Solution	Marks	Remarks
$f(1+h)$		
$=((1+h)^2-1)e^{1+h}$		
$=(2h+h^2)e^{1+h}$	1A	
$f'(1)$		
$=\lim_{h\to 0}\dfrac{f(1+h)-f(1)}{h}$	1M	
$=\lim_{h\to 0}\dfrac{(2h+h^2)e^{1+h}-0}{h}$		
$=\lim_{h\to 0}(2+h)e^{1+h}$	1M	withhold 1M if this step is skipped
$=2e$	1A	
	----------(4)	
$(x+3)^5$		
$=x^5+5(3)x^4+10(3^2)x^3+10(3^3)x^2+5(3^4)x+3^5$	1M	
$=x^5+15x^4+90x^3+270x^2+405x+243$	1A	
$\left(x-\dfrac{4}{x}\right)^2$		
$=x^2-2x\left(\dfrac{4}{x}\right)+\left(\dfrac{4}{x}\right)^2$	1M	
$=x^2-8+\dfrac{16}{x^2}$		
The coefficient of x^3		
$=(1)(16)+(90)(-8)+(405)(1)$	1M	withhold 1M if this step is skipped
$=-299$	1A	
	----------(5)	

Solution	Marks	Remarks
3. (a) $\cot A = 3\cot B$		
$\frac{\cos A}{\sin A} = \frac{3\cos B}{\sin B}$		
$3\sin A\cos B = \cos A\sin B$	1M	
$\sin(A+B) - 2\sin(B-A)$		
$= (\sin A\cos B + \cos A\sin B) - 2(\sin B\cos A - \cos B\sin A)$		
$= 3\sin A\cos B - \cos A\sin B$		
$= 0$		
Thus, we have $\sin(A+B) = 2\sin(B-A)$.	1	
(b) $\cot\left(x+\frac{4\pi}{9}\right) = 3\cot\left(x+\frac{5\pi}{18}\right)$		
By letting $A = x+\frac{4\pi}{9}$ and $B = x+\frac{5\pi}{18}$, we have $\cot A = 3\cot B$.	1M	
By (a), we have $\sin(A+B) = 2\sin(B-A)$.		
With the help of $\sin\left(\frac{-\pi}{6}\right) = \frac{-1}{2}$, we have $\sin\left(2x+\frac{13\pi}{18}\right) = -1$.	1M	
Noting that $0 \le x \le \frac{\pi}{2}$, we have $x = \frac{7\pi}{18}$.	1A	
Since $\cot\left(\frac{7\pi}{18}+\frac{4\pi}{9}\right) = -\sqrt{3} = 3\cot\left(\frac{7\pi}{18}+\frac{5\pi}{18}\right)$, the required solution of the equation $\cot\left(x+\frac{4\pi}{9}\right) = 3\cot\left(x+\frac{5\pi}{18}\right)$ is $x = \frac{7\pi}{18}$.		
	-----------(5)	
4. (a) $\int u(5^u)\,\mathrm{d}u$		
$= \frac{1}{\ln 5}\left(u(5^u) - \int 5^u\,\mathrm{d}u\right)$	1M	
$= \frac{1}{\ln 5}\left(u(5^u) - \frac{5^u}{\ln 5}\right) + \text{constant}$	1A	
$= \frac{5^u(u\ln 5 - 1)}{(\ln 5)^2} + \text{constant}$		
(b) The required area		
$= \int_0^1 x(5^{2x})\,\mathrm{d}x$	1M	
$= \frac{1}{4}\int_0^2 u(5^u)\,\mathrm{d}u$ (by letting $u = 2x$)	1M	
$= \frac{1}{4(\ln 5)^2}\left[5^u(u\ln 5 - 1)\right]_0^2$ (by (a))	1M	for using the result of (a)
$= \frac{50\ln 5 - 24}{4(\ln 5)^2}$		
$= \frac{25\ln 5 - 12}{2(\ln 5)^2}$	1A	
	-----------(6)	

