Module 2 (Algebra and Calculus) Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. In the marking scheme, marks are classified into the following three categories:

   'M' marks – awarded for applying correct methods
   'A' marks – awarded for the accuracy of the answers
   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

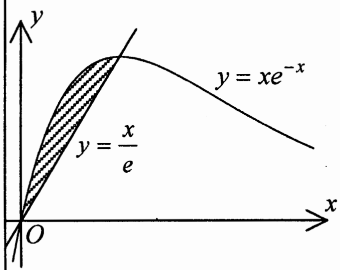
   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.

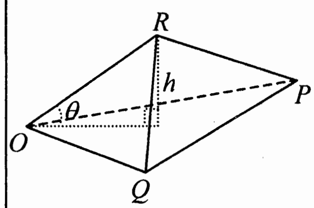
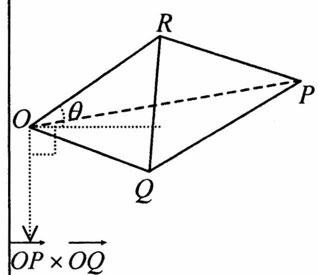
6. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.

7. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted.

Solution	Marks	Remarks
(a) $(1-4x)^2(1+x)^n$		
$=(1-8x+16x^2)\left[1+nx+\frac{n(n-1)}{2}x^2+\cdots\right]$ ---------------------------------- (*)	1M	For binomial expansion of $(1+x)^n$ up to the x^2 term
Coefficient of x $=n-8$		
$\therefore\ n-8=1$		
i.e. $n=9$	1A	
(b) $\therefore\ (1-4x)^2(1+x)^9=(1-8x+16x^2)(1+9x+36x^2+\cdots)$		For binomial expansion of $(1+x)^n$ up to the x^2 term
Coefficient of $x^2=36-8\cdot 9+16$	1M	
Alternative Solution Coefficient of $x^2=\frac{n(n-1)}{2}-8n+16$ by (*)	1M	
$=-20$	1A	
	(4)	
(a) $y=x^3-3x$		
$\frac{dy}{dx}=\lim_{h\to 0}\frac{[(x+h)^3-3(x+h)]-(x^3-3x)}{h}$	1M	
$=\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-3x-3h-x^3+3x}{h}$	1M	OR $\frac{h[(x+h)^2+(x+h)x+x^2]-3h}{h}$
$=\lim_{h\to 0}(3x^2+3xh+h^2-3)$		
$=3x^2-3$	1A	
(b) When C is decreasing, $\frac{dy}{dx}\le 0$.		
$3x^2-3\le 0$	1M	Accept $3x^2-3<0$
$(x+1)(x-1)\le 0$		
$-1\le x\le 1$	1A	Accept $-1<x<1$
	(5)	
$x\ln y+y=2$		
$\ln y+x\cdot\frac{1}{y}\frac{dy}{dx}+\frac{dy}{dx}=0$	1M+1M	1M for product rule 1M for chain rule
$\frac{dy}{dx}=\frac{-y\ln y}{x+y}$		
Alternative Solution $x=\frac{2-y}{\ln y}$		
$\frac{dx}{dy}=\frac{\ln y\cdot(-1)-(2-y)\frac{1}{y}}{(\ln y)^2}$	1M	For quotient rule
$\therefore\ \frac{dy}{dx}=\frac{y(\ln y)^2}{y-2-y\ln y}$	1M	For $\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$
When the curve cuts the y-axis, $x=0$.		
$\therefore\ y=2$	1A	

Solution	Marks	Remarks
$\left.\frac{dy}{dx}\right\vert_{(0,2)}=\frac{-2\ln 2}{0+2}$	1M	OR $\frac{2(\ln 2)^2}{2-2-2\ln 2}$
$=-\ln 2$		
Hence the equation of the tangent is $y=-x\ln 2+2$.	1A	
	(5)	
4. $x=2y+\sin y$		
$\frac{dx}{dy}=2+\cos y$	1M	
$\frac{dy}{dx}=\frac{1}{2+\cos y}$		
$\frac{d^2y}{dx^2}=-1\cdot(2+\cos y)^{-2}(-\sin y)\frac{dy}{dx}$	1M	OR $\frac{0-1(-\sin y)\frac{dy}{dx}}{(2+\cos y)^2}$
Alternative Solution		
$1=2\frac{dy}{dx}+\cos y\cdot\frac{dy}{dx}$	1M	
$0=2\frac{d^2y}{dx^2}+\left[\cos y\frac{d^2y}{dx^2}+(-\sin y)\left(\frac{dy}{dx}\right)^2\right]$	1M	
$\sin y\left(\frac{1}{2+\cos y}\right)^2=(2+\cos y)\frac{d^2y}{dx^2}$		
$\frac{d^2y}{dx^2}=\frac{\sin y}{(2+\cos y)^3}$	1A	
	(3)	
5. (a) $\int\frac{dx}{\sqrt{9-x}}=\int-(9-x)^{\frac{-1}{2}}\,d(9-x)$	1M+1A	
Alternative Solution		
Let $u=9-x$.	1M	
$du=-dx$		
$\int\frac{dx}{\sqrt{9-x}}=\int-u^{\frac{-1}{2}}\,du$	1A	
$=-2u^{\frac{1}{2}}+C$		
$=-2\sqrt{9-x}+C$	1A	
(b) Let $x=3\sin\theta$.	1M	
$dx=3\cos\theta\,d\theta$		
$\int\frac{dx}{\sqrt{9-x^2}}=\int\frac{3\cos\theta\,d\theta}{\sqrt{9-9\sin^2\theta}}$		
$=\int d\theta$	1A	
$=\theta+C$		
$=\sin^{-1}\frac{x}{3}+C$	1A	
	(6)	

