Marking Scheme

Module 2 (Algebra and Calculus)

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

6. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

Solution	Marks	Remarks
1. $(5+x)^4$		
$=5^4+4(5^3)x+6(5^2)x^2+4(5)x^3+x^4$	1M	
$=625+500x+150x^2+20x^3+x^4$	1A	
$\left(1-\dfrac{2}{x}\right)^3$		
$=1-3\left(\dfrac{2}{x}\right)+3\left(\dfrac{2}{x}\right)^2-\left(\dfrac{2}{x}\right)^3$	1M	
$=1-\dfrac{6}{x}+\dfrac{12}{x^2}-\dfrac{8}{x^3}$		
The constant term		
$=(625)(1)+(500)(-6)+(150)(12)+(20)(-8)$	1M	withhold 1M if the step is skipped
$=-735$	1A	
	----------(5)	
2. $\dfrac{1}{\sqrt{x}}-\dfrac{1}{\sqrt{x+h}}$		
$=\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x(x+h)}}$		
$=\left(\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x(x+h)}}\right)\left(\dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$	1M	
$=\dfrac{x+h-x}{(x+h)\sqrt{x}+x\sqrt{x+h}}$		
$=\dfrac{h}{(x+h)\sqrt{x}+x\sqrt{x+h}}$	1	
$\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{\dfrac{3}{x}}$		
$=\lim\limits_{h\to 0}\dfrac{1}{h}\left(\dfrac{\sqrt{3}}{\sqrt{x+h}}-\dfrac{\sqrt{3}}{\sqrt{x}}\right)$	1M	
$=\lim\limits_{h\to 0}\dfrac{1}{h}\left(\dfrac{-h\sqrt{3}}{(x+h)\sqrt{x}+x\sqrt{x+h}}\right)$		
$=\lim\limits_{h\to 0}\dfrac{-\sqrt{3}}{(x+h)\sqrt{x}+x\sqrt{x+h}}$	1M	withhold 1M if the step is skipped
$=\dfrac{-\sqrt{3}}{2x\sqrt{x}}$	1A	
	----------(5)	

Solution	Marks	Remarks
3. (a) Note that the coordinates of Q are $(0, 2e^u)$, where $u>0$.		
The area of ΔOPQ		
$= \dfrac{u(2e^u)}{2}$		
$= ue^u$	1A	
(b) Let v be the y-coordinate of P .		
Since $v = 2e^u$, we have $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 2e^u \dfrac{\mathrm{d}u}{\mathrm{d}t}$.	1M	
Therefore, we have $\dfrac{\mathrm{d}u}{\mathrm{d}t} = \dfrac{1}{2e^u}\left(\dfrac{\mathrm{d}v}{\mathrm{d}t}\right)$.		
Let A square units be the area of ΔOPQ .		
By (a), we have $A = ue^u$.		
$\dfrac{\mathrm{d}A}{\mathrm{d}t}$		
$= (e^u + ue^u)\dfrac{\mathrm{d}u}{\mathrm{d}t}$	1M+1M	
$= \left(\dfrac{e^u + ue^u}{2e^u}\right)\dfrac{\mathrm{d}v}{\mathrm{d}t}$		
$= \left(\dfrac{1+u}{2}\right)\dfrac{\mathrm{d}v}{\mathrm{d}t}$		
So, we have $\left.\dfrac{\mathrm{d}A}{\mathrm{d}t}\right\vert_{u=4} = \left(\dfrac{1+4}{2}\right)(6) = 15$.	1A	
Thus, the required rate of change is 15 square units per second.		
Let v be the y-coordinate of P .		
Since $v = 2e^u$, we have $u = \ln\left(\dfrac{v}{2}\right)$.	1M	
Let A square units be the area of ΔOPQ .		
By (a), we have $A = \dfrac{v}{2}\ln\left(\dfrac{v}{2}\right)$.		
$\dfrac{\mathrm{d}A}{\mathrm{d}t}$		
$= \left(\left(\dfrac{v}{2}\right)\left(\dfrac{2}{v}\right)\left(\dfrac{1}{2}\right) + \dfrac{1}{2}\ln\left(\dfrac{v}{2}\right)\right)\dfrac{\mathrm{d}v}{\mathrm{d}t}$	1M+1M	
$= \left(\dfrac{1}{2} + \dfrac{1}{2}\ln\left(\dfrac{v}{2}\right)\right)\dfrac{\mathrm{d}v}{\mathrm{d}t}$		
When $u = 4$, we have $v = 2e^4$.		
$\left.\dfrac{\mathrm{d}A}{\mathrm{d}t}\right\vert_{u=4}$		
$= \left(\dfrac{1}{2} + \dfrac{1}{2}\ln\left(\dfrac{2e^4}{2}\right)\right)(6)$		
$= 15$	1A	
Thus, the required rate of change is 15 square units per second.		
	----------(5)	