Solution	Marks	Remarks
(a) Let $u=1+x^2$.	1M	
Then, we have $\frac{du}{dx}=2x$.		
$\int x^3\sqrt{1+x^2}\,dx$		
$=\int \frac{1}{2}(u-1)u^{\frac{1}{2}}\,du$	1M	
$=\frac{1}{2}\left(\int u^{\frac{3}{2}}\,du-\int u^{\frac{1}{2}}\,du\right)$		
$=\frac{1}{5}\left(\sqrt{1+x^2}\right)^5-\frac{1}{3}\left(\sqrt{1+x^2}\right)^3+\text{constant}$	1A	
Let $x=\tan\theta$.	1M	
Then, we have $\frac{dx}{d\theta}=\sec^2\theta$.		
$\int x^3\sqrt{1+x^2}\,dx$		
$=\int \tan^3\theta\sec\theta(\sec^2\theta)\,d\theta$	1M	
$=\int \tan^3\theta\sec^3\theta\,d\theta$		
$=\int (\sec^2\theta-1)\sec^2\theta\,d\sec\theta$		
$=\int \sec^4\theta\,d\sec\theta-\int\sec^2\theta\,d\sec\theta$		
$=\frac{\sec^5\theta}{5}-\frac{\sec^3\theta}{3}+\text{constant}$		
$=\frac{1}{5}\left(\sqrt{1+x^2}\right)^5-\frac{1}{3}\left(\sqrt{1+x^2}\right)^3+\text{constant}$	1A	
(b) y		
$=\int 15x^3\sqrt{1+x^2}\,dx$	1M	
$=15\int x^3\sqrt{1+x^2}\,dx$		
$=15\left(\frac{1}{5}\left(\sqrt{1+x^2}\right)^5-\frac{1}{3}\left(\sqrt{1+x^2}\right)^3\right)+C$ (by (a))	1M	for using the result of (a)
$=3\left(\sqrt{1+x^2}\right)^5-5\left(\sqrt{1+x^2}\right)^3+C$, where C is a constant		
Since the y-intercept of Γ is 2, we have $3-5+C=2$. Solving, we have $C=4$.	1M	
Thus, the equation of Γ is $y=3\left(\sqrt{1+x^2}\right)^5-5\left(\sqrt{1+x^2}\right)^3+4$.	1A	
	----------(7)	

6\. (a) Note that $(1)(1+4)=5=\frac{(1)(2)(15)}{6}$. — 1

So, the statement is true for $n=1$.

Assume that $\sum_{k=1}^{m} k(k+4)=\frac{m(m+1)(2m+13)}{6}$ for some positive integer m . — 1M

$$\sum_{k=1}^{m+1} k(k+4)$$

$$=\sum_{k=1}^{m} k(k+4)+(m+1)(m+5)$$

$$=\frac{m(m+1)(2m+13)}{6}+(m+1)(m+5) \quad \text{(by induction assumption)}$$ — 1M (for using induction assumption)

$$=\frac{(m+1)(2m^2+13m+6m+30)}{6}$$

$$=\frac{(m+1)(2m^2+19m+30)}{6}$$

$$=\frac{(m+1)(m+2)(2m+15)}{6}$$

So, the statement is true for $n=m+1$ if it is true for $n=m$.

By mathematical induction, the statement is true for all positive integers n . — 1

(b) Putting $n=555$ in (a), we have

$$\sum_{k=1}^{555} k(k+4)=\frac{(555)(556)(1123)}{6}=57\ 755\ 890 .$$ — 1M (either one)

Putting $n=332$ in (a), we have

$$\sum_{k=1}^{332} k(k+4)=\frac{(332)(333)(677)}{6}=12\ 474\ 402 .$$

$$\sum_{k=333}^{555}\left(\frac{k}{112}\right)\left(\frac{k+4}{223}\right)$$

$$=\left(\frac{1}{112}\right)\left(\frac{1}{223}\right)\sum_{k=333}^{555} k(k+4)$$ — 1M (either one)

$$=\frac{1}{(112)(223)}\left(\sum_{k=1}^{555} k(k+4)-\sum_{k=1}^{332} k(k+4)\right)$$

$$=\frac{1}{24\ 976}(57\ 755\ 890-12\ 474\ 402)$$

$$=1813$$ — 1A

(7)

