Learning Content of Module 2 (Algebra and Calculus)

Notes:

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Algebra" and "Calculus") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge			
1. Odd and even functions	1.1 recognise odd and even functions and their graphs	2	Students are required to recognise that the absolute value function is an example of even functions.
2. Mathematical induction	2.1 understand the principle of mathematical induction	3	The First Principle of Mathematical Induction is required. Students are required to prove propositions related to the summation of a finite sequence. Proving propositions involving inequalities is **not** required.

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3. The binomial theorem	3.1 expand binomials with positive integral indices using the binomial theorem	3	Proving the binomial theorem is required. Students are required to recognise the summation notation ($\sum$). The following contents are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation
4. More about trigonometric functions	4.1 understand the concept of radian measure	15	
	4.2 understand the functions cosecant, secant and cotangent		The formulae that students are required to use include: $1+\tan^2\theta=\sec^2\theta$ and $1+\cot^2\theta=\operatorname{cosec}^2\theta$ Simplifying trigonometric expressions by identities is required.
	4.3 understand compound angle formulae and double angle formulae for the functions sine, cosine and tangent, and product-to-sum and sum-to-product formulae for the functions sine and cosine		The formulae include: • $\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$ • $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

Learning Unit	**Learning Objective**	**Time**	**Remarks**
			• $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ • $\sin 2A = 2 \sin A \cos A$ • $\cos 2A = \cos^2 A - \sin^2 A$ $= 1 - 2\sin^2 A = 2\cos^2 A - 1$ • $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$ • $\sin^2 A = \dfrac{1}{2}(1 - \cos 2A)$ • $\cos^2 A = \dfrac{1}{2}(1 + \cos 2A)$ • $2\sin A \cos B = \sin(A+B) + \sin(A-B)$ • $2\cos A \cos B = \cos(A+B) + \cos(A-B)$ • $2\sin A \sin B = \cos(A-B) - \cos(A+B)$ • $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$ • $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$ • $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

Learning Unit	Learning Objective	Time	Remarks
			• $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$ "Subsidiary angle form" is **not** required. $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ and $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$ can be considered as formulae derived from the double angle formulae.
5. Introduction to *e*	5.1 recognise the definitions and notations of *e* and the natural logarithm	2	Two approaches for the introduction to *e* can be considered: • $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$ (proving the existence of this limit is **not** required) • $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ These definitions may be introduced in Learning Objective 6.1.
	Subtotal in hours	25	

Learning Unit	Learning Objective	Time	Remarks
Calculus			
6. Limits	6.1 understand the intuitive concept of the limit of a function	3	Student are required to recognise the theorems on the limits of sum, difference, product, quotient, scalar multiplication of functions and the limits of composite functions (the proofs are **not** required).
	6.2 find the limit of a function		The formulae that students are required to use include: • $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ • $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ Finding the limit of a rational function at infinity is required.

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7. Differentiation	7.1 understand the concept of the derivative of a function	13	Students are required to find the derivatives of elementary functions from first principles, for example: C , x^n (n is a positive integer), $\sqrt{x}$, $\sin x$, $\cos x$, e^x and ln x. Students are required to recognise the notations: y' , $f'(x)$ and $\frac{dy}{dx}$. Testing differentiability of functions is **not** required.
	7.2 understand the addition rule, product rule, quotient rule and chain rule of differentiation		The rules include: • $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$ • $\frac{d}{dx}(uv)=v\frac{du}{dx}+u\frac{dv}{dx}$ • $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ • $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

Learning Unit	Learning Objective	Time	Remarks
	7.3 find the derivatives of functions involving algebraic functions, trigonometric functions, exponential functions and logarithmic functions		The formulae that students are required to use include: • $(C)' = 0$ • $(x^n)' = nx^{n-1}$ • $(\sin x)' = \cos x$ • $(\cos x)' = -\sin x$ • $(\tan x)' = \sec^2 x$ • $(e^x)' = e^x$ • $(\ln x)' = \frac{1}{x}$ The following algebraic functions are required: • polynomial functions • rational functions • power functions x^α • functions formed from the above functions through addition, subtraction, multiplication, division and composition, such as $\sqrt{x^2+1}$

