2014-DSE
MATH EP
M2

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Candidate Number									

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2014

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9, 11, 13 and 15.

2. Answer **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

3. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this Book.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$
$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$
$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (50 marks)

1. In the expansion of $(1-4x)^2(1+x)^n$, the coefficient of x is 1.

   (a) Find the value of n.

   (b) Find the coefficient of x^2.

(4 marks)

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2. Consider the curve $C : y = x^3 - 3x$.

(a) Find $\frac{dy}{dx}$ from first principles.

(b) Find the range of x where C is decreasing.

(5 marks)

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3. Find the equation of tangent to the curve $x \ln y + y = 2$ at the point where the curve cuts the y-axis.

(5 marks)

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4. Let $x = 2y + \sin y$. Find $\dfrac{d^2y}{dx^2}$ in terms of y.

(3 marks)

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5. (a) Find $\int \frac{dx}{\sqrt{9-x}}$, where $x < 9$.

(b) Using integration by substitution, find $\int \frac{dx}{\sqrt{9-x^2}}$, where $-3 < x < 3$.

(6 marks)

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6. (a) Find $\int xe^{-x}dx$.

(b)

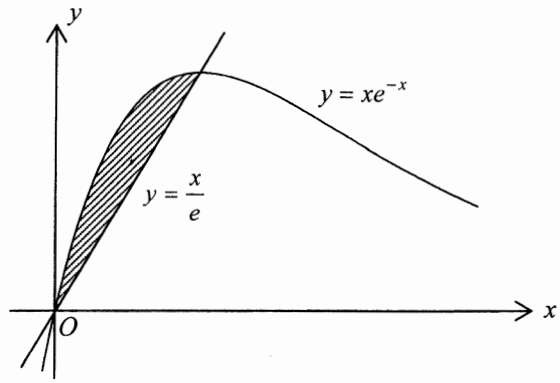

Figure 1

Figure 1 shows the shaded region bounded by the curve $y = xe^{-x}$ and the straight line $y = \frac{x}{e}$. Find the area of the shaded region.

(6 marks)

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7. Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$.

(a) Prove, by mathematical induction, that for all positive integers n, $A^{n+1} = 2^n A$.

(b) Using the result of (a), Willy proceeds in the following way:

$$A^2 = 2A$$
$$A^2 A^{-1} = 2AA^{-1}$$
$$A = 2I$$

Explain why Willy arrives at a wrong conclusion.

(7 marks)

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8. Let $\overrightarrow{OP} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OQ} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OR} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$.

(a) Find $\overrightarrow{OP} \times \overrightarrow{OQ}$.
Hence find the volume of tetrahedron $OPQR$.

(b) Find the acute angle between the plane OPQ and the line OR, correct to the nearest $0.1°$.

(8 marks)

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9. (a) Solve the system of linear equations $\begin{cases} x + y + z = 100 \\ x + 6y + 10z = 200 \end{cases}$

(b) In a store, the prices of each of small, medium and large marbles are \$0.5 ,- \$3 and \$5 respectively. Aubrey plans to spend all \$100 for exactly 100 marbles, which include m small marbles, n medium marbles and k large marbles.
Aubrey claims that there is only one set of combination of m, n and k. Do you agree? Explain your answer.

(6 marks)

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Section B (50 marks)

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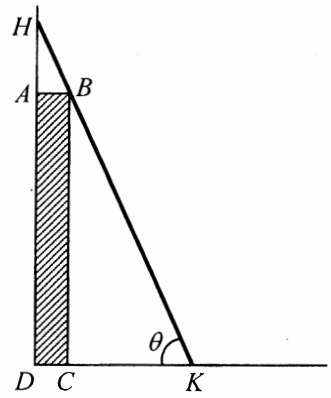

Figure 2

Thomas has a bookcase of dimensions 100 cm × 24 cm × 192 cm at the corner in his room. He wants to hang a decoration on the wall above the bookcase. Therefore, he finds a ladder to climb up. Initially, the ladder touches the wall, the edge of the top of the bookcase and the floor at the same time. Let rectangle $ABCD$ be the side-view of the bookcase and HK be the side-view of the ladder, so that $AB = 24$ cm and $BC = 192$ cm (see Figure 2). Let $\angle HKD = \theta$.

(a) Find the length of HK in terms of θ.

(1 mark)

(b) Prove that the shortest length of the ladder is $120\sqrt{5}$ cm.

(5 marks)

(c)

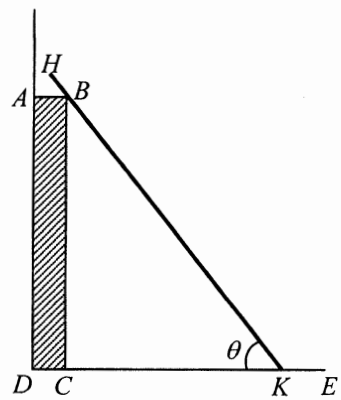

Figure 3

Suppose the length of the ladder is 270 cm. Suddenly, the ladder slides down so that the end of the ladder, K, moves towards E (see Figure 3). The ladder touches the edge of the top of the bookcase and the floor at the same time. Let x cm and y cm be the horizontal distances from H and K respectively to the wall.