Solution	Marks	Remarks
1. (a) $MX = XM$ $\begin{pmatrix}7 & 3\\ -1 & 5\end{pmatrix}\begin{pmatrix}a & 6a\\ b & c\end{pmatrix}=\begin{pmatrix}a & 6a\\ b & c\end{pmatrix}\begin{pmatrix}7 & 3\\ -1 & 5\end{pmatrix}$		
$\begin{pmatrix}7a+3b & 42a+3c\\ -a+5b & -6a+5c\end{pmatrix}=\begin{pmatrix}a & 33a\\ 7b-c & 3b+5c\end{pmatrix}$	1M	
$\begin{cases}7a+3b=a\\ -a+5b=7b-c\\ 42a+3c=33a\\ -6a+5c=3b+5c\end{cases}$	1M	
$\begin{cases}b=-2a\\ c=-3a\end{cases}$	1A	for both correct
(b) $\lvert X\rvert$ $=\begin{vmatrix}a & 6a\\ -2a & -3a\end{vmatrix}$	1M	for considering the determinant
$=(a)(-3a)-(6a)(-2a)$ $=9a^2$ Note that X is a non-zero real matrix. By (a), a is a non-zero real number. So, we have $\lvert X\rvert>0$. Therefore, we have $\lvert X\rvert\neq 0$. Thus, X is a non-singular matrix.	1	
(c) $(X^T)^{-1}$ $=(X^{-1})^T$	1M	
$=\left(\dfrac{1}{\lvert X\rvert}\begin{pmatrix}-3a & -6a\\ 2a & a\end{pmatrix}\right)^T$	1M	
$=\left(\dfrac{1}{9a}\begin{pmatrix}-3 & -6\\ 2 & 1\end{pmatrix}\right)^T$ $=\dfrac{1}{9a}\begin{pmatrix}-3 & 2\\ -6 & 1\end{pmatrix}$	1A	
$(X^T)^{-1}$ $=\begin{pmatrix}a & -2a\\ 6a & -3a\end{pmatrix}^{-1}$ $=\dfrac{1}{\lvert X^T\rvert}\begin{pmatrix}-3a & 2a\\ -6a & a\end{pmatrix}$	1M	
$=\dfrac{1}{\lvert X\rvert}\begin{pmatrix}-3a & 2a\\ -6a & a\end{pmatrix}$	1M	
$=\dfrac{1}{9a}\begin{pmatrix}-3 & 2\\ -6 & 1\end{pmatrix}$	1A	
	----------(8)	

8. (a) Note that $A \neq 0$.

$f'(x)$

$= \dfrac{-A(2x-4)}{(x^2-4x+7)^2}$ — 1M

So, we have $f'(x)=0 \Leftrightarrow x=2$. — 1M

Since the equation $f'(x)=0$ has only one solution $x=2$ and the extreme value of $f(x)$ is 4, we have $f(2)=4$.

Hence, we have $\dfrac{A}{2^2-4(2)+7}=4$.

Therefore, we have $A=12$.

Thus, we have $f'(x)=\dfrac{24(2-x)}{(x^2-4x+7)^2}$. — 1A

(b) Note that $x^2-4x+7=(x-2)^2+3>0$ for all real values of x .

So, there are no vertical asymptotes of the graph of $y=f(x)$.

Also note that $f(x)=\dfrac{12}{x^2-4x+7}$.

Therefore, $y=0$ is the only asymptote of the graph of $y=f(x)$. — 1M

Hence, there is only one asymptote of the graph of $y=f(x)$.

Thus, the claim is disagreed. — 1A — f.t.

