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香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2022年香港中學文憑考試
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2022

數學　　延伸部分　　單元二（代數與微積分）

MATHEMATICS　　EXTENDED PART　　MODULE 2 (ALGEBRA AND CALCULUS)

評卷參考
MARKING SCHEME

Hong Kong Diploma of Secondary Education Examination
Mathematics Extended Part Module 2 (Algebra and Calculus)

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

6. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

Solution	Marks	Remarks
1. $g(1+h)-g(1)$		
$=\dfrac{1}{\sqrt{5(1+h)+4}}-\dfrac{1}{\sqrt{5(1)+4}}$		
$=\dfrac{3-\sqrt{5h+9}}{3\sqrt{5h+9}}$		
$=\dfrac{3^2-(5h+9)}{3\sqrt{5h+9}(3+\sqrt{5h+9})}$		
$=\dfrac{-5h}{3\sqrt{5h+9}(3+\sqrt{5h+9})}$	1	
$g'(1)$		
$=\lim\limits_{h\to 0}\dfrac{g(1+h)-g(1)}{h}$	1M	
$=\lim\limits_{h\to 0}\dfrac{1}{h}\left(\dfrac{-5h}{3\sqrt{5h+9}(3+\sqrt{5h+9})}\right)$		
$=\lim\limits_{h\to 0}\dfrac{-5}{3\sqrt{5h+9}(3+\sqrt{5h+9})}$	1M	withhold 1M if this step is skipped
$=\dfrac{-5}{54}$	1A	
	----------(4)	
2. (a) $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$		
$=\dfrac{\tan^2\theta}{\tan\theta-1}+\dfrac{1}{\tan\theta(1-\tan\theta)}$	1M	
$=\dfrac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$		
$=\dfrac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta(\tan\theta-1)}$	1M	
$=\dfrac{\sec^2\theta+\tan\theta}{\tan\theta}$		
$=1+\dfrac{\sec^2\theta}{\tan\theta}$		
$=1+\sec\theta\csc\theta$	1	
(b) $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=5$		
$1+\sec\theta\csc\theta=5$		
$\sin\theta\cos\theta=\dfrac{1}{4}$		
$\sin 2\theta=\dfrac{1}{2}$	1M	
$2\theta=\dfrac{5\pi}{6}$		
$\theta=\dfrac{5\pi}{12}$	1A	
	----------(5)	

Solution	Marks	Remarks
3. (a) For $n=1$,		
L.H.S. $=(-1)(1^2)+(-1)^2(2^2)=3$		
R.H.S. $=(1)(2+1)=3$		
Therefore, the statement is true for $n=1$.	1	
Assume that $\sum_{k=1}^{2m}(-1)^k k^2=m(2m+1)$, where m is a positive integer.	1M	
$\sum_{k=1}^{2(m+1)}(-1)^k k^2$		
$=\sum_{k=1}^{2m}(-1)^k k^2-(2m+1)^2+(2m+2)^2$		
$=m(2m+1)-(2m+1)^2+(2m+2)^2$ (by induction assumption)	1M	for using induction assumption
$=2m^2+5m+3$		
$=(m+1)(2m+3)$		
So, the statement is true for $n=m+1$ if it is true for $n=m$.		
By mathematical induction, the statement is true for all positive integers n.	1	
(b) $\sum_{k=11}^{100}(-1)^k k^2$		
$=\sum_{k=1}^{100}(-1)^k k^2-\sum_{k=1}^{10}(-1)^k k^2$	1M	
$=50(2(50)+1)-5(2(5)+1)$	1M	
$=4995$	1A	
	----------(7)	

Solution	Marks	Remarks
4. (a) $\frac{dy}{dx}$		
$= (7-4x)e^{-x} + (7x-2x^2)(e^{-x})(-1)$	1M	
$= (2x^2-11x+7)e^{-x}$	1A	
$\frac{d^2y}{dx^2}$		
$= (4x-11)e^{-x} + (2x^2-11x+7)(e^{-x})(-1)$		
$= (-2x^2+15x-18)e^{-x}$	1A	
(b) $\frac{d^2y}{dx^2} = 0$		
$(-2x^2+15x-18)e^{-x} = 0$		
$-2x^2+15x-18 = 0$		
$x = \frac{3}{2}$ or $x = 6$	1M	
(see table below)	1M	for testing
Therefore, there are two points of inflexion of the graph of $y = (7x-2x^2)e^{-x}$. Thus, the claim is agreed.	1A	f.t.
	----------(6)	

x	$\left(-\infty, \frac{3}{2}\right)$	$\frac{3}{2}$	$\left(\frac{3}{2}, 6\right)$	6	$(6, \infty)$
$\frac{d^2y}{dx^2}$	$-$	0	$+$	0	$-$