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	7.4 find derivatives by implicit differentiation		Logarithmic differentiation is required.
	7.5 find the second derivative of an explicit function		Students are required to recognise the notations: y'', $f''(x)$ and $\frac{d^2y}{dx^2}$. Students are required to recognise the second derivative test and concavity. Third and higher order derivatives are **not** required.
8. Applications of differentiation	8.1 find the equations of tangents to a curve	14	
	8.2 find the maximum and minimum values of a function		Local and global extrema are required.
	8.3 sketch curves of polynomial functions and rational functions		The following points should be considered in curve sketching: • symmetry of the curve • limitations on the values of x and y • intercepts with the axes • maximum and minimum points

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	8.4 solve the problems relating to rate of change, maximum and minimum		• points of inflexion • vertical, horizontal and oblique asymptotes to the curve Students are required to deduce the equation of the oblique asymptote to the curve of a rational function by division.
9. Indefinite integration and its applications	9.1 recognise the concept of indefinite integration 9.2 understand the properties of indefinite integrals and use the integration formulae of algebraic functions, trigonometric functions and exponential functions to find indefinite integrals	15	Indefinite integration as the reverse process of differentiation should be introduced. The formulae include: • $\int k \ dx = kx + C$ • $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ • $\int \frac{1}{x} dx = \ln\lvert x \rvert + C$ • $\int e^x dx = e^x + C$

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			• $\int \sin x\,dx = -\cos x + C$ • $\int \cos x\,dx = \sin x + C$ • $\int \sec^2 x\,dx = \tan x + C$
	9.3 understand the applications of indefinite integrals in mathematical contexts		Applications of indefinite integrals in some fields such as geometry is required.
	9.4 use integration by substitution to find indefinite integrals		
	9.5 use trigonometric substitutions to find the indefinite integrals involving $\sqrt{a^2-x^2}$, $\frac{1}{\sqrt{a^2-x^2}}$ or $\frac{1}{x^2+a^2}$		Students are required to recognise the notations: $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$, and the concept of their related principal values.
	9.6 use integration by parts to find indefinite integrals		Teachers can use $\int \ln x\,dx$ as an example to illustrate the method of integration by parts. The use of integration by parts is limited to at most two times in finding an integral.

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10. Definite integration	10.1 recognise the concept of definite integration	10	The definite integral as the limit of a sum and finding a definite integral from the definition should be introduced. The concept of dummy variables is required, for example, $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$. Using definite integration to find the sum to infinity of a sequence is **not** required.
	10.2 understand the properties of definite integrals		The properties include: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

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			• $\int_{-a}^{a} f(x)\,dx = 0$ if $f(x)$ is an odd function • $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ if $f(x)$ is an even function
	10.3 find definite integrals of algebraic functions, trigonometric functions and exponential functions		The Fundamental Theorem of Calculus that students are required to recognise is: $\int_{a}^{b} f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$.
	10.4 use integration by substitution to find definite integrals		
	10.5 use integration by parts to find definite integrals		The use of integration by parts is limited to at most two times in finding an integral.
11. Applications of definite integration	11.1 understand the application of definite integrals in finding the area of a plane figure	4	

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	11.2 understand the application of definite integrals in finding the volume of a solid of revolution about a coordinate axis or a line parallel to a coordinate axis		"Disc method" is required.
	Subtotal in hours	59	
Algebra			
12. Determinants	12.1 recognise the concept of determinants of order 2 and order 3	2	Students are required to recognise the notations: $\|A\|$ and $\det A$.
13. Matrices	13.1 understand the concept, operations and properties of matrices	10	The addition, scalar multiplication and multiplication of matrices are required. The properties include: • $A+B=B+A$ • $A+(B+C)=(A+B)+C$ • $(\lambda+\mu)A=\lambda A+\mu A$ • $\lambda(A+B)=\lambda A+\lambda B$ • $A(BC)=(AB)C$ • $A(B+C)=AB+AC$

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			• $(A+B)C = AC + BC$ • $(\lambda A)(\mu B) = (\lambda\mu)AB$ • $\lvert AB\rvert = \lvert A\rvert\lvert B\rvert$
	13.2 understand the concept, operations and properties of inverses of square matrices of order 2 and order 3		The properties include: • the inverse of A is unique • $(A^{-1})^{-1} = A$ • $(\lambda A)^{-1} = \lambda^{-1}A^{-1}$ • $(A^n)^{-1} = (A^{-1})^n$ • $(A^T)^{-1} = (A^{-1})^T$ • $\lvert A^{-1}\rvert = \lvert A\rvert^{-1}$ • $(AB)^{-1} = B^{-1}A^{-1}$ where A and B are invertible matrices and λ is a non-zero scalar.