Solution	Marks	Remarks
4. (a) Note that the equation of the vertical asymptote is $x=1$.	1A	
$\mathrm{f}(x)$		
$=\dfrac{2x^2+x+1}{x-1}$		
$=2x+3+\dfrac{4}{x-1}$	1M	
Thus, the equation of the oblique asymptote is $y=2x+3$.	1A	
(b) $\mathrm{f}'(x)$		
$=\dfrac{(x-1)(4x+1)-(2x^2+x+1)}{(x-1)^2}$	1M	
$=\dfrac{2(x^2-2x-1)}{(x-1)^2}$		
$\mathrm{f}'(2)$		
$=\dfrac{2(2^2-2(2)-1)}{(2-1)^2}$	1M	
$=-2$		
The slope of the normal to G at the point $(2,11)$		
$=\dfrac{-1}{\mathrm{f}'(2)}$	1M	
$=\dfrac{1}{2}$	1A	
$\mathrm{f}'(x)$		
$=\dfrac{\mathrm{d}}{\mathrm{d}x}\left(2x+3+\dfrac{4}{x-1}\right)$		
$=2-\dfrac{4}{(x-1)^2}$	1M	
$\mathrm{f}'(2)$		
$=2-\dfrac{4}{(2-1)^2}$	1M	
$=-2$		
The slope of the normal to G at the point $(2,11)$		
$=\dfrac{-1}{\mathrm{f}'(2)}$	1M	
$=\dfrac{1}{2}$	1A	
	----------(7)	

Solution	Marks	Remarks
5. (a) Note that $(-1)^1(1^2) = -1 = \frac{(-1)^1(1)(2)}{2}$.		
So, the statement is true for $n = 1$.	1	
Assume that $\sum_{k=1}^{m}(-1)^k k^2 = \frac{(-1)^m m(m+1)}{2}$ for some positive integer m .	1M	
$\sum_{k=1}^{m+1}(-1)^k k^2$		
$= \sum_{k=1}^{m}(-1)^k k^2 + (-1)^{m+1}(m+1)^2$		
$= \frac{(-1)^m m(m+1)}{2} + (-1)^{m+1}(m+1)^2$ (by induction assumption)	1M	for using induction assumption
$= \frac{(-1)^m m(m+1) + (-1)^{m+1}(m+1)(2m+2)}{2}$		
$= \frac{(-1)^m (m+1)(m-2m-2)}{2}$		
$= \frac{(-1)^{m+1}(m+1)(m+2)}{2}$		
So, the statement is true for $n = m+1$ if it is true for $n = m$. By mathematical induction, the statement is true for all positive integers n .	1	
(b) Putting $n = 333$ in (a), we have $\sum_{k=1}^{333}(-1)^k k^2 = \frac{(-1)^{333}(333)(334)}{2}$.	1M	
So, we have $-1 + 4 + \sum_{k=3}^{333}(-1)^k k^2 = \frac{(-1)^{333}(333)(334)}{2}$.		
Thus, we have $\sum_{k=3}^{333}(-1)^{k+1} k^2 = 55\,614$.	1A	
	-----------(6)	