Solution	Marks	Remarks
5. (a) $(a+x)^n$		
$= a^n + C_1^n a^{n-1}x + C_2^n a^{n-2}x^2 + \cdots + x^n$		
$= a^n + a^{n-1}nx + \dfrac{a^{n-2}n(n-1)}{2}x^2 + \cdots + x^n$	1M	
$\mu_2 = -10$		
$\dfrac{a^{n-2}n(n-1)}{2} = -10$	1M	
$a^{n-2}n(n-1) = -20$		
Since n is greater than 1, a^{n-2} is a negative number. Thus, a is a negative number and n is an odd number.	1	
(b) $(bx-1)^n$		
$= (-1)^n + C_1^n(-1)^{n-1}(bx) + C_2^n(-1)^{n-2}(bx)^2 + \cdots + (bx)^n$		
$= (-1)^n + (-1)^{n-1}bnx + (-1)^{n-2}\dfrac{b^2n(n-1)}{2}x^2 + \cdots + b^nx^n$		
$\lambda_0 = \mu_0$		
$(-1)^n = a^n$		
$a = -1$	1A	
$\lambda_1 = 2\mu_1$		
$(-1)^{n-1}bn = 2a^{n-1}n$		
$b = 2$	1A	
$a^{n-2}n(n-1) = -20$		
$(-1)^{n-2}n(n-1) = -20$		
$n(n-1) = 20$ (since n is an odd number)		
$n^2 - n - 20 = 0$		
$(n-5)(n+4) = 0$		
$n = 5$ or $n = -4$ (rejected)	1A	
	------------(6)	

Solution	Marks	Remarks
6. (a) Note that $x^2+2x+5\equiv(x+1)^2+2^2$.		
Let $x+1=2\tan\theta$.	1M	
So, we have $\dfrac{dx}{d\theta}=2\sec^2\theta$.		
$\int\frac{1}{x^2+2x+5}dx$		
$=\int\frac{1}{4\tan^2\theta+4}(2\sec^2\theta)d\theta$	1M	
$=\int\frac{1}{2}d\theta$		
$=\frac{1}{2}\theta+\text{constant}$		
$=\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)+\text{constant}$	1	
(b) y		
$=\int\frac{2x+1}{x^2+2x+5}dx$		
$=\int\frac{2x+2}{x^2+2x+5}dx-\int\frac{1}{x^2+2x+5}dx$		
$=\int\frac{1}{x^2+2x+5}d(x^2+2x+5)-\int\frac{1}{x^2+2x+5}dx$	1M	
$=\ln\lvert x^2+2x+5\rvert-\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)+C$, where C is a constant	1M	
When $x=-3$, $y=\ln 2$.		
$\ln 2=\ln\lvert(-3)^2+2(-3)+5\rvert-\frac{1}{2}\tan^{-1}\left(\frac{(-3)+1}{2}\right)+C$		
$\ln 2=\ln 8-(\frac{1}{2})(\frac{-\pi}{4})+C$	1M	
$C=-2\ln 2-\frac{\pi}{8}$		
The equation of G is $y=\ln\lvert x^2+2x+5\rvert-\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)-2\ln 2-\frac{\pi}{8}$.		
When $x=-1$,		
y		
$=\ln\lvert(-1)^2+2(-1)+5\rvert-\frac{1}{2}\tan^{-1}\left(\frac{(-1)+1}{2}\right)-2\ln 2-\frac{\pi}{8}$		
$=\frac{-\pi}{8}$		
Thus, G passes through the point $\left(-1,\frac{-\pi}{8}\right)$.	1A	f.t.
	----------(7)	