(c) $f''(x)$

$= \dfrac{(x^2-4x+7)^2(-24)-(-24x+48)(2)(x^2-4x+7)(2x-4)}{(x^2-4x+7)^4}$ — 1M

$= \dfrac{72(x-3)(x-1)}{(x^2-4x+7)^3}$

So, we have $f''(x)=0 \Leftrightarrow x=1$ or $x=3$.

x	$(-\infty,1)$	1	$(1,3)$	3	$(3,\infty)$
$f''(x)$	+	0	−	0	+

1M — for testing

Thus, the points of inflexion are $(1,3)$ and $(3,3)$. — 1A — for both correct

----------(8)

Solution	Marks	Remarks
(a) $y=\frac{\ln x}{2}$		
$\frac{dy}{dx}=\frac{1}{2x}$	1M	
The slope of the tangent at P is $\frac{1}{2r}$.		
The slope of the normal at P is $-2r$.		
Let a be the x-coordinate of Q.		
$\frac{0-\ln\sqrt{r}}{a-r}=-2r$	1M	
$\frac{-1}{2}\ln r=2r^2-2ar$		
$2ar=2r^2+\frac{1}{2}\ln r$		
$a=\frac{4r^2+\ln r}{4r}$	1	
Thus, the x-coordinate of Q is $\frac{4r^2+\ln r}{4r}$.		
	----------(3)	
(b) Let A square units be the area of ΔPQR.		
A		
$=\frac{1}{2}\left(\frac{4r^2+\ln r}{4r}-r\right)\ln\sqrt{r}$	1M	
$=\frac{(\ln r)^2}{16r}$	1A	
$\frac{dA}{dr}$		
$=\frac{r(2\ln r)\frac{1}{r}-(\ln r)^2}{16r^2}$	1M	
$=\frac{2\ln r-(\ln r)^2}{16r^2}$		
$=\frac{(2-\ln r)\ln r}{16r^2}$		
So, we have $\frac{dA}{dr}=0 \Leftrightarrow \ln r=2$ or $\ln r=0$ (rejected).		
Hence, we have $\frac{dA}{dr}=0 \Leftrightarrow r=e^2$.		
(see table below)	1M	for testing
Therefore, A attains its greatest value when $r=e^2$.		
Thus, the greatest area of ΔPQR is $\frac{1}{4e^2}$ square units.	1A	
	----------(5)	

r	$(1,e^2)$	e^2	(e^2,∞)
$\frac{dA}{dr}$	+	0	−
A	↗	$\frac{1}{4e^2}$	↘

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Solution	Marks	Remarks
(c) OP $=\sqrt{r^2+(\ln\sqrt{r})^2}$ $=\frac{1}{2}\sqrt{4r^2+(\ln r)^2}$	1M	
$\frac{\mathrm{d}OP}{\mathrm{d}t}$ $=\left(\frac{4r^2+\ln r}{2r\sqrt{4r^2+(\ln r)^2}}\right)\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)$	1M	
$\frac{\mathrm{d}A}{\mathrm{d}t}$ $=\left(\frac{\mathrm{d}A}{\mathrm{d}r}\right)\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)$ $=\left(\frac{(2-\ln r)\ln r}{16r^2}\right)\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)$ (by (b)) $=\left(\frac{(2-\ln r)\ln r}{16r^2}\right)\left(\frac{2r\sqrt{4r^2+(\ln r)^2}}{4r^2+\ln r}\right)\left(\frac{\mathrm{d}OP}{\mathrm{d}t}\right)$ $=\frac{(2-\ln r)(\ln r)\sqrt{4r^2+(\ln r)^2}}{8r(4r^2+\ln r)}\left(\frac{\mathrm{d}OP}{\mathrm{d}t}\right)$		
$\left.\frac{\mathrm{d}A}{\mathrm{d}t}\right\|_{r=e}$ $=\frac{(2-\ln e)(\ln e)\sqrt{4e^2+(\ln e)^2}}{8e(4e^2+\ln e)}\left(\left.\frac{\mathrm{d}OP}{\mathrm{d}t}\right\|_{r=e}\right)$ $=\frac{\sqrt{4e^2+1}}{8e(4e^2+1)}\left(\left.\frac{\mathrm{d}OP}{\mathrm{d}t}\right\|_{r=e}\right)$	1M	
Since $0\le\left.\frac{\mathrm{d}OP}{\mathrm{d}t}\right\|_{r=e}\le 32e^2$, we have $0\le\left.\frac{\mathrm{d}A}{\mathrm{d}t}\right\|_{r=e}\le\frac{32e^2\sqrt{4e^2+1}}{8e(4e^2+1)}$. So, we have $0\le\left.\frac{\mathrm{d}A}{\mathrm{d}t}\right\|_{r=e}\le\frac{4e}{\sqrt{4e^2+1}}$. Therefore, we have $0\le\left.\frac{\mathrm{d}A}{\mathrm{d}t}\right\|_{r=e}<\frac{4e}{\sqrt{4e^2}}$. Hence, we have $0\le\left.\frac{\mathrm{d}A}{\mathrm{d}t}\right\|_{r=e}<2$. Thus, the claim is correct.	1A ----------(4)	f.t.

