2017-DSE
MATH EP
M2

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Candidate Number | | | | | | | |

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2017

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)
Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9, 11 and 13.

(2) This paper consists of TWO sections, A and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers must be exact.

(7) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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SECTION A (50 marks)

1. Find $\frac{d}{d\theta}\sec 6\theta$ from first principles. (5 marks)

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2. Let $(1+ax)^8 = \sum_{k=0}^{8} \lambda_k x^k$ and $(b+x)^9 = \sum_{k=0}^{9} \mu_k x^k$, where a and b are constants. It is given that $\lambda_2 : \mu_7 = 7:4$ and $\lambda_1 + \mu_8 + 6 = 0$. Find a . (5 marks)

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3. P is a point lying on AB such that $AP : PB = 3 : 2$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, where O is the origin.

(a) Express $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) It is given that $|\mathbf{a}| = 45$, $|\mathbf{b}| = 20$ and $\cos \angle AOB = \frac{1}{4}$. Find

(i) $\mathbf{a} \cdot \mathbf{b}$,

(ii) $\left|\overrightarrow{OP}\right|$.

(5 marks)

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4. (a) Using integration by parts, find $\int x^2 e^{-x}\, dx$.

(b) Find the area of the region bounded by the graph of $y = x^2 e^{-x}$, the x-axis and the straight line $x = 6$.

(6 marks)

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5. Consider the following system of linear equations in real variables x, y, z

$$(E): \begin{cases} x + 2y - z = 11 \\ 3x + 8y - 11z = 49 \\ 2x + 3y + hz = k \end{cases}, \text{ where } h, k \in \mathbf{R} .$$

(a) Assume that (E) has a unique solution.

(i) Find the range of values of h.

(ii) Express z in terms of h and k.

(b) Assume that (E) has infinitely many solutions. Solve (E).

(6 marks)

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6. A container in the form of an inverted right circular cone is held vertically. The height and the base radius of the container are 20 cm and 15 cm respectively. Water is now poured into the container.

(a) Let A cm^2 be the wet curved surface area of the container and h cm be the depth of water in the container. Prove that $A=\dfrac{15}{16}\pi h^2$.

(b) The depth of water in the container increases at a constant rate of $\dfrac{3}{\pi}$ cm/s . Find the rate of change of the wet curved surface area of the container when the volume of water in the container is 96π cm^3 .

(7 marks)

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7. (a) Prove that $\sin 3x = 3\sin x - 4\sin^3 x$.

(b) Let $\frac{\pi}{4} < x < \frac{\pi}{2}$.

(i) Prove that $\dfrac{\sin 3\left(x - \frac{\pi}{4}\right)}{\sin\left(x - \frac{\pi}{4}\right)} = \dfrac{\cos 3x + \sin 3x}{\cos x - \sin x}$.

(ii) Solve the equation $\dfrac{\cos 3x + \sin 3x}{\cos x - \sin x} = 2$.

(8 marks)

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8. Let $f(x)$ be a continuous function defined on $\mathbf{R}^+$, where $\mathbf{R}^+$ is the set of positive real numbers. Denote the curve $y = f(x)$ by Γ. It is given that Γ passes through the point $P(e^3, 7)$ and $f'(x) = \frac{1}{x} \ln x^2$ for all $x > 0$. Find

(a) the equation of the tangent to Γ at P,

(b) the equation of Γ,

(c) the point(s) of inflexion of Γ.

(8 marks)

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SECTION B (50 marks)

9. Define $f(x)=\dfrac{x^2-5x}{x+4}$ for all $x\neq -4$. Denote the graph of $y=f(x)$ by G .

(a) Find the asymptote(s) of G . (3 marks)

(b) Find $f'(x)$. (2 marks)

(c) Find the maximum point(s) and the minimum point(s) of G . (4 marks)

(d) Let R be the region bounded by G and the x-axis. Find the volume of the solid of revolution generated by revolving R about the x-axis. (4 marks)

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10. ABC is a triangle. D is the mid-point of AC. E is a point lying on BC such that $BE:EC=1:r$. AB produced and DE produced meet at the point F. It is given that $DE:EF=1:10$. Let $\overrightarrow{OA}=2\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, $\overrightarrow{OB}=4\mathbf{i}+4\mathbf{j}-\mathbf{k}$ and $\overrightarrow{OC}=8\mathbf{i}-3\mathbf{j}-2\mathbf{k}$, where O is the origin.

(a) By expressing $\overrightarrow{AE}$ and $\overrightarrow{AF}$ in terms of r, find r. (4 marks)

(b) (i) Find $\overrightarrow{AD}\cdot\overrightarrow{DE}$.

(ii) Are B, D, C and F concyclic? Explain your answer. (5 marks)

(c) Let $\overrightarrow{OP}=3\mathbf{i}+10\mathbf{j}-4\mathbf{k}$. Denote the circumcentre of ΔBCF by Q. Find the volume of the tetrahedron $ABPQ$. (3 marks)

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11. (a) Using $\tan^{-1}\sqrt{2}-\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)=\tan^{-1}\left(\frac{\sqrt{2}}{4}\right)$, evaluate $\int_0^1 \frac{1}{x^2+2x+3}\,\mathrm{d}x$. (3 marks)

(b) (i) Let $0\le\theta\le\frac{\pi}{4}$. Prove that $\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$ and $\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$.

(ii) Using the substitution $t=\tan\theta$, evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{\sin 2\theta+\cos 2\theta+2}\,\mathrm{d}\theta$.

(5 marks)

(c) Prove that $\int_0^{\frac{\pi}{4}} \frac{\sin 2\theta+1}{\sin 2\theta+\cos 2\theta+2}\,\mathrm{d}\theta=\int_0^{\frac{\pi}{4}} \frac{\cos 2\theta+1}{\sin 2\theta+\cos 2\theta+2}\,\mathrm{d}\theta$. (2 marks)

(d) Evaluate $\int_0^{\frac{\pi}{4}} \frac{8\sin 2\theta+9}{\sin 2\theta+\cos 2\theta+2}\,\mathrm{d}\theta$. (3 marks)

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12. Let $A=\begin{pmatrix}3 & 1\\ 0 & 3\end{pmatrix}$. Denote the 2×2 identity matrix by I.

(a) Using mathematical induction, prove that $A^n = 3^n I + 3^{n-1} n\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$ for all positive integers n.

(4 marks)

(b) Let $B=\begin{pmatrix}5 & 1\\ -4 & 1\end{pmatrix}$.

(i) Define $P=\begin{pmatrix}-1 & 0\\ 2 & -1\end{pmatrix}$. Evaluate $P^{-1}BP$.

(ii) Prove that $B^n = 3^n I + 3^{n-1} n\begin{pmatrix}2 & 1\\ -4 & -2\end{pmatrix}$ for any positive integer n.

(iii) Does there exist a positive integer m such that $|A^m - B^m| = 4m^2$? Explain your answer.

(8 marks)

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