Solution	Marks	Remarks
6. (a) Note that $4(-1)^3+2(-1)^2-3(-1)-1=-4+2+3-1=0$.		
Thus, $x+1$ is a factor of $4x^3+2x^2-3x-1$.	1	
(b) $\cos 3\theta$		
$=\cos(\theta+2\theta)$		
$=\cos\theta\cos 2\theta-\sin\theta\sin 2\theta$	1M	
$=\cos\theta(\cos^2\theta-\sin^2\theta)-\sin\theta(2\sin\theta\cos\theta)$		
$=\cos\theta(\cos^2\theta-(1-\cos^2\theta))-2\cos\theta(1-\cos^2\theta)$		
$=4\cos^3\theta-3\cos\theta$	1A	
(c) Note that $\theta=\frac{3\pi}{5}$ satisfies $\cos 3\theta=\cos(3\pi-2\theta)$.	1M	
Therefore, $\theta=\frac{3\pi}{5}$ satisfies $\cos 3\theta=-\cos 2\theta$.		
By (b), $\theta=\frac{3\pi}{5}$ satisfies $4\cos^3\theta-3\cos\theta=-(2\cos^2\theta-1)$.	1M	for using (b)
Hence, $\cos\frac{3\pi}{5}$ is a root of the equation $4x^3+2x^2-3x-1=0$.		
By (a), we have $4x^3+2x^2-3x-1=(x+1)(4x^2-2x-1)$.		
So, we have $4x^3+2x^2-3x-1=4(x+1)\left(x-\frac{1+\sqrt{5}}{4}\right)\left(x-\frac{1-\sqrt{5}}{4}\right)$.		
Since $\frac{\pi}{2}<\frac{3\pi}{5}<\pi$, we have $-1<\cos\frac{3\pi}{5}<0$.		
Thus, we have $\cos\frac{3\pi}{5}=\frac{1-\sqrt{5}}{4}$.	1	
	----------(6)	

Solution	Marks	Remarks
7. (a) Let $u=\sqrt{t+1}$.	1M	
So, we have $2u\dfrac{du}{dt}=1$.		
$\int(1+\sqrt{t+1})^2\,dt$		
$=\int(1+u)^2\,2u\,du$		
$=\int(2u+4u^2+2u^3)\,du$	1M	
$=u^2+\dfrac{4}{3}u^3+\dfrac{1}{2}u^4+\text{constant}$		
$=(t+1)+\dfrac{4}{3}(t+1)^{\frac{3}{2}}+\dfrac{1}{2}(t+1)^2+\text{constant}$	1A	
$=2t+\dfrac{t^2}{2}+\dfrac{4}{3}(t+1)^{\frac{3}{2}}+\text{constant}$		
$\int(1+\sqrt{t+1})^2\,dt$		
$=\int(1+2\sqrt{t+1}+t+1)\,dt$		
$=\int(2+t+2\sqrt{t+1})\,dt$	1M	
$=2t+\dfrac{t^2}{2}+\int 2\sqrt{t+1}\,dt$		
$=2t+\dfrac{t^2}{2}+\int 2\sqrt{u}\,du$ (by letting $u=t+1$)	1M	
$=2t+\dfrac{t^2}{2}+\dfrac{4}{3}(t+1)^{\frac{3}{2}}+\text{constant}$	1A	
(b) Note that $y=4x^2-4x$, where $1\le x\le 4$.		
So, we have $x=\dfrac{1}{2}(1+\sqrt{y+1})$.	1M	
The required volume		
$=\int_0^{48}\pi\left(\dfrac{1}{2}(1+\sqrt{y+1})\right)^2 dy$	1M	
$=\dfrac{\pi}{4}\int_0^{48}(1+\sqrt{y+1})^2\,dy$		
$=\dfrac{\pi}{4}\int_0^{48}(1+\sqrt{t+1})^2\,dt$	1M	
$=\dfrac{\pi}{4}\left[2t+\dfrac{t^2}{2}+\dfrac{4}{3}(t+1)^{\frac{3}{2}}\right]_0^{48}$ (by (a))	1M	for using (a)
$=426\pi$	1A	
	----------(8)	