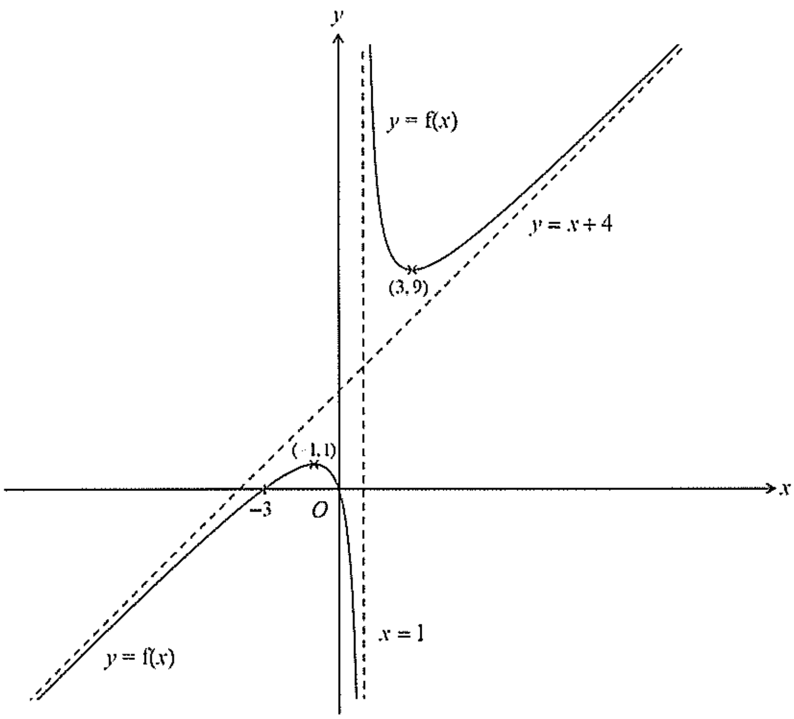
Solution	Marks	Remarks
7. (a) $\dfrac{dy}{dx}=\dfrac{1}{x+2}$		
The equation of L is $y=\dfrac{1}{h+2}(x-h)+\ln(h+2)$.	1M	
A $=\displaystyle\int_0^h\left(\frac{1}{h+2}(x-h)+\ln(h+2)-\ln(x+2)\right)dx$	1M	
$=\left[\dfrac{1}{h+2}\left(\dfrac{1}{2}x^2-hx\right)+x\ln(h+2)\right]_0^h-[x\ln(x+2)]_0^h+\displaystyle\int_0^h\frac{x}{x+2}dx$		
$=\dfrac{1}{h+2}\left(\dfrac{1}{2}h^2-h^2\right)+h\ln(h+2)-h\ln(h+2)+\displaystyle\int_0^h\left(1-\frac{2}{x+2}\right)dx$	1M	
$=\dfrac{-h^2}{2h+4}+\left[x-2\ln\lvert x+2\rvert\right]_0^h$		
$=\dfrac{h^2+4h}{2h+4}-2\ln(h+2)+2\ln 2$	1	
(b) $h=3^{-t}=e^{-t\ln 3}$ $\dfrac{dh}{dt}=(-\ln 3)e^{-t\ln 3}=(-\ln 3)3^{-t}$	1M	
$\dfrac{dA}{dh}$ $=\dfrac{(2h+4)(2h+4)-(h^2+4h)(2)}{(2h+4)^2}-\dfrac{2}{h+2}$	1M	
$=\dfrac{h^2}{2(h+2)^2}$		
$\dfrac{dA}{dt}$ $=\dfrac{dA}{dh}\dfrac{dh}{dt}$		
$=\dfrac{(3^{-t})^2}{2(3^{-t}+2)^2}(-\ln 3)3^{-t}$	1M	
$=\dfrac{-\ln 3}{2(3^t)(1+2(3^t))^2}$		
$\left.\dfrac{dA}{dt}\right\vert_{t=1}$ $=\dfrac{-\ln 3}{2(3^1)(1+2(3^1))^2}$		
$=\dfrac{-\ln 3}{294}$	1A	
Thus, the rate of change of A is $\dfrac{-\ln 3}{294}$ when $t=1$.		
	----------(8)	

