2015-DSE MATH EP M2

> HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2015

MATHEMATICS Extended Part Module 2 (Algebra and Calculus) Question-Answer Book

 $8.30 \text{ am} - 11.00 \text{ am} (2\frac{1}{2} \text{ hours})$ This paper must be answered in English

INSTRUCTIONS

- 1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
- 2. This paper consists of TWO sections, A and B.
- 3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
- 4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.
- 5. Unless otherwise specified, all working must be clearly shown.
- 6. Unless otherwise specified, numerical answers must be exact.
- 7. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

Please stick the barcode label here.

Candidate Number



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FORMULAS FOR REFERENCE

 $\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ $2 \sin A \cos B = \sin (A + B) + \sin (A - B)$ $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$ $2 \sin A \sin B = \cos (A - B) - \cos (A + B)$

$$\sin A + \sin B = 2\sin \frac{A+B}{2}\cos \frac{A-B}{2}$$
$$\sin A - \sin B = 2\cos \frac{A+B}{2}\sin \frac{A-B}{2}$$
$$\cos A + \cos B = 2\cos \frac{A+B}{2}\cos \frac{A-B}{2}$$
$$\cos A - \cos B = -2\sin \frac{A+B}{2}\sin \frac{A-B}{2}$$

SECTION A (50 marks)

Find $\frac{d}{dx}(x^5+4)$ from first principles. 1.

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(a)	Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
(b)	Let k be a constant such that $x \frac{d^2 y}{dx^2} + k \frac{dy}{dx} + xy = 0$ for all real values of x. Find the v
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(a)	Using integration by parts, find $\int x^2 \ln x dx$.
(b)	At any point (x, y) on the curve Γ , the slope of the tangent to Γ is $9x^2 \ln x$. It is given that Γ passes through the point $(1, 4)$. Find the equation of Γ . (7 marks)
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								equations in real variables x, y, z :	
(a)	$\begin{cases} x \\ 2x \end{cases}$	++	у 3у	+	z 3z	=	2; 4;		
(b)	$\begin{cases} x \\ 2x \\ 3x \end{cases}$	+ + +	y 3y 2y	+ +	z 3z kz	=	2 4 , 6	where k is a real constant.	(6 montro)
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6.	(a)	Let M be a 3×3 real matrix such that $M^T = -M$, where M^T is the transpose of M.
		Prove that $ M = 0$.
	(b)	Let $A = \begin{pmatrix} -1 & a & b \\ -a & -1 & -8 \\ -b & 8 & -1 \end{pmatrix}$, where <i>a</i> and <i>b</i> are real numbers. Denote the 3×3 identity matrix
		by I.
		(i) Using (a), or otherwise, prove that $ A + I = 0$.
		(ii) Someone claims that $A^3 + I$ is a singular matrix. Do you agree? Explain your answer. (6 marks)
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(a)	Prove that $\sin^2 x \cos^2 x = \frac{1 - \cos 4x}{8}$.
(b)	Let $f(x) = \sin^4 x + \cos^4 x$.
	(i) Express $f(x)$ in the form $A \cos Bx + C$, where A, B and C are constants.
	(ii) Solve the equation $8f(x) = 7$, where $0 \le x \le \frac{\pi}{2}$. (7 marks)
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8.	(a)	Using mathematical induction, prove that $\sin \frac{x}{2} \sum_{k=1}^{n} \cos kx = \sin \frac{nx}{2} \cos \frac{(n+1)x}{2}$	for all positive
		integers n.	
	(b)	Using (a), evaluate $\sum_{k=1}^{567} \cos \frac{k\pi}{7}$.	
			(8 marks)
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9. Define $f(x) = \frac{x^3 + 12}{x - 2}$ for all $x \neq 2$. (a) Find f'(x). (2 marks) (b) Prove that the maximum value and the minimum value of $f(x)$ are -4 and 12 respectively. (4 marks) (c) Find the asymptote(s) of the graph of $y = f(x)$. (3 marks) (d) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (4 marks) (4 marks) (5) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (4 marks) (5) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (4 marks) (5) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (4 marks) (5) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (4 marks) (5) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (5) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (6) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (7) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (6) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (7) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (8) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (9) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (9) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (9) Find the area of the region bounded by the graph of $y = 16$. (9) Find the area of the region bounded by the graph of $y = 16$.	SEC	FION B	(50 marks)	
 (b) Prove that the maximum value and the minimum value of f(x) are -4 and 12 respectively. (4 marks) (c) Find the asymptote(s) of the graph of y = f(x). (3 marks) (d) Find the area of the region bounded by the graph of y = f(x) and the horizontal line y = 14. (4 marks) 	9.	Define	e $f(x) = \frac{x^2 + 12}{x - 2}$ for all $x \neq 2$.	
 (c) Find the asymptote(s) of the graph of y = f(x). (d) Find the area of the region bounded by the graph of y = f(x) and the horizontal line y = 14. (4 marks) 		(a)	Find $f'(x)$.	(2 marks)
(d) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$. (4 marks)		(b)	Prove that the maximum value and the minimum value of $f(x)$ are -4 and 12 resp	
(4 marks)		(c)	Find the asymptote(s) of the graph of $y = f(x)$.	(3 marks)
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10.			iangle. P is the mid-point of $OA \cdot Q$ is a point lying on AB such that $AQ : QB = 1 : A$ a point lying on OB such that $OR : RB = 3 : 1 \cdot PR$ and OQ intersect at C.
	(a)	(i)	Let t be a constant such that $PC: CR = t: (1-t)$.
			By expressing \overrightarrow{OQ} in terms of \overrightarrow{OA} and \overrightarrow{OB} , find the value of t.
		(ii)	Find $CQ:OQ$. (7 marks
	(b)	Supp Find	ose that $\overrightarrow{OA} = 20\mathbf{i} - 6\mathbf{j} - 12\mathbf{k}$, $\overrightarrow{OB} = 16\mathbf{i} - 16\mathbf{j}$ and $\overrightarrow{OD} = \mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$, where <i>O</i> is the origin
		(i)	the area of $\triangle OAB$,
		(ii)	the volume of the tetrahedron <i>ABCD</i> . (5 marks