Solution	Marks	Remarks
8. (a) (i) A		
$=\begin{pmatrix}1&0\\1&1\end{pmatrix}$		
A^2		
$=\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}$		
$=\begin{pmatrix}1&0\\2&1\end{pmatrix}$	1A	
(ii) A^3		
$=\begin{pmatrix}1&0\\2&1\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}$	1M	
$=\begin{pmatrix}1&0\\3&1\end{pmatrix}$		
A^n		
$=\begin{pmatrix}1&0\\n&1\end{pmatrix}$	1A	
(iii) $\det(A^n)$		
$=\begin{vmatrix}1&0\\n&1\end{vmatrix}$		
$=1$		
$(A^{-1})^n$		
$=(A^n)^{-1}$		
$=\dfrac{1}{\det(A^n)}\begin{pmatrix}1&0\\-n&1\end{pmatrix}$	1M	
$=\begin{pmatrix}1&0\\-n&1\end{pmatrix}$	1A	
$A^{-1}=\begin{pmatrix}1&0\\-1&1\end{pmatrix}$		
$(A^{-1})^2=\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}1&0\\-1&1\end{pmatrix}=\begin{pmatrix}1&0\\-2&1\end{pmatrix}$		
$(A^{-1})^3=\begin{pmatrix}1&0\\-2&1\end{pmatrix}\begin{pmatrix}1&0\\-1&1\end{pmatrix}=\begin{pmatrix}1&0\\-3&1\end{pmatrix}$	1M	
Thus, we have $(A^{-1})^n=\begin{pmatrix}1&0\\-n&1\end{pmatrix}$.	1A	
	----------(5)	

Solution	Marks	Remarks
(b) (i) $\sum_{k=0}^{n-1} 2^k$		
$= \frac{2^n - 1}{2-1}$		
$= 2^n - 1$	1A	
(ii) $\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2^0 & 2^1 \end{pmatrix}$		
$\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 2^0 & 2^1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2^0 & 2^1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2^0 + 2^1 & 2^2 \end{pmatrix}$		
$\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}^3 = \begin{pmatrix} 1 & 0 \\ 2^0 + 2^1 & 2^2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2^0 & 2^1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2^0 + 2^1 + 2^2 & 2^3 \end{pmatrix}$	1M	
So, we have $\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ \sum_{k=0}^{n-1} 2^k & 2^n \end{pmatrix}$.		
By (b)(i), we have $\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ 2^n - 1 & 2^n \end{pmatrix}$.	1A	
$\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2^1 - 1 & 2^1 \end{pmatrix}$		
$\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 2^1 - 1 & 2^1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2^2 - 1 & 2^2 \end{pmatrix}$		
$\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}^3 = \begin{pmatrix} 1 & 0 \\ 2^2 - 1 & 2^2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2^3 - 1 & 2^3 \end{pmatrix}$	1M	
Thus, we have $\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ 2^n - 1 & 2^n \end{pmatrix}$.	1A	
	----------(3)	

Solution	Marks	Remarks
9. (a) Since $f(x) = x^3 + ax^2 + bx + 5$, we have $f'(x) = 3x^2 + 2ax + b$.	1A	
Note that $P(-1, 10)$ is a turning point of C .		
So, we have $-1 + a - b + 5 = 10$ and $3 - 2a + b = 0$.	1M	for either one
Therefore, we have $a - b - 6 = 0$ and $-2a + b + 3 = 0$.		
Solving, we have $a = -3$ and $b = -9$.	1A	for both correct
	----------(3)	
(b) Note that $f''(x) = 6x - 6$.		
$f''(-1)$		
$= -12$		
< 0	1M	
Thus, P is a maximum point of C .	1A	f.t.
Note that $f'(x) = 3(x+1)(x-3)$.		

x	$(-\infty, -1)$	-1	$(-1, 3)$
$f'(x)$	$+$	0	$-$
$f(x)$	↗	10	↘

Solution	Marks	Remarks
(table above)	1M	
Thus, P is a maximum point of C .	1A	f.t.
	----------(2)	
(c) Note that $f'(x) = 3(x+1)(x-3)$.		
So, we have $f'(3) = 0$.	1M	
Also note that $f''(x) = 6x - 6$.		
$f''(3)$		
$= 12$		
> 0		
Further note that $f(3) = -22$.		
Thus, the minimum value of $f(x)$ is -22 .	1A	
Note that $f'(x) = 3(x+1)(x-3)$.		
So, we have $f'(3) = 0$.	1M	

x	$(-1, 3)$	3	$(3, \infty)$
$f'(x)$	$-$	0	$+$
$f(x)$	↘	-22	↗

Solution	Marks	Remarks
Thus, the minimum value of $f(x)$ is -22 .	1A	
	----------(2)	
(d) Note that $f''(x) = 6(x-1)$.		
Therefore, we have $f''(x) = 0$ when $x = 1$.		

x	$(-\infty, 1)$	1	$(1, \infty)$
$f''(x)$	$-$	0	$+$

Solution	Marks	Remarks
(table above)	1M	
Thus, the point of inflexion of C is $(1, -6)$.	1A	
	----------(2)	