Solution	Marks	Remarks
8. (a) Δ $= \begin{vmatrix} a & 2 & -1 \\ -1 & a & 2 \\ 2 & -1 & a \end{vmatrix}$ $= a^3 + 6a + 7$	1M	
Δ_y $= \begin{vmatrix} a & 4k & -1 \\ -1 & 4 & 2 \\ 2 & k^2 & a \end{vmatrix}$ $= 4a^2 + 4ak - 2ak^2 + k^2 + 16k + 8$		
Note that $\Delta \neq 0$. y $= \dfrac{\Delta_y}{\Delta}$ $= \dfrac{4a^2 + 4ak - 2ak^2 + k^2 + 16k + 8}{a^3 + 6a + 7}$	1M+1A	1M for Cramer's rule
(b) As (E) has infinitely many solutions, $\Delta = 0$ $a^3 + 6a + 7 = 0$	1M	
$(a+1)(a^2 - a + 7) = 0$ $a = -1$		
The augmented matrix of (E) becomes $\left(\begin{array}{ccc\|c} -1 & 2 & -1 & 4k \\ -1 & -1 & 2 & 4 \\ 2 & -1 & -1 & k^2 \end{array}\right) \sim \left(\begin{array}{ccc\|c} -1 & 2 & -1 & 4k \\ 0 & -3 & 3 & 4-4k \\ 0 & 0 & 0 & k^2+4k+4 \end{array}\right)$	1M	
$k^2 + 4k + 4 = 0$	1M	
$(k+2)^2 = 0$ $k = -2$		either one
Hence, the augmented matrix of (E) becomes $\left(\begin{array}{ccc\|c} -1 & 2 & -1 & -8 \\ 0 & -3 & 3 & 12 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc\|c} -1 & 2 & -1 & -8 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right)$		
Thus, the solution set of (E) is $\{(t, t-4, t) : t \in \mathbf{R}\}$.	1A -----------(7)	

Solution	Marks	Remarks
9. (a) The equation of the vertical asymptote is $x-1=0$.	1A	
Note that $\dfrac{x^2+3x}{x-1}=x+4+\dfrac{4}{x-1}$.	1M	
Thus, the equation of the oblique asymptote is $y=x+4$.	1A	
	----------(3)	
(b) $f'(x)=1-\dfrac{4}{(x-1)^2}$	1M	
$f'(x)=0$ $1-\dfrac{4}{(x-1)^2}=0$ $x^2-2x-3=0$ $x=-1$ or $x=3$	1M	
(table below)	1M	
Thus, the maximum point and minimum point of H are $(-1, 1)$ and $(3, 9)$ respectively.	1A	
	----------(4)	

x	$(-\infty, -1)$	-1	$(-1, 1)$	$(1, 3)$	3	$(3, \infty)$
$f'(x)$	$+$	0	$-$	$-$	0	$+$

(c)



Solution	Marks	Remarks
	1M	for shape
	1M	for asymptotes
	1A	for all correct
	----------(3)	

Solution	Marks	Remarks
(d) $f(x)=10$		
$\frac{x^2+3x}{x-1}=10$		
$x^2-7x+10=0$		
$x=2$ or $x=5$		
The required volume		
$=\pi\int_2^5(f(x)-10)^2\,dx$	1M+1A	
$=\pi\int_2^5\left(x+4+\frac{4}{x-1}-10\right)^2 dx$		
$=\pi\int_2^5\left(x-6+\frac{4}{x-1}\right)^2 dx$		
$=\pi\int_2^5\left((x-6)^2+\frac{8(x-6)}{x-1}+\frac{16}{(x-1)^2}\right)dx$		
$=\pi\int_2^5\left((x-6)^2+8-\frac{40}{x-1}+\frac{16}{(x-1)^2}\right)dx$		
$=\pi\left[\frac{(x-6)^3}{3}+8x-40\ln\lvert x-1\rvert-\frac{16}{x-1}\right]_2^5$		
$=\pi(57-80\ln 2)$	1A	
	-----------(3)	

