefore \frac{(6.67\times10^{-11}) m_2}{(4.13\times10^9 + 2.75\times10^9)^2} = (4.13 \times 10^9) \times (\frac{2\pi}{40\times3600})^2$

$\therefore m_2 = 5.58 \times 10^{30}$ kg < accept 5.6×10^{30} kg > [1]

(d) $\frac{\Delta\lambda}{\lambda_o} = \frac{v_r}{c}$

$\therefore \frac{\Delta\lambda}{(656.28)} = \frac{(180\times10^3)}{(3\times10^8)}$ [1]

$\therefore \Delta\lambda = 0.394$ nm

Since the Doppler shift $\Delta\lambda$ is less than 0.5 nm, the instrument is not suitable. [1]

Q6. Red shift of the light emitted from distant galaxies. [1]

The spectrum emitted from distant galaxies is found to shift towards the red end, i.e. longer wavelength [1]

By Doppler effect, the distant galaxies must be moving away from us, thus the universe is expanding. [1]

Q7. (a) x stands for F. [1]

(b) By $L = 4\pi R^2 \sigma T^4$

$\therefore (3.9 \times 10^{26}) = 4\pi R^2 (5.67 \times 10^{-8}) (5800)^4$ [1]

$\therefore R = 6.95 \times 10^8$ m [1]

Q7. (c) The chemical composition of the star can be known by analysing the absorption lines. [1]

(d) Absolute magnitude of a star is defined as the apparent magnitude that the star would have if it is placed at a distance of 10 pc from the Earth. [1]

(e) (i) Red giants [1]

(ii) $\frac{L}{L_S} = \left(\frac{R}{R_S}\right)^2 \times \left(\frac{T}{T_S}\right)^4$ [1]

$(100) = \left(\frac{R}{R_S}\right)^2 \times \left(\frac{3000}{5800}\right)^4$

$\therefore \; R = 37.4\, R_S$ [1]

(iii) $p = 1.25 \times 10^{-6} \times \frac{180}{\pi} \times 60 \times 60'' = 0.2578''$

$d = \frac{1}{p} = \frac{1}{0.2578} = 3.88$ pc [1]

Since the actual distance of star Y is less than 10 pc, its apparent brightness should be greater, thus its apparent magnitude is smaller than 0. [1]

Q8. (a) $d = \frac{1}{p} = \frac{1}{0.572} = 1.75$ pc [1]

(b) $b = \frac{L}{4\pi d^2}$

$\therefore \; (8.45 \times 10^{-10}) = \frac{L}{4\pi (1.75 \times 3.09 \times 10^{16})^2}$ [1]

$\therefore \; L = 3.10 \times 10^{25}$ W m^{-2} [1]

(c) (i) Absolute magnitude of a star is the apparent magnitude that the star would have if it is placed at a distance of 10 pc from the Earth. [1]

(ii) The distance of star W is 1.75 pc.

If it is placed at a further distance of 10 pc, its brightness would decrease. [1]

Thus, it would have the absolute magnitude greater than its apparent magnitude. [1]

(d) By Stefan's Law :

$L = \sigma A T^4 = 4\pi R^2 \sigma T^4$

$\therefore \; (3.10 \times 10^{25}) = 4\pi R^2 \times (5.67 \times 10^{-8})(12500)^4$ [1]

$\therefore \; R = 4.22 \times 10^7$ m [1]

Diameter of star $W = 2R = 8.44 \times 10^7$ m [1]

Q9. (a) (i) It is the gravitational force. [1]

(ii) $F = \dfrac{G m_1 m_2}{r^2}$

$= \dfrac{(6.67\times10^{-11})(1.6\times10^{39})(4.0\times10^{37})}{(3.2\times10^{15})^2}$ [1]

$= 4.17 \times 10^{35}$ N [1]

(iii) $F = m r \omega^2$

$\therefore\ (4.17 \times 10^{35}) = (1.6 \times 10^{39})\,(7.7 \times 10^{13})\,\omega^2$ [1]

$\therefore\ \omega = 1.84 \times 10^{-9}$ rad s^{-1} [1]

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{(1.84\times10^{-9})}$

$= 3.42 \times 10^9$ s $= \dfrac{3.42\times10^9}{365\times24\times3600}$ year

$= 108$ years [1]

(b) (i) Red shift means that the radiation observed has a longer wavelength than those observed in the laboratory. [1]

This indicates that all distant galaxies are moving away from us. [1]

This gives the evidence that the Universe is expanding. [1]

(ii) The rotational motion of the black hole is small compared with the receding motion. [1]

Hence both black holes are still moving away to give red shift. [1]

(iii) Red shift $= \dfrac{v}{c}$

$\therefore\ v = (0.38) \times (3 \times 10^8) = 1.14 \times 10^8$ m s^{-1} [1]

By $v = H d$ [1]

$\therefore\ (1.14 \times 10^8) = (1.6 \times 10^{-18})\, d$

$\therefore\ d = 7.13 \times 10^{25}$ m [1]

Q10. (a) $v = 50$ km s^{-1} [1]

$T = 80$ days [1]

(b) By $v = r\,\omega = r \times \dfrac{2\pi}{T}$ [1]

$\therefore\ r = \dfrac{v \cdot T}{2\pi}$

$= \dfrac{(50\times10^3)\cdot(80\times24\times3600)}{2\pi}$ [1]

$= 5.5 \times 10^{10}$ m [1]

Q10. (c) By $\frac{GMm}{r^2} = \frac{mv^2}{r}$ [1]

$\therefore \quad M = \frac{v^2 r}{G} = \frac{(50\times10^3)^2(5.5\times10^{10})}{(6.67\times10^{-11})}$ [1]

$= 2.1 \times 10^{30}$ kg [1]

(d) $\frac{\Delta\lambda}{\lambda_o} = \frac{v}{c}$ [1]

$= \frac{50\times10^3}{3\times10^8}$

$= 1.67 \times 10^{-4}$ [1]