Solution	Marks	Remarks
(e) Note that the equation of L is $y=10$.		
$x^3-3x^2-9x+5=10$	1M	
$x^3-3x^2-9x-5=0$		
$(x+1)^2(x-5)=0$		
$x=-1$ or $x=5$		
The required area		
$=\int_{-1}^{5}(10-(x^3-3x^2-9x+5))\,dx$	1M	
$=\int_{-1}^{5}(-x^3+3x^2+9x+5)\,dx$		
$=\left[\frac{-x^4}{4}+x^3+\frac{9x^2}{2}+5x\right]_{-1}^{5}$	1M	
$=108$	1A	
	----------(4)	

Solution	Marks	Remarks
10. (a) Let $x=a-y$.	1M	
So, we have $\frac{dx}{dy}=-1$.		
$\int_0^a f(x)\,dx$		
$=-\int_a^0 f(a-y)\,dy$		
$=\int_0^a f(a-y)\,dy$	1M	
$=\int_0^a f(a-x)\,dx$	1	
	----------(3)	
(b) $\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx$		
$=\int_0^{\frac{\pi}{4}} \ln\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$ (by (a) with $a=\frac{\pi}{4}>0$)	1M	for using (a)
$=\int_0^{\frac{\pi}{4}} \ln\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$	1M	
$=\int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan x}\right)dx$	1	
	----------(3)	
(c) $\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx=\int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan x}\right)dx$ (by (b))		
$\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx=\int_0^{\frac{\pi}{4}} (\ln 2-\ln(1+\tan x))\,dx$	1M	
$\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx=\int_0^{\frac{\pi}{4}} \ln 2\,dx-\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx$		
$2\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx=\int_0^{\frac{\pi}{4}} \ln 2\ dx$	1M	
$\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx=\frac{1}{2}\int_0^{\frac{\pi}{4}} \ln 2\ dx$		
$\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx=\frac{\pi\ln 2}{8}$	1	
	----------(3)	
(d) $\int_0^{\frac{\pi}{4}} \frac{x\sec^2 x}{1+\tan x}\,dx$		
$=\left[x\ln(1+\tan x)\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx$	1M	
$=\left(\frac{\pi\ln 2}{4}-0\right)-\frac{\pi\ln 2}{8}$ (by (c))	1M	for using (c)
$=\frac{\pi\ln 2}{8}$	1A	
	----------(3)	

Solution	Marks	Remarks
11. (a) (i) (1) Note that		
$\begin{vmatrix} 1 & 1 & -1 \\ 4 & 6 & a \\ 5 & 1-a & 3a-1 \end{vmatrix}$		
$= 6(3a-1)+(a)(5)+(-1)(4)(1-a)-(a)(1-a)-4(3a-1)-(-1)(6)(5)$	1A	
$= (a+2)(a+12)$		
As (E) has a unique solution, we have $\begin{vmatrix} 1 & 1 & -1 \\ 4 & 6 & a \\ 5 & 1-a & 3a-1 \end{vmatrix} \neq 0$.	1M	
So, we have $(a+2)(a+12) \neq 0$.		
Thus, we have $a \neq -2$ and $a \neq -12$.	1	
The augmented matrix of (E) is		
$\left(\begin{array}{ccc\|c} 1 & 1 & -1 & 3 \\ 4 & 6 & a & b \\ 5 & 1-a & 3a-1 & b-1 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & -1 & 3 \\ 0 & 2 & a+4 & b-12 \\ 0 & -a-4 & 3a+4 & b-16 \end{array}\right)$	1M	
$\sim \left(\begin{array}{ccc\|c} 1 & 1 & -1 & 3 \\ 0 & 2 & a+4 & b-12 \\ 0 & 0 & (a+2)(a+12) & ab-12a+6b-80 \end{array}\right)$	1A	
As (E) has a unique solution, we have $(a+2)(a+12) \neq 0$.		
Thus, we have $a \neq -2$ and $a \neq -12$.	1	
(2) Since (E) has a unique solution, we have		
$x = \dfrac{\begin{vmatrix} 3 & 1 & -1 \\ b & 6 & a \\ b-1 & 1-a & 3a-1 \end{vmatrix}}{(a+2)(a+12)}$	1M	for Cramer's Rule
$= \dfrac{3a^2-ab+50a+6b-24}{(a+2)(a+12)}$		
$y = \dfrac{\begin{vmatrix} 1 & 3 & -1 \\ 4 & b & a \\ 5 & b-1 & 3a-1 \end{vmatrix}}{(a+2)(a+12)}$		
$= \dfrac{2(ab-10a+8)}{(a+2)(a+12)}$		
$z = \dfrac{\begin{vmatrix} 1 & 1 & 3 \\ 4 & 6 & b \\ 5 & 1-a & b-1 \end{vmatrix}}{(a+2)(a+12)}$		
$= \dfrac{ab-12a+6b-80}{(a+2)(a+12)}$	1A+1A	1A for any one + 1A for all

