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Candidate Number									

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

PRACTICE PAPER

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question-Answer Book

(2½ hours)
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5 and 7.
2. This paper consists of Section A and Section B.
3. Answer **ALL** questions in Section A. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Answer **ALL** questions in Section B. Write your answers in the other answer book. Start each question (not part of a question) on a new page.
5. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** the book.
6. The Question-Answer book and the answer book will be collected separately at the end of the examination.
7. Unless otherwise specified, all working must be clearly shown.
8. Unless otherwise specified, numerical answers must be exact.
9. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
10. The diagrams in this paper are not necessarily drawn to scale.
11. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (50 marks)
Answer **ALL** questions in this section and write your answers in the spaces provided in this Question-Answer Book.

Answers written in the margins will not be marked.

1. Find the coefficient of x^5 in the expansion of $(2-x)^9$.

(4 marks)

2. Consider the following system of linear equations in x , y , z

$$\begin{cases} x - 7y + 7z = 0 \\ x - ky + 3z = 0 \\ 2x + y + kz = 0 \end{cases}$$, where k is a real number.

If the system has non-trivial solutions, find the two possible values of k .

(4 marks)

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3. Prove by mathematical induction that $4^n + 15n - 1$ is divisible by 9 for all positive integers n.

(5 marks)

4. (a) Let $x = \tan\theta$, show that $\dfrac{2x}{1+x^2} = \sin 2\theta$.

(b) Using (a), find the greatest value of $\dfrac{(1+x)^2}{1+x^2}$, where x is real.

(5 marks)

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5. (a) It is given that $\cos(x+1)+\cos(x-1)=k\cos x$ for any real x. Find the value of k.

(b) Without using a calculator, find the value of $\begin{vmatrix} \cos 1 & \cos 2 & \cos 3 \\ \cos 4 & \cos 5 & \cos 6 \\ \cos 7 & \cos 8 & \cos 9 \end{vmatrix}$.

(6 marks)

6. Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{x}\right)$ from first principles.

(4 marks)

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7. Let $\mathrm{f}(x) = e^{x}(\sin x + \cos x)$.

(a) Find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$.

(b) Find the value of x such that $\mathrm{f}''(x) - \mathrm{f}'(x) + \mathrm{f}(x) = 0$ for $0 \le x \le \pi$.

(5 marks)

8. (a) Using integration by substitution, find $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{4 - x^{2}}}$.

(b) Using integration by parts, find $\displaystyle\int \ln x \, \mathrm{d}x$.

(5 marks)

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9. Find the equations of the two tangents to the curve $x^2 - xy - 2y^2 - 1 = 0$ which are parallel to the straight line $y = 2x + 1$.

(6 marks)

10. (a) Find $\int xe^{-x^2}\,dx$.

(b)

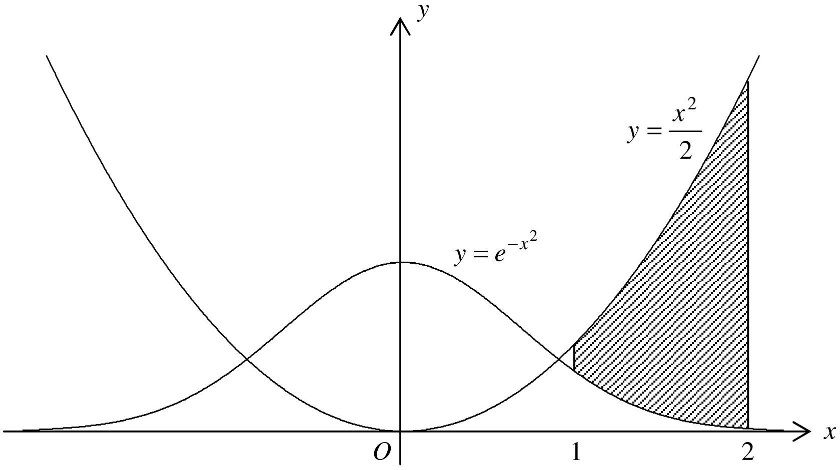

Figure 1

In Figure 1, the shaded region is bounded by the curves $y = \frac{x^2}{2}$ and $y = e^{-x^2}$, where $1 \le x \le 2$. Find the volume of the solid generated by revolving the shaded region about the y-axis.

(6 marks)

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Section B (50 marks)
Answer **ALL** questions in this section and write your answers in the other answer book.

11. Let $A = \begin{pmatrix} \alpha+\beta & -\alpha\beta \\ 1 & 0 \end{pmatrix}$ where α and β are distinct real numbers. Let I be the 2×2 identity matrix.

(a) Show that $A^2 = (\alpha+\beta)A - \alpha\beta I$.

(2 marks)

(b) Using (a), or otherwise, show that $(A-\alpha I)^2 = (\beta-\alpha)(A-\alpha I)$ and $(A-\beta I)^2 = (\alpha-\beta)(A-\beta I)$.

(3 marks)

(c) Let $X = s(A-\alpha I)$ and $Y = t(A-\beta I)$ where s and t are real numbers.
Suppose $A = X+Y$.

(i) Find s and t in terms of α and β .

(ii) For any positive integer n , prove that

$$X^n = \frac{\beta^n}{\beta-\alpha}(A-\alpha I) \quad \text{and} \quad Y^n = \frac{\alpha^n}{\alpha-\beta}(A-\beta I) .$$

(iii) For any positive integer n , express A^n in the form of $pA+qI$, where p and q are real numbers.
[Note: It is known that for any 2×2 matrices H and K ,
if $HK = KH = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, then $(H+K)^n = H^n + K^n$ for any positive integer n .]

