2016-DSE
MATH EP
M2

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Candidate Number									

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2016

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)
Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of TWO sections, A and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers must be exact.

(7) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Answers written in the margins will not be marked.

SECTION A (50 marks)

1. Expand $(5+x)^4$. Hence, find the constant term in the expansion of $(5+x)^4\left(1-\dfrac{2}{x}\right)^3$. (5 marks)

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2. Prove that $\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{x+h}}=\frac{h}{(x+h)\sqrt{x}+x\sqrt{x+h}}$. Hence, find $\frac{d}{dx}\sqrt{\frac{3}{x}}$ from first principles.

(5 marks)

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3. Consider the curve $C: y = 2e^x$, where $x > 0$. It is given that P is a point lying on C . The horizontal line which passes through P cuts the y-axis at the point Q . Let O be the origin. Denote the x-coordinate of P by u .

(a) Express the area of ΔOPQ in terms of u .

(b) If P moves along C such that OQ increases at a constant rate of 6 units per second, find the rate of change of the area of ΔOPQ when $u = 4$.

(5 marks)

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4. Define $\mathrm{f}(x)=\dfrac{2x^2+x+1}{x-1}$ for all $x \neq 1$. Denote the graph of $y=\mathrm{f}(x)$ by G . Find

(a) the asymptote(s) of G ,

(b) the slope of the normal to G at the point $(2,11)$.

(7 marks)

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5. (a) Using mathematical induction, prove that $\sum_{k=1}^{n}(-1)^k k^2 = \frac{(-1)^n n(n+1)}{2}$ for all positive integers n.

(b) Using (a), evaluate $\sum_{k=3}^{333}(-1)^{k+1}k^2$.

(6 marks)

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6. (a) Prove that $x+1$ is a factor of $4x^3+2x^2-3x-1$.

(b) Express $\cos 3\theta$ in terms of $\cos\theta$.

(c) Using the results of (a) and (b), prove that $\cos\frac{3\pi}{5}=\frac{1-\sqrt{5}}{4}$.

(6 marks)

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7. (a) Using integration by substitution, find $\int (1+\sqrt{t+1})^2 \, \mathrm{d}t$.

(b) Consider the curve Γ: $y = 4x^2 - 4x$, where $1 \le x \le 4$. Let R be the region bounded by Γ , the straight line $y = 48$ and the two axes. Find the volume of the solid of revolution generated by revolving R about the y-axis.

(8 marks)

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8. Let n be a positive integer.

(a) Define $A=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$. Evaluate

(i) A^2 ,

(ii) A^n ,

(iii) $(A^{-1})^n$.

(b) Evaluate

(i) $\sum_{k=0}^{n-1} 2^k$,

(ii) $\begin{pmatrix}1 & 0\\ 1 & 2\end{pmatrix}^n$.

(8 marks)

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SECTION B (50 marks)

9. Let a and b be constants. Define $f(x) = x^3 + ax^2 + bx + 5$ for all real numbers x. Denote the curve $y = f(x)$ by C. It is given that $P(-1, 10)$ is a turning point of C.

(a) Find a and b. (3 marks)

(b) Is P a maximum point of C? Explain your answer. (2 marks)

(c) Find the minimum value(s) of $f(x)$. (2 marks)

(d) Find the point(s) of inflexion of C. (2 marks)

(e) Let L be the tangent to C at P. Find the area of the region bounded by C and L. (4 marks)

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10. (a) Let $f(x)$ be a continuous function defined on the interval $[0, a]$, where a is a positive constant. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. (3 marks)

(b) Prove that $\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx = \int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan x}\right)dx$. (3 marks)

(c) Using (b), prove that $\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx = \frac{\pi \ln 2}{8}$. (3 marks)

(d) Using integration by parts, evaluate $\int_0^{\frac{\pi}{4}} \frac{x\sec^2 x}{1+\tan x}\,dx$. (3 marks)

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11. (a) Consider the system of linear equations in real variables x , y , z

$$(E):\begin{cases} x + y - z = 3 \\ 4x + 6y + az = b \\ 5x + (1-a)y + (3a-1)z = b-1 \end{cases}$$, where a and b are real numbers.

(i) Assume that (E) has a unique solution.

(1) Prove that $a \neq -2$ and $a \neq -12$.

(2) Solve (E) .

(ii) Assume that $a=-2$ and (E) is consistent.

(1) Find b .

(2) Solve (E) .

(9 marks)

(b) Is there a real solution of the system of linear equations

$$\begin{cases} x + y - z = 3 \\ 2x + 3y - z = 7 \\ 5x + 3y - 7z = 13 \end{cases}$$

satisfying $x^2+y^2-6z^2>14$? Explain your answer. (3 marks)

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12. Let $\overrightarrow{OA} = 2\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\overrightarrow{OP} = \mathbf{i} + t\mathbf{j}$, where t is a constant and O is the origin. It is given that P is equidistant from A and B .

(a) Find t . (3 marks)

(b) Let $\overrightarrow{OC} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{OD} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$. Denote the plane which contains A , B and C by Π .

(i) Find a unit vector which is perpendicular to Π .

(ii) Find the angle between CD and Π .

(iii) It is given that E is a point lying on Π such that $\overrightarrow{DE}$ is perpendicular to Π . Let F be a point such that $\overrightarrow{PF} = \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$. Describe the geometric relationship between D , E and F . Explain your answer.

(10 marks)

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