Solution	Marks	Remarks
Since (E) has a unique solution, the augmented matrix of (E) $\sim\left(\begin{array}{ccc\|c} 1 & 1 & -1 & 3 \\ 0 & 2 & a+4 & b-12 \\ 0 & 0 & (a+2)(a+12) & ab-12a+6b-80 \end{array}\right)$	1M	
$\sim\left(\begin{array}{ccc\|c} 1 & 0 & 0 & \frac{3a^2-ab+50a+6b-24}{(a+2)(a+12)} \\ 0 & 1 & 0 & \frac{2(ab-10a+8)}{(a+2)(a+12)} \\ 0 & 0 & 1 & \frac{ab-12a+6b-80}{(a+2)(a+12)} \end{array}\right)$		
Thus, we have $\begin{cases} x=\frac{3a^2-ab+50a+6b-24}{(a+2)(a+12)} \\ y=\frac{2(ab-10a+8)}{(a+2)(a+12)} \\ z=\frac{ab-12a+6b-80}{(a+2)(a+12)} \end{cases}$	1A+1A	1A for any one + 1A for all
(ii) (1) When $a=-2$, the augmented matrix of (E) is $\left(\begin{array}{ccc\|c} 1 & 1 & -1 & 3 \\ 4 & 6 & -2 & b \\ 5 & 3 & -7 & b-1 \end{array}\right)\sim\left(\begin{array}{ccc\|c} 1 & 1 & -1 & 3 \\ 0 & 2 & 2 & b-12 \\ 0 & 0 & 0 & 2b-28 \end{array}\right)$	1M	either one
Since (E) is consistent, we have $b=14$.	1A	
(2) When $a=-2$ and $b=14$, the augmented matrix of (E) $\sim\left(\begin{array}{ccc\|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$		
Thus, the solution set of (E) is $\{(2t+2,1-t,t):\ t\in\mathbf{R}\}$.	1A ----------(9)	
(b) Putting $a=-2$ and $b=14$, (E) becomes $\begin{cases} x+y-z=3 \\ 2x+3y-z=7 \\ 5x+3y-7z=13 \end{cases}$		
By (b)(ii), the solution set is $\{(2t+2,1-t,t):\ t\in\mathbf{R}\}$.	1M	
So, we have $x^2+y^2-6z^2$ $=(2t+2)^2+(1-t)^2-6t^2$ $=-t^2+6t+5$ $=-(t^2-6t+3^2)+3^2+5$ $=-(t-3)^2+14$	1M	
Therefore, the greatest value of $x^2+y^2-6z^2$ is 14 . Thus, there is no real solution of the system of linear equations satisfying $x^2+y^2-6z^2>14$.	1A ----------(3)	f.t.