(i) When $CK = 160$ cm, the rate of change of θ is -0.1 rad s^{-1}. Find the rate of change of x at this moment, correct to 4 significant figures.

(ii) Thomas claims that K is moving towards E at a speed faster than the horizontal speed H is leaving the wall. Do you agree? Explain your answer.

(6 marks)

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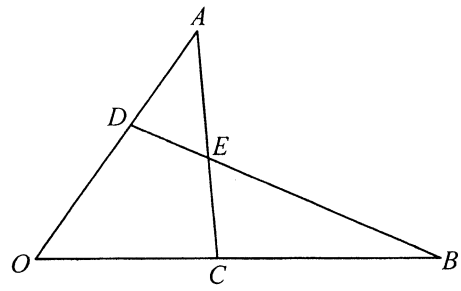

Figure 4

In Figure 4, C and D are points on OB and OA respectively such that $AD:DO = OC:CB = t:1-t$, where $0 < t < 1$. BD and AC intersect at E such that $AE:EC = m:1$ and $BE:ED = n:1$, where m and n are positive. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) (i) By considering ΔOAC, express $\overrightarrow{OE}$ in terms of m, t, $\mathbf{a}$ and $\mathbf{b}$.

(ii) By considering ΔOBD, express $\overrightarrow{OE}$ in terms of n, t, $\mathbf{a}$ and $\mathbf{b}$.

(iii) Show that $m = \dfrac{t}{(1-t)^2}$ and $n = \dfrac{1-t}{t^2}$.

(iv) Chris claims that

"if $m = n$, then E is the centroid of ΔOAB".

Do you agree? Explain your answer.

(9 marks)

(b) It is given that $OA = 1$ and $OB = 2$. Francis claims that

"if AC is perpendicular to OB, then BD is always perpendicular to OA".

Do you agree? Explain your answer.

(4 marks)

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12. Let $M=\begin{pmatrix} k-1 & k \\ 1 & 0 \end{pmatrix}$ and $A=\begin{pmatrix} 1 & p \\ -1 & 1 \end{pmatrix}$, where k and p are real numbers and $p \neq -1$.

(a) (i) Find A^{-1} in terms of p.

(ii) Show that $A^{-1}MA=\begin{pmatrix} -1 & k-p \\ 0 & k \end{pmatrix}$.

(iii) Suppose $p=k$. Using (ii), find M^n in terms of k and n, where n is a positive integer.

(8 marks)

(b) A sequence is defined by

$x_1=0$, $x_2=1$ and $x_n=x_{n-1}+2x_{n-2}$ for $n=3,4,5,\ldots$.

It is known that this sequence can be expressed in the matrix form $\begin{pmatrix} x_n \\ x_{n-1} \end{pmatrix}=\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_{n-1} \\ x_{n-2} \end{pmatrix}$.

Using the result of (a)(iii), express x_n in terms of n.

(3 marks)

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13. (a) Prove that $1-\cos 4\theta-2\cos 2\theta\sin^2 2\theta=16\cos^2\theta\sin^4\theta$.

(2 marks)

(b) Show that $\int_0^{n\pi}\cos^2 x\sin^4 x\,\mathrm{d}x=\frac{n\pi}{16}$, where n is a positive integer.

(4 marks)

(c) Let $\mathrm{f}(x)$ be a continuous function such that $\mathrm{f}(k-x)=\mathrm{f}(x)$, where k is a constant.

Show that $\int_0^k x\mathrm{f}(x)\,\mathrm{d}x=\frac{k}{2}\int_0^k \mathrm{f}(x)\,\mathrm{d}x$.

(4 marks)

(d)

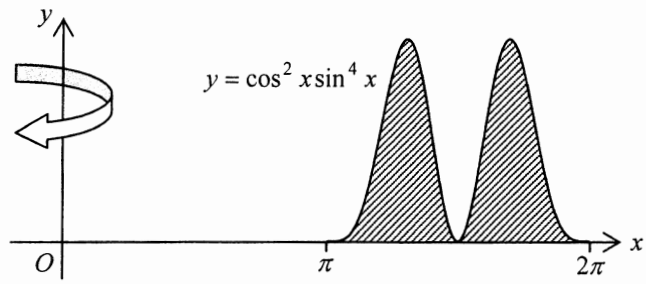

Figure 5

Figure 5 shows the shaded region bounded by the curve $y=\cos^2 x\sin^4 x$ and the x-axis, where $\pi\le x\le 2\pi$. Find the volume of the solid of revolution when the shaded region is revolved about the y-axis.

(4 marks)

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