Solution	Marks	Remarks
10. (a) $\int g(x)\,dx$		
$=\frac{\sin 2x\cos^2 x}{2}-\int \frac{\sin 2x}{2}(2\cos x)(-\sin x)\,dx$	1M	
$=\frac{\sin 2x\cos^2 x}{2}+\frac{1}{2}\int \sin^2 2x\,dx$	1	
$\frac{d}{dx}\left(\frac{\sin 2x\cos^2 x}{2}\right)$		
$=\frac{1}{2}(\cos 2x)(2)(\cos^2 x)+\frac{1}{2}(\sin 2x)(2\cos x)(-\sin x)$		
$=\cos^2 x\cos 2x-\frac{1}{2}\sin^2 2x$		
$\therefore \cos^2 x\cos 2x=\frac{d}{dx}\left(\frac{\sin 2x\cos^2 x}{2}\right)+\frac{1}{2}\sin^2 2x$	1M	
$\int \cos^2 x\cos 2x\,dx=\int\left(\frac{d}{dx}\left(\frac{\sin 2x\cos^2 x}{2}\right)+\frac{1}{2}\sin^2 2x\right)dx$		
$\int g(x)\,dx=\frac{\sin 2x\cos^2 x}{2}+\frac{1}{2}\int \sin^2 2x\,dx$	1	
	----------(2)	
(b) $\int_0^{\pi} g(x)\,dx$		
$=\left[\frac{\sin 2x\cos^2 x}{2}\right]_0^{\pi}+\frac{1}{2}\int_0^{\pi}\sin^2 2x\,dx$ (by (a))	1M	1M for using (a)
$=\left[\frac{\sin 2x\cos^2 x}{2}\right]_0^{\pi}+\frac{1}{2}\int_0^{\pi}\frac{1-\cos 4x}{2}\,dx$		
$=0+\frac{1}{4}\left[x-\frac{\sin 4x}{4}\right]_0^{\pi}$		
$=\frac{\pi}{4}$	1A	
	----------(2)	

Solution	Marks	Remarks
(c) Let $u=\pi-x$.	1M	
$\int_0^\pi xg(x)\,dx$		
$=\int_\pi^0 -(\pi-u)\cos^2(\pi-u)\cos 2(\pi-u)\,du$	1M	
$=\int_0^\pi (\pi-u)\cos^2 u\cos 2u\,du$		
$=\int_0^\pi (\pi-x)\cos^2 x\cos 2x\,dx$		
$=\pi\int_0^\pi g(x)\,dx-\int_0^\pi xg(x)\,dx$		
$\int_0^\pi xg(x)\,dx$		
$=\frac{\pi}{2}\int_0^\pi g(x)\,dx$	1M	
$=\frac{\pi}{2}\left(\frac{\pi}{4}\right)$		
$=\frac{\pi^2}{8}$	1A	
	----------(4)	

Solution	Marks	Remarks
(d) $(-x)\mathrm{g}(-x)$		
$=(-x)\cos^2(-x)\cos 2(-x)$		
$=-x\cos^2 x\cos 2x$		
$=-x\mathrm{g}(x)$		
$\therefore$ $x\mathrm{g}(x)$ is an odd function.	1M	
So, we have $\int_{-\pi}^{\pi} x\mathrm{g}(x)\,\mathrm{d}x=0$.		
Let $y=x-\pi$.	1M	
$\int_{\pi}^{2\pi} x\mathrm{g}(x)\,\mathrm{d}x$		
$=\int_0^{\pi}(\pi+y)\cos^2(\pi+y)\cos 2(\pi+y)\,\mathrm{d}y$		
$=\int_0^{\pi}(\pi+y)\cos^2 y\cos 2y\,\mathrm{d}y$		
$=\int_0^{\pi}(\pi+x)\cos^2 x\cos 2x\,\mathrm{d}x$		
$=\pi\int_0^{\pi}\mathrm{g}(x)\,\mathrm{d}x+\int_0^{\pi}x\mathrm{g}(x)\,\mathrm{d}x$		
$=\pi\left(\frac{\pi}{4}\right)+\frac{\pi^2}{8}$ (by (b) and (c))	1M	
$=\frac{3\pi^2}{8}$		
$\int_{-\pi}^{2\pi} x\mathrm{g}(x)\,\mathrm{d}x$		
$=\int_{-\pi}^{\pi} x\mathrm{g}(x)\,\mathrm{d}x+\int_{\pi}^{2\pi} x\mathrm{g}(x)\,\mathrm{d}x$		
$=0+\frac{3\pi^2}{8}$		
$=\frac{3\pi^2}{8}$	1A	
	----------(4)	