Solution	Marks	Remarks
12. (a) $\lvert\overrightarrow{PA}\rvert = \lvert\overrightarrow{PB}\rvert$		
$\lvert\overrightarrow{OA}-\overrightarrow{OP}\rvert = \lvert\overrightarrow{OB}-\overrightarrow{OP}\rvert$	1M	
$\lvert -\mathbf{i}+(2-t)\mathbf{j}+2\mathbf{k}\rvert = \lvert 3\mathbf{i}+(1-t)\mathbf{j}+\mathbf{k}\rvert$		
$\sqrt{(-1)^2+(2-t)^2+2^2} = \sqrt{3^2+(1-t)^2+1^2}$	1M	
$t^2-4t+9 = t^2-2t+11$		
$t=-1$	1A	
	----------(3)	
(b) (i) $\overrightarrow{CA}\times\overrightarrow{CB}$		
$=(-2\mathbf{i}+3\mathbf{j}-2\mathbf{k})\times(2\mathbf{i}+2\mathbf{j}-3\mathbf{k})$		
$=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -2 & 3 & -2\\ 2 & 2 & -3\end{vmatrix}$		
$=-5\mathbf{i}-10\mathbf{j}-10\mathbf{k}$	1M	
$\lvert\overrightarrow{CA}\times\overrightarrow{CB}\rvert$		
$=\sqrt{(-5)^2+(-10)^2+(-10)^2}$		
$=15$		
A unit vector which is perpendicular to Π		
$=\dfrac{\overrightarrow{CA}\times\overrightarrow{CB}}{\lvert\overrightarrow{CA}\times\overrightarrow{CB}\rvert}$	1M	
$=\dfrac{-1}{3}\mathbf{i}-\dfrac{2}{3}\mathbf{j}-\dfrac{2}{3}\mathbf{k}$	1A	accept $\dfrac{1}{3}\mathbf{i}+\dfrac{2}{3}\mathbf{j}+\dfrac{2}{3}\mathbf{k}$
(ii) Note that $\overrightarrow{CD}=\mathbf{i}+3\mathbf{j}+\mathbf{k}$.		
Let $\mathbf{n}=\dfrac{-1}{3}\mathbf{i}-\dfrac{2}{3}\mathbf{j}-\dfrac{2}{3}\mathbf{k}$.		
$\overrightarrow{CD}\cdot\mathbf{n}$		
$=(\mathbf{i}+3\mathbf{j}+\mathbf{k})\cdot\left(\dfrac{-1}{3}\mathbf{i}-\dfrac{2}{3}\mathbf{j}-\dfrac{2}{3}\mathbf{k}\right)$		
$=\dfrac{-1}{3}-2-\dfrac{2}{3}$	1M	
$=-3$		
Let θ be the angle between CD and Π .		
Since $\overrightarrow{CD}\cdot\mathbf{n}<0$, the angle between $\overrightarrow{CD}$ and $\mathbf{n}$ is $\dfrac{\pi}{2}+\theta$.		
$\overrightarrow{CD}\cdot\mathbf{n}=\lvert\overrightarrow{CD}\rvert\lvert\mathbf{n}\rvert\cos\left(\dfrac{\pi}{2}+\theta\right)$	1M	
$3=\sqrt{1^2+3^2+1^2}\sin\theta$		
$\sin\theta=\dfrac{3\sqrt{11}}{11}$		
$\theta=\sin^{-1}\left(\dfrac{3\sqrt{11}}{11}\right)$	1A	
Thus, the angle between CD and Π is $\sin^{-1}\left(\dfrac{3\sqrt{11}}{11}\right)$.		

Solution	Marks	Remarks
(iii) $\overrightarrow{DE}$		
$=(\overrightarrow{DC}\cdot\mathbf{n})\mathbf{n}$	1M	
$=(3)\left(\frac{-1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}\right)$ (by (b)(ii))		
$=-\mathbf{i}-2\mathbf{j}-2\mathbf{k}$		
$\overrightarrow{PF}$		
$=\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$		
$=(-\mathbf{i}+3\mathbf{j}+2\mathbf{k})+(3\mathbf{i}+2\mathbf{j}+\mathbf{k})+(\mathbf{i}+4\mathbf{k})$		
$=3\mathbf{i}+5\mathbf{j}+7\mathbf{k}$		
$\overrightarrow{FD}$		
$=\overrightarrow{OD}-(\overrightarrow{OP}+\overrightarrow{PF})$		
$=(3\mathbf{i}+2\mathbf{j}+5\mathbf{k})-((\mathbf{i}-\mathbf{j})+(3\mathbf{i}+5\mathbf{j}+7\mathbf{k}))$	1M	
$=-\mathbf{i}-2\mathbf{j}-2\mathbf{k}$		
Therefore, we have $\overrightarrow{DE}=\overrightarrow{FD}$.		
So, we have $DE // FD$ and $DE=DF$.		
Hence, D, E and F are collinear and $DE=DF$.	1A+1A	f.t.
Thus, D is the mid-point of the line segment joining E and F .		
	----------(10)	