(9 marks)

12.

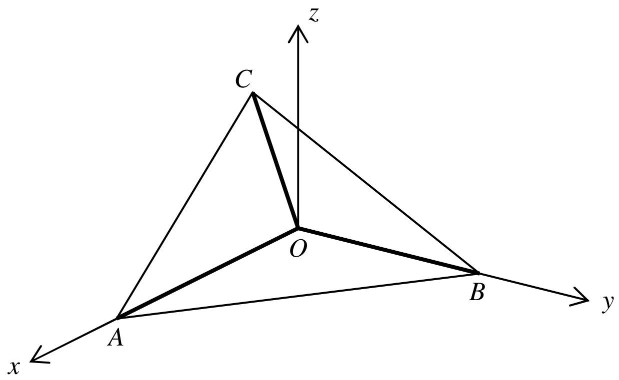

Figure 2

Let $\overrightarrow{OA} = \mathbf{i}$, $\overrightarrow{OB} = \mathbf{j}$ and $\overrightarrow{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ (see Figure 2). Let M and N be points on the straight lines AB and OC respectively such that $AM : MB = a : (1-a)$ and $ON : NC = b : (1-b)$, where $0 < a < 1$ and $0 < b < 1$. Suppose that MN is perpendicular to both AB and OC .

(a) (i) Show that $\overrightarrow{MN} = (a+b-1)\mathbf{i} + (b-a)\mathbf{j} + b\mathbf{k}$.

(ii) Find the values of a and b .

(iii) Find the shortest distance between the straight lines AB and OC .

(8 marks)

(b) (i) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

(ii) Let G be the projection of O on the plane ABC , find the coordinates of the intersecting point of the two straight lines OG and MN .

(5 marks)

13. (a) Let $f(x)$ be an odd function for $-p \le x \le p$, where p is a positive constant.

Prove that $\int_0^{2p} f(x-p)\,dx = 0$.

Hence evaluate $\int_0^{2p} [f(x-p)+q]\,dx$, where q is a constant.

(4 marks)

(b) Prove that $\dfrac{\sqrt{3}+\tan\left(x-\dfrac{\pi}{6}\right)}{\sqrt{3}-\tan\left(x-\dfrac{\pi}{6}\right)} = \dfrac{1+\sqrt{3}\tan x}{2}$.

(2 marks)

(c) Using (a) and (b), or otherwise, evaluate $\int_0^{\frac{\pi}{3}} \ln(1+\sqrt{3}\tan x)\,dx$.

(4 marks)

14. (a)

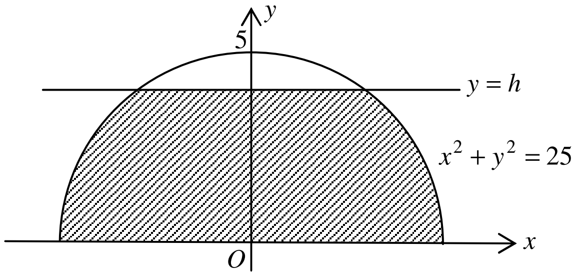

Figure 3

In Figure 3, the shaded region enclosed by the circle $x^2 + y^2 = 25$, the x-axis and the straight line $y = h$ (where $0 \le h \le 5$) is revolved about the y-axis. Show that the volume of the solid of revolution is $\left(25h - \frac{h^3}{3}\right)\pi$.

(2 marks)

(b)

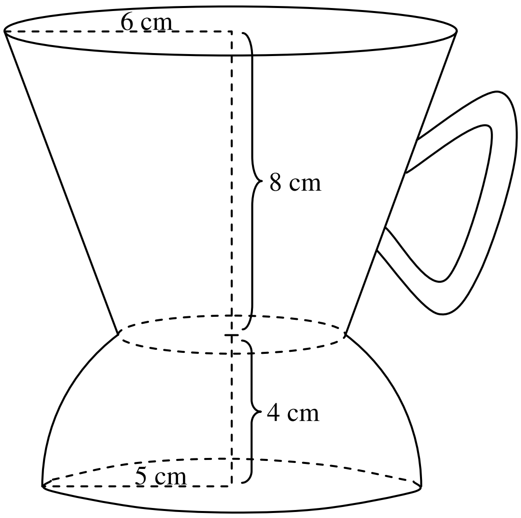

Figure 4

In Figure 4, an empty coffee cup consists of two portions. The lower portion is in the shape of the solid described in (a) with height 4 cm. The upper portion is a frustum of a circular cone. The height of the frustum is 8 cm. The radius of the top of the cup is 6 cm. Hot coffee is poured into the cup to a depth h cm at a rate of 8 $\mathrm{cm}^3\mathrm{s}^{-1}$, where $0 \le h \le 12$. Let V cm^3 be the volume of coffee in the cup.

(i) Find the rate of increase of the depth of coffee when the depth is 3 cm.

(ii) Show that $V = \frac{164\pi}{3} + \frac{3\pi}{64}(h+4)^3$ for $4 \le h \le 12$.

(iii) After the cup is fully filled, suddenly it cracks at the bottom. The coffee leaks at a rate of 2 $\mathrm{cm}^3\mathrm{s}^{-1}$. Find the rate of decrease of the depth of coffee after 15 seconds of leaking, giving your answer correct to 3 significant figures.

(11 marks)

END OF PAPER

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Answers written on this page will not be marked.