Solution	Marks	Remarks
11. (a) (i) $(I-A)(I+A+A^2+\cdots+A^n)$		
$=I+A+A^2+\cdots+A^n-(A+A^2+A^3+\cdots+A^{n+1})$		
$=I-A^{n+1}$	1A	
(ii) (1) $I-A$		
$=\begin{pmatrix}1&0\\0&1\end{pmatrix}-\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}$		
$=\begin{pmatrix}1-\cos\theta&\sin\theta\\-\sin\theta&1-\cos\theta\end{pmatrix}$		
$\lvert I-A\rvert$		
$=(1-\cos\theta)^2+\sin^2\theta$		
$=2(1-\cos\theta)$	1M	
$(I-A)^{-1}$		
$=\dfrac{1}{2(1-\cos\theta)}\begin{pmatrix}1-\cos\theta&-\sin\theta\\\sin\theta&1-\cos\theta\end{pmatrix}$	1M	
$=\dfrac{1}{2}\begin{pmatrix}1&\dfrac{-\sin\theta}{1-\cos\theta}\\\dfrac{\sin\theta}{1-\cos\theta}&1\end{pmatrix}$		
$=\dfrac{1}{2}\begin{pmatrix}1&\dfrac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}\\\dfrac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}&1\end{pmatrix}$		
$=\dfrac{1}{2\sin\frac{\theta}{2}}\begin{pmatrix}\sin\frac{\theta}{2}&-\cos\frac{\theta}{2}\\\cos\frac{\theta}{2}&\sin\frac{\theta}{2}\end{pmatrix}$	1	

Solution	Marks	Remarks
(2) $I+A+A^2+\cdots+A^n$		
$=(I-A)^{-1}(I-A^{n+1})$	1M	
$=\dfrac{1}{2\sin\frac{\theta}{2}}\begin{pmatrix}\sin\frac{\theta}{2} & -\cos\frac{\theta}{2}\\ \cos\frac{\theta}{2} & \sin\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}1-\cos(n+1)\theta & \sin(n+1)\theta\\ -\sin(n+1)\theta & 1-\cos(n+1)\theta\end{pmatrix}$		
$=\dfrac{1}{2\sin\frac{\theta}{2}}\begin{pmatrix}\sin\frac{\theta}{2} & -\cos\frac{\theta}{2}\\ \cos\frac{\theta}{2} & \sin\frac{\theta}{2}\end{pmatrix}\left(2\sin\frac{(n+1)\theta}{2}\right)\begin{pmatrix}\sin\frac{(n+1)\theta}{2} & \cos\frac{(n+1)\theta}{2}\\ -\cos\frac{(n+1)\theta}{2} & \sin\frac{(n+1)\theta}{2}\end{pmatrix}$	1M	
$=\dfrac{\sin\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}}\begin{pmatrix}\sin\frac{\theta}{2}\sin\frac{(n+1)\theta}{2}+\cos\frac{\theta}{2}\cos\frac{(n+1)\theta}{2} & \sin\frac{\theta}{2}\cos\frac{(n+1)\theta}{2}-\cos\frac{\theta}{2}\sin\frac{(n+1)\theta}{2}\\ \cos\frac{\theta}{2}\sin\frac{(n+1)\theta}{2}-\sin\frac{\theta}{2}\cos\frac{(n+1)\theta}{2} & \cos\frac{\theta}{2}\cos\frac{(n+1)\theta}{2}+\sin\frac{\theta}{2}\sin\frac{(n+1)\theta}{2}\end{pmatrix}$		
$=\dfrac{\sin\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}}\begin{pmatrix}\cos\frac{n\theta}{2} & -\sin\frac{n\theta}{2}\\ \sin\frac{n\theta}{2} & \cos\frac{n\theta}{2}\end{pmatrix}$	1	
	----------(7)	
(b) (i) $I+A+A^2+\cdots+A^n$		
$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}+\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}^2+\cdots+\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}^n$		
$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}+\begin{pmatrix}\cos2\theta & -\sin2\theta\\ \sin2\theta & \cos2\theta\end{pmatrix}+\cdots+\begin{pmatrix}\cos n\theta & -\sin n\theta\\ \sin n\theta & \cos n\theta\end{pmatrix}$		
$=\begin{pmatrix}1+\cos\theta+\cos2\theta+\cdots+\cos n\theta & -\sin\theta-\sin2\theta-\cdots-\sin n\theta\\ \sin\theta+\sin2\theta+\cdots+\sin n\theta & 1+\cos\theta+\cos2\theta+\cdots+\cos n\theta\end{pmatrix}$	1M	
$\therefore 1+\cos\theta+\cos2\theta+\cdots+\cos n\theta=\dfrac{\sin\frac{(n+1)\theta}{2}\cos\frac{n\theta}{2}}{\sin\frac{\theta}{2}}$	1M	
Putting $\theta=\frac{5\pi}{18}$ and $n=90$,		
$\cos\frac{5\pi}{18}+\cos\frac{5\pi}{9}+\cos\frac{5\pi}{6}+\cdots+\cos25\pi$		
$=\dfrac{\sin\frac{445\pi}{36}\cos\frac{25\pi}{2}}{\sin\frac{5\pi}{36}}-1$		
$=-1$	1A	