Solution	Marks	Remarks
(a) (i) $\int \sin^4 x \, dx$		
$= -\cos x \sin^3 x + \int \cos x (3\sin^2 x \cos x) \, dx$	1M	
$= -\cos x \sin^3 x + 3\int (1-\sin^2 x)(\sin^2 x) \, dx$		
So, we have $\int \sin^4 x \, dx = -\cos x \sin^3 x + 3\int \sin^2 x \, dx - 3\int \sin^4 x \, dx$.		
Hence, we have $4\int \sin^4 x \, dx = -\cos x \sin^3 x + 3\int \sin^2 x \, dx$.		
Thus, we have $\int \sin^4 x \, dx = \frac{-\cos x \sin^3 x}{4} + \frac{3}{4}\int \sin^2 x \, dx$.	1	
(ii) $\int \sin^4 x \, dx$		
$= \frac{-\cos x \sin^3 x}{4} + \frac{3}{4}\int \sin^2 x \, dx$ (by (a)(i))		
$= \frac{-\cos x \sin^3 x}{4} + \frac{3}{4}\int \frac{1-\cos 2x}{2} \, dx$	1M	
$= \frac{-\cos x \sin^3 x}{4} + \frac{3}{4}\left(\frac{x}{2} - \frac{\sin 2x}{4}\right) + \text{constant}$	1M	
$= \frac{3x}{8} - \frac{\cos x \sin^3 x}{4} - \frac{3\sin 2x}{16} + \text{constant}$		
$\int_0^{\pi} \sin^4 x \, dx$		
$= \left[\frac{3x}{8} - \frac{\cos x \sin^3 x}{4} - \frac{3\sin 2x}{16}\right]_0^{\pi}$		
$= \frac{3\pi}{8}$	1A	
$\int_0^{\pi} \sin^4 x \, dx$		
$= \int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right)^2 dx$	1M	
$= \frac{1}{4}\int_0^{\pi} (1 - 2\cos 2x + \cos^2 2x) \, dx$		
$= \frac{1}{4}\int_0^{\pi} \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right) dx$		
$= \frac{1}{8}\int_0^{\pi} (3 - 4\cos 2x + \cos 4x) \, dx$		
$= \frac{1}{8}\left[3x - 2\sin 2x + \frac{\sin 4x}{4}\right]_0^{\pi}$	1M	
$= \frac{3\pi}{8}$	1A	
	----------(5)	