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11.	(a)	Let λ and μ be real numbers such that $\mu - \lambda \neq 2$. Denote the 2×2 identity matrix by <i>I</i> .	7
		Define $A = \frac{1}{\lambda - \mu + 2}(I - \mu I + M)$ and $B = \frac{1}{\lambda - \mu + 2}(I + \lambda I - M)$, where	
		$M = \begin{pmatrix} \lambda & 1 \\ \lambda - \mu + 1 & \mu \end{pmatrix} .$	
		(i) Evaluate AB , BA and $A+B$.	
		(ii) Prove that $A^2 = A$ and $B^2 = B$.	
		(iii) Prove that $M^n = (\lambda + 1)^n A + (\mu - 1)^n B$ for all positive integers n . (8 marks)	
	(b)	Using (a), or otherwise, evaluate $\begin{pmatrix} 4 & 2 \\ 0 & 6 \end{pmatrix}^{315}$. (4 marks)	
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12. (a) In the figure, the curve Γ consists of the curve AB, the line segments BC and CO, where O is the origin, B lies in the first quadrant and C lies on the x-axis. The equations of AB and BCare $x^2 - 4y + 8 = 0$ and 3x + y - 9 = 0 respectively. R 0 (i) Find the coordinates of B. (ii) Let h be the y-coordinate of A, where h > 3. A cup is formed by revolving Γ about the y-axis. Prove that the capacity of the cup is $\pi(2h^2 - 8h + 25)$. (7 marks) Answers written in the margins will not be marked. (b) A cup described in (a)(ii) is placed on a horizontal table. The radii of the base and the lip of the cup are 3 cm and 6 cm respectively. Find the capacity of the cup. (i) Water is poured into the cup at a constant rate of 24π cm³/s . Find the rate of change of (ii) the depth of water when the volume of water in the cup is 35π cm³. (6 marks)

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