Solution	Marks	Remarks
(ii) $\cos^2\frac{\pi}{7}+\cos^2\frac{2\pi}{7}+\cos^2\frac{3\pi}{7}+\cdots+\cos^2 7\pi$		
$=\frac{1}{2}\left(1+\cos\frac{2\pi}{7}\right)+\frac{1}{2}\left(1+\cos\frac{4\pi}{7}\right)+\frac{1}{2}\left(1+\cos\frac{6\pi}{7}\right)+\cdots$		
$+\frac{1}{2}(1+\cos 14\pi)$		
$=\frac{1}{2}\left(49+\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}+\cdots+\cos 14\pi\right)$	1M	
$\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}+\cdots+\cos 14\pi$		
$=\frac{\sin\frac{50\pi}{7}\cos 7\pi}{\sin\frac{\pi}{7}}-1$ (putting $\theta=\frac{2\pi}{7}$ and $n=49$ in (b)(i))	1M	
$=0$		
$\therefore \cos^2\frac{\pi}{7}+\cos^2\frac{2\pi}{7}+\cos^2\frac{3\pi}{7}+\cdots+\cos^2 7\pi=\frac{49}{2}$	1A	
	----------(6)	

Solution	Marks	Remarks
12. (a) (i) $\overrightarrow{OD}$		
$=\frac{b}{b+c}\overrightarrow{OB}+\frac{c}{b+c}\overrightarrow{OC}$	1M	
$\overrightarrow{AD}$		
$=\overrightarrow{OD}-\overrightarrow{OA}$		
$=\frac{b}{b+c}\overrightarrow{OB}+\frac{c}{b+c}\overrightarrow{OC}-\overrightarrow{OA}$		
$=-\overrightarrow{OA}+\frac{b}{b+c}\overrightarrow{OB}+\frac{c}{b+c}\overrightarrow{OC}$	1	
(ii) $\overrightarrow{AJ}$		
$=\overrightarrow{OJ}-\overrightarrow{OA}$		
$=\frac{a}{a+b+c}\overrightarrow{OA}+\frac{b}{a+b+c}\overrightarrow{OB}+\frac{c}{a+b+c}\overrightarrow{OC}-\overrightarrow{OA}$		
$=\frac{-(b+c)}{a+b+c}\overrightarrow{OA}+\frac{b}{a+b+c}\overrightarrow{OB}+\frac{c}{a+b+c}\overrightarrow{OC}$		
$=\left(\frac{b+c}{a+b+c}\right)\left(-\overrightarrow{OA}+\frac{b}{b+c}\overrightarrow{OB}+\frac{c}{b+c}\overrightarrow{OC}\right)$	1M	
$=\frac{b+c}{a+b+c}\overrightarrow{AD}$		
Since $0<\frac{b+c}{a+b+c}<1$,	1M	withhold 1M if this step is skipped
J lies on AD.	1	
$\overrightarrow{OE}$		
$=\frac{a}{a+c}\overrightarrow{OA}+\frac{c}{a+c}\overrightarrow{OC}$		
$\overrightarrow{BE}$		
$=\overrightarrow{OE}-\overrightarrow{OB}$		
$=\frac{a}{a+c}\overrightarrow{OA}-\overrightarrow{OB}+\frac{c}{a+c}\overrightarrow{OC}$		
$\overrightarrow{BJ}$		
$=\overrightarrow{OJ}-\overrightarrow{OB}$		
$=\frac{a}{a+b+c}\overrightarrow{OA}+\frac{b}{a+b+c}\overrightarrow{OB}+\frac{c}{a+b+c}\overrightarrow{OC}-\overrightarrow{OB}$		
$=\frac{a}{a+b+c}\overrightarrow{OA}-\frac{a+c}{a+b+c}\overrightarrow{OB}+\frac{c}{a+b+c}\overrightarrow{OC}$		
$=\left(\frac{a+c}{a+b+c}\right)\left(\frac{a}{a+c}\overrightarrow{OA}-\overrightarrow{OB}+\frac{c}{a+c}\overrightarrow{OC}\right)$		
$=\frac{a+c}{a+b+c}\overrightarrow{BE}$	1M	
Since $0<\frac{a+c}{a+b+c}<1$, J lies on BE.		
Thus, AD and BE intersect at J.	1	
	----------(7)	