Solution	Marks	Remarks
(b) (i) Let $x=\beta-u$. Then, we have $\frac{dx}{du}=-1$.		
$\int_0^\beta x f(x)\,dx$		
$=\int_\beta^0 -(\beta-u) f(\beta-u)\,du$	1M	
$=\int_0^\beta (\beta f(\beta-u)-u f(\beta-u))\,du$		
$=\int_0^\beta \beta f(x)\,dx-\int_0^\beta x f(x)\,dx$		
So, we have $2\int_0^\beta x f(x)\,dx=\beta\int_0^\beta f(x)\,dx$.		
Thus, we have $\int_0^\beta x f(x)\,dx=\frac{\beta}{2}\int_0^\beta f(x)\,dx$.	1	
(ii) Note that $\sin^4(\pi-x)=\sin^4 x$ for all real numbers x .	1M	withhold 1M if checking is skipped
$\int_0^\pi x\sin^4 x\,dx$		
$=\frac{\pi}{2}\int_0^\pi \sin^4 x\,dx$ (by (b)(i))	1M	for using the result of (b)(i)
$=\frac{\pi}{2}\left(\frac{3\pi}{8}\right)$ (by (a)(ii))		
$=\frac{3\pi^2}{16}$	1M	for $\frac{\pi}{2}$(a)(ii)
	----------(5)	
(c) The required volume		
$=\int_\pi^{2\pi} \pi(\sqrt{x}\sin^2 x)^2\,dx$	1M	
$=\pi\int_\pi^{2\pi} x\sin^4 x\,dx$		
$=\pi\int_0^\pi (\pi+y)\sin^4(\pi+y)\,dy$ (by letting $x=\pi+y$)	1M	accept $x=2\pi-y$
$=\pi\int_0^\pi (\pi\sin^4 y+y\sin^4 y)\,dy$		
$=\pi\int_0^\pi (\pi\sin^4 x+x\sin^4 x)\,dx$		
$=\pi\left(\pi\int_0^\pi \sin^4 x\,dx+\int_0^\pi x\sin^4 x\,dx\right)$		
$=\pi\left(\pi\left(\frac{3\pi}{8}\right)+\frac{3\pi^2}{16}\right)$ (by (a)(ii) and (b)(ii))		
$=\frac{9\pi^3}{16}$	1A	
	----------(3)	

Solution	Marks	Remarks

(a) (i) (1) Note that

$$\begin{vmatrix} 1 & a & 4(a+1) \\ 2 & a-1 & 2(a-1) \\ 1 & -1 & -12 \end{vmatrix}$$

$= (a-1)(-12) + a(2)(a-1) + 4(a+1)(2)(-1) - 4(a-1)(a+1) + 2(a-1) - 2a(-12)$

$= -2(a-3)(a+1)$ — **1A**

Since (E) has a unique solution, we have $\begin{vmatrix} 1 & a & 4(a+1) \\ 2 & a-1 & 2(a-1) \\ 1 & -1 & -12 \end{vmatrix} \neq 0$.

So, we have $-2(a-3)(a+1) \neq 0$. — **1M**

Therefore, we have $a \neq 3$ and $a \neq -1$.

Thus, we have $a < -1$, $-1 < a < 3$ or $a > 3$. — **1A**

The augmented matrix of (E) is

$$\left(\begin{array}{ccc|c} 1 & a & 4(a+1) & 18 \\ 2 & a-1 & 2(a-1) & 20 \\ 1 & -1 & -12 & b \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & a & 4(a+1) & 18 \\ 0 & -a-1 & -6a-10 & -16 \\ 0 & -a-1 & -4a-16 & b-18 \end{array}\right)$$ — **1M**

$$\sim \left(\begin{array}{ccc|c} 1 & a & 4(a+1) & 18 \\ 0 & -a-1 & -6a-10 & -16 \\ 0 & 0 & 2a-6 & b-2 \end{array}\right)$$ — **1A**

Since (E) has a unique solution, we have $2a-6 \neq 0$ and $-a-1 \neq 0$.

Therefore, we have $a \neq 3$ and $a \neq -1$.

Thus, we have $a < -1$, $-1 < a < 3$ or $a > 3$. — **1A**

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(2) Since (E) has a unique solution, we have

$$x = \frac{\begin{vmatrix} 18 & a & 4(a+1) \\ 20 & a-1 & 2(a-1) \\ b & -1 & -12 \end{vmatrix}}{-2(a-3)(a+1)} = \frac{a^2b + ab + 10a - 2b - 50}{(a-3)(a+1)}$$ — **1M** (for Cramer's Rule)

$$y = \frac{\begin{vmatrix} 1 & 18 & 4(a+1) \\ 2 & 20 & 2(a-1) \\ 1 & b & -12 \end{vmatrix}}{-2(a-3)(a+1)} = \frac{-3ab + 22a - 5b - 38}{(a-3)(a+1)}$$

$$z = \frac{\begin{vmatrix} 1 & a & 18 \\ 2 & a-1 & 20 \\ 1 & -1 & b \end{vmatrix}}{-2(a-3)(a+1)} = \frac{b-2}{2(a-3)}$$ — **1A+1A** (1A for any one + 1A for all)