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14. Systems of linear equations	14.1 solve the systems of linear equations in two and three variables by Cramer's rule, inverse matrices and Gaussian elimination	6	The following theorem is required: A system of homogeneous linear equations has nontrivial solutions if and only if the coefficient matrix is singular
15. Introduction to vectors	15.1 understand the concepts of vectors and scalars	5	The concepts of magnitudes of vectors, zero vector and unit vectors are required. Students are required to recognise some common notations of vectors in printed form (including $\mathbf{a}$ and $\overrightarrow{AB}$) and in written form (including $\vec{a}$, $\overrightarrow{AB}$ and $\underline{a}$); and some notations for magnitude (including $\lvert\mathbf{a}\rvert$ and $\lvert\vec{a}\rvert$).
	15.2 understand the operations and properties of vectors		The addition, subtraction and scalar multiplication of vectors are required. The properties include: • $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$ • $\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$ • $\mathbf{a}+\mathbf{0}=\mathbf{a}$ • $0\mathbf{a}=\mathbf{0}$

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			• $\lambda(\mu\mathbf{a}) = (\lambda\mu)\mathbf{a}$ • $(\lambda+\mu)\mathbf{a} = \lambda\mathbf{a}+\mu\mathbf{a}$ • $\lambda(\mathbf{a}+\mathbf{b}) = \lambda\mathbf{a}+\lambda\mathbf{b}$ • If $\alpha\mathbf{a}+\beta\mathbf{b} = \alpha_1\mathbf{a}+\beta_1\mathbf{b}$ (**a** and **b** are non-zero and are not parallel to each other), then $\alpha = \alpha_1$ and $\beta = \beta_1$
	15.3 understand the representation of a vector in the rectangular coordinate system		The formulae that students are required to use include: • $\lvert\overrightarrow{OP}\rvert = \sqrt{x^2+y^2+z^2}$ in $\mathbf{R}^3$ • $\sin\theta = \dfrac{y}{\sqrt{x^2+y^2}}$ and $\cos\theta = \dfrac{x}{\sqrt{x^2+y^2}}$ in $\mathbf{R}^2$ The representation of vectors in the rectangular coordinate system can be used to discuss those properties listed in the Remarks against Learning Objective 15.2. The concept of direction cosines is **not** required.

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16. Scalar product and vector product	16.1 understand the definition and properties of the scalar product (dot product) of vectors	5	The properties include: • $\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}$ • $\mathbf{a}\cdot(\lambda\mathbf{b})=\lambda(\mathbf{a}\cdot\mathbf{b})$ • $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})=\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}$ • $\mathbf{a}\cdot\mathbf{a}=\lvert\mathbf{a}\rvert^2\geq 0$ • $\mathbf{a}\cdot\mathbf{a}=0$ if and only if $\mathbf{a}=\mathbf{0}$ • $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\geq\lvert\mathbf{a}\cdot\mathbf{b}\rvert$ • $\lvert\mathbf{a}-\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2-2(\mathbf{a}\cdot\mathbf{b})$
	16.2 understand the definition and properties of the vector product (cross product) of vectors in $\mathbf{R}^3$		The properties include: • $\mathbf{a}\times\mathbf{a}=\mathbf{0}$ • $\mathbf{b}\times\mathbf{a}=-(\mathbf{a}\times\mathbf{b})$ • $(\mathbf{a}+\mathbf{b})\times\mathbf{c}=\mathbf{a}\times\mathbf{c}+\mathbf{b}\times\mathbf{c}$ • $\mathbf{a}\times(\mathbf{b}+\mathbf{c})=\mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$ • $(\lambda\mathbf{a})\times\mathbf{b}=\mathbf{a}\times(\lambda\mathbf{b})=\lambda(\mathbf{a}\times\mathbf{b})$ • $\lvert\mathbf{a}\times\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2\lvert\mathbf{b}\rvert^2-(\mathbf{a}\cdot\mathbf{b})^2$

Learning Unit	Learning Objective	Time	Remarks
17. Applications of vectors	17.1 understand the applications of vectors	6	Division of a line segment, parallelism and orthogonality are required. Finding angles between two vectors, the projection of a vector onto another vector and the area of a triangle are required.
	Subtotal in hours	34	
Further Learning Unit			
18. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	7	

Grand total: 125 hours