Solution	Marks	Remarks
(b) (i) Define $BC = a$, $AC = b$ and $AB = c$.		
a $= \left\vert \overrightarrow{OC} - \overrightarrow{OB} \right\vert$ $= \left\vert (-3\mathbf{j}+\mathbf{k}) - (40\mathbf{i} - 3\mathbf{j} + \mathbf{k}) \right\vert$ $= \left\vert -40\mathbf{i} \right\vert$ $= 40$	1M	any one
b $= \left\vert \overrightarrow{OC} - \overrightarrow{OA} \right\vert$ $= \left\vert (-3\mathbf{j}+\mathbf{k}) - (35\mathbf{i} + 9\mathbf{j} + \mathbf{k}) \right\vert$ $= \left\vert -35\mathbf{i} - 12\mathbf{j} \right\vert$ $= 37$		
c $= \left\vert \overrightarrow{OB} - \overrightarrow{OA} \right\vert$ $= \left\vert (40\mathbf{i} - 3\mathbf{j}+\mathbf{k}) - (35\mathbf{i} + 9\mathbf{j} + \mathbf{k}) \right\vert$ $= \left\vert 5\mathbf{i} - 12\mathbf{j} \right\vert$ $= 13$		
$\overrightarrow{OI}$ $= \dfrac{40}{40+37+13}\overrightarrow{OA} + \dfrac{37}{40+37+13}\overrightarrow{OB} + \dfrac{13}{40+37+13}\overrightarrow{OC}$		by putting $J = I$ in (a)(ii)
$= 32\mathbf{i} + \dfrac{7}{3}\mathbf{j} + \mathbf{k}$	1A	

Solution	Marks	Remarks
(ii) $\overrightarrow{AI}$		
$=\overrightarrow{OI}-\overrightarrow{OA}$		
$=\left(32\mathbf{i}+\dfrac{7}{3}\mathbf{j}+\mathbf{k}\right)-(35\mathbf{i}+9\mathbf{j}+\mathbf{k})$		
$=-3\mathbf{i}-\dfrac{20}{3}\mathbf{j}$		
$\overrightarrow{AI}\times\overrightarrow{AB}$		
$=\left(-3\mathbf{i}-\dfrac{20}{3}\mathbf{j}\right)\times(5\mathbf{i}-12\mathbf{j})$		
$=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -3 & \dfrac{-20}{3} & 0\\ 5 & -12 & 0\end{vmatrix}$		
$=\dfrac{208}{3}\mathbf{k}$	1M	
The radius of the inscribed circle		
$=AI\sin\angle BAI$		
$=\dfrac{(AI)(AB)\sin\angle BAI}{AB}$		
$=\dfrac{\left\lvert\overrightarrow{AI}\times\overrightarrow{AB}\right\rvert}{c}$	1M	
$=\dfrac{16}{3}$	1A	
	-----------(5)	