Solution	Marks	Remarks
Since (E) has a unique solution, the augmented matrix of (E) $\sim\left(\begin{array}{ccc\|c}1 & a & 4(a+1) & 18\\0 & -a-1 & -6a-10 & -16\\0 & 0 & 2a-6 & b-2\end{array}\right)$	1M	
$\sim\left(\begin{array}{ccc\|c}1 & 0 & 0 & \frac{a^2b+ab+10a-2b-50}{(a-3)(a+1)}\\0 & 1 & 0 & \frac{-3ab+22a-5b-38}{(a-3)(a+1)}\\0 & 0 & 1 & \frac{b-2}{2(a-3)}\end{array}\right)$		
Thus, we have $\begin{cases}x=\frac{a^2b+ab+10a-2b-50}{(a-3)(a+1)}\\y=\frac{-3ab+22a-5b-38}{(a-3)(a+1)}\\z=\frac{b-2}{2(a-3)}\end{cases}$.	1A+1A	1A for any one + 1A for all
(ii) (1) When $a=3$, the augmented matrix of (E) is $\left(\begin{array}{ccc\|c}1 & 3 & 16 & 18\\2 & 2 & 4 & 20\\1 & -1 & -12 & b\end{array}\right)\sim\left(\begin{array}{ccc\|c}1 & 3 & 16 & 18\\0 & 1 & 7 & 4\\0 & 0 & 0 & b-2\end{array}\right)$	1M	either one
Since (E) is consistent, we have $b=2$.	1A	either one
(2) When $a=3$ and $b=2$, the augmented matrix of (E) $\sim\left(\begin{array}{ccc\|c}1 & 3 & 16 & 18\\0 & 1 & 7 & 4\\0 & 0 & 0 & 0\end{array}\right)$		
Thus, the solution set of (E) is $\{(5u+6, -7u+4, u):\ u\in\mathbf{R}\}$.	1A	
	----------(9)	
(b) When $a=3$ and $b=s$, (E) becomes $(G):\begin{cases}x+3y+16z=18\\x+y+2z=10\\x-y-12z=s\end{cases}$.		
Since (F) is consistent, (G) is consistent.		
By (a)(ii), we have $s=2$.		
When $s=2$, the solution set of (G) is $\{(5u+6, -7u+4, u):\ u\in\mathbf{R}\}$.	1M	
Therefore, we have $2(5u+6)-5(-7u+4)-45u=t$.	1M	
Solving, we have $t=-8$.	1A	for both correct
Thus, we have $s=2$ and $t=-8$.		
	----------(3)	

Solution	Marks	Remarks
(a) (i) Note that $\overrightarrow{AB}=-5\mathbf{i}+6\mathbf{j}-4\mathbf{k}$ and $\overrightarrow{AC}=3\mathbf{i}+2\mathbf{j}+4\mathbf{k}$.		
$\overrightarrow{AB}\times\overrightarrow{AC}$		
$=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -5 & 6 & -4\\ 3 & 2 & 4\end{vmatrix}$		
$=32\mathbf{i}+8\mathbf{j}-28\mathbf{k}$	1A	
(ii) Note that $\overrightarrow{AD}=-\mathbf{i}+\mathbf{j}-6\mathbf{k}$.		
The required volume		
$=\frac{1}{6}\left\|(\overrightarrow{AB}\times\overrightarrow{AC})\cdot\overrightarrow{AD}\right\|$	1M	
$=\frac{1}{6}\left\|(32\mathbf{i}+8\mathbf{j}-28\mathbf{k})\cdot(-\mathbf{i}+\mathbf{j}-6\mathbf{k})\right\|$		
$=\frac{1}{6}\left\|(32)(-1)+(8)(1)+(-28)(-6)\right\|$		
$=24$	1A	
(iii) $\overrightarrow{DE}$		
$=\left(\overrightarrow{DA}\cdot\frac{32\mathbf{i}+8\mathbf{j}-28\mathbf{k}}{\sqrt{32^2+8^2+(-28)^2}}\right)\left(\frac{32\mathbf{i}+8\mathbf{j}-28\mathbf{k}}{\sqrt{32^2+8^2+(-28)^2}}\right)$	1M	
$=\left((\mathbf{i}-\mathbf{j}+6\mathbf{k})\cdot\frac{32\mathbf{i}+8\mathbf{j}-28\mathbf{k}}{\sqrt{32^2+8^2+(-28)^2}}\right)\left(\frac{32\mathbf{i}+8\mathbf{j}-28\mathbf{k}}{\sqrt{32^2+8^2+(-28)^2}}\right)$		
$=\frac{-32}{13}\mathbf{i}-\frac{8}{13}\mathbf{j}+\frac{28}{13}\mathbf{k}$	1A	
	----------(5)	
(b) (i) Let $\overrightarrow{BF}=t\overrightarrow{BC}$, where $0<t<1$.		
$\overrightarrow{DF}$		
$=(1-t)\overrightarrow{DB}+t\overrightarrow{DC}$	1M	
$=(1-t)(-4\mathbf{i}+5\mathbf{j}+2\mathbf{k})+t(4\mathbf{i}+\mathbf{j}+10\mathbf{k})$		
$=(8t-4)\mathbf{i}+(5-4t)\mathbf{j}+(8t+2)\mathbf{k}$		
Since $DF\perp BC$, we have $\overrightarrow{DF}\cdot\overrightarrow{BC}=0$.	1M	
Note that $\overrightarrow{BC}=8\mathbf{i}-4\mathbf{j}+8\mathbf{k}$.		
Hence, we have $(8t-4)(8)+(5-4t)(-4)+(8t+2)(8)=0$.		
So, we have $144t-36=0$.		
Solving, we have $t=\frac{1}{4}$.		
Thus, we have $\overrightarrow{DF}=-2\mathbf{i}+4\mathbf{j}+4\mathbf{k}$.	1A	

Solution	Marks	Remarks
(ii) $\overrightarrow{EF}$ $=\overrightarrow{DF}-\overrightarrow{DE}$ $=-2\mathbf{i}+4\mathbf{j}+4\mathbf{k}-\left(\frac{-32}{13}\mathbf{i}-\frac{8}{13}\mathbf{j}+\frac{28}{13}\mathbf{k}\right)$ $=\frac{6}{13}\mathbf{i}+\frac{60}{13}\mathbf{j}+\frac{24}{13}\mathbf{k}$	1M	
$\overrightarrow{BC}\cdot\overrightarrow{EF}$ $=(8\mathbf{i}-4\mathbf{j}+8\mathbf{k})\cdot\left(\frac{6}{13}\mathbf{i}+\frac{60}{13}\mathbf{j}+\frac{24}{13}\mathbf{k}\right)$ $=8\left(\frac{6}{13}\right)-4\left(\frac{60}{13}\right)+8\left(\frac{24}{13}\right)$ $=0$ Thus, $\overrightarrow{BC}$ is perpendicular to $\overrightarrow{EF}$.	1A ----------(5)	f.t.
(c) Note that the required angle is $\angle DFE$.	1M	for identifying the required angle
$\cos\angle DFE$ $=\frac{\overrightarrow{DF}\cdot\overrightarrow{EF}}{\left\|\overrightarrow{DF}\right\|\left\|\overrightarrow{EF}\right\|}$ $=\frac{(-2\mathbf{i}+4\mathbf{j}+4\mathbf{k})\cdot\left(\frac{6}{13}\mathbf{i}+\frac{60}{13}\mathbf{j}+\frac{24}{13}\mathbf{k}\right)}{\sqrt{(-2)^2+4^2+4^2}\sqrt{\left(\frac{6}{13}\right)^2+\left(\frac{60}{13}\right)^2+\left(\frac{24}{13}\right)^2}}$ $=\frac{324}{(6)(18\sqrt{13})}$ $=\frac{3}{\sqrt{13}}$ $=\frac{3\sqrt{13}}{13}$	1M	
Thus, the required angle is $\cos^{-1}\left(\frac{3\sqrt{13}}{13}\right)$.	1A ----------(3)	