Solution	Marks	Remarks
6. (a) $\int xe^{-x}\mathrm{d}x = x(-e^{-x}) - \int (-e^{-x})\,\mathrm{d}x$	1M	
$= -xe^{-x} - e^{-x} + C$	1A	
(b) $\begin{cases} y = xe^{-x} \\ y = \dfrac{x}{e} \end{cases}$		
$xe^{-x} = \dfrac{x}{e}$		
$x\left(e^{-x} - \dfrac{1}{e}\right) = 0$		
$x = 0$ or 1	1A	For $x = 1$
$\therefore$ the area $= \int_0^1 \left(xe^{-x} - \dfrac{x}{e}\right)\mathrm{d}x$	1M	For $\int_a^b (y_1 - y_2)\,\mathrm{d}x$
$= \left[-xe^{-x} - e^{-x} - \dfrac{x^2}{2e}\right]_0^1$	1M	For using (a)
$= \left(-e^{-1} - e^{-1} - \dfrac{1}{2e}\right) - (-1)$		
$= 1 - \dfrac{5}{2e}$	1A	
	(6)	
7. (a) $A^2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$		
$= \begin{pmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \\ 2 & 0 & 2 \end{pmatrix}$		
$= 2A$	1	
Hence the statement is true for $n = 1$.		
Assume the statement is true for $n = k$, i.e. $A^{k+1} = 2^k A$.	1	
$A^{k+2} = A^{k+1}A$		
$= (2^k A)A$ by assumption	1	
$= 2^k A^2$		
$= 2^k \cdot 2A$ by the statement for $n = 1$		
$= 2^{k+1}A$	1	
Hence the statement is also true for $n = k + 1$.		
By the principle of mathematical induction, the statement is true for all positive integers n .	1	
(b) $\lvert A \rvert = 0$	1A	
Hence A^{-1} does not exist and so Willy arrives at a wrong conclusion by using A^{-1}	1	
	(7)	

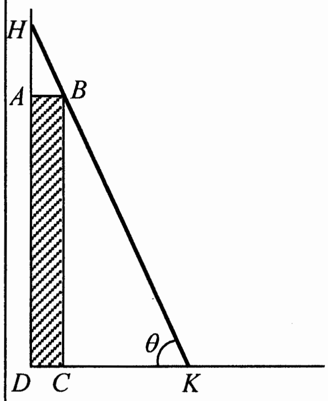
Solution	Marks	Remarks
8. (a) $\overrightarrow{OP}\times\overrightarrow{OQ}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -1 & 2 & 2\\ 1 & -1 & 2\end{vmatrix}$		
$=6\mathbf{i}+4\mathbf{j}-\mathbf{k}$	1A	
The volume of tetrahedron $OPQR=\frac{1}{6}\left\|(\overrightarrow{OP}\times\overrightarrow{OQ})\cdot\overrightarrow{OR}\right\|$		
$=\frac{1}{6}\left\|(6\mathbf{i}+4\mathbf{j}-\mathbf{k})\cdot(2\mathbf{i}-3\mathbf{j}+6\mathbf{k})\right\|$	1M	
$=1$	1A	
(b) $OR=\sqrt{2^2+(-3)^2+6^2}$		
$=7$	1A	
The area of $\Delta OPQ=\frac{1}{2}\left\|\overrightarrow{OP}\times\overrightarrow{OQ}\right\|$		
$=\frac{1}{2}\sqrt{6^2+4^2+(-1)^2}$		
$=\frac{\sqrt{53}}{2}$	1A	
Let h be the height of the tetrahedron with OPQ as base.		
$\therefore\ \frac{1}{3}\cdot\frac{\sqrt{53}}{2}h=1$	1M	
$h=\frac{6}{\sqrt{53}}$		
Let θ be the angle between the plane OPQ and the line OR.		
$\therefore\ \sin\theta=\frac{\frac{6}{\sqrt{53}}}{7}$	1M	
Alternative Solution		
$\left\|\overrightarrow{OP}\times\overrightarrow{OQ}\right\|=\sqrt{6^2+4^2+(-1)^2}$		
$=\sqrt{53}$	1A	
Let θ be the angle between the plane OPQ and the line OR.		
$\overrightarrow{OR}\cdot(\overrightarrow{OP}\times\overrightarrow{OQ})=\left\|\overrightarrow{OR}\right\|\cdot\left\|\overrightarrow{OP}\times\overrightarrow{OQ}\right\|\cos(\theta+90^\circ)$	1M+1M	1M for dot product formula 1M for $\theta+90^\circ$
$\cos(\theta+90^\circ)=\frac{2\cdot6-3\cdot4+6(-1)}{7\sqrt{53}}$		
$\theta\approx6.8^\circ$	1A	
	(8)	

Solution	Marks	Remarks
9. (a) The augmented matrix is $\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 100 \\ 1 & 6 & 10 & 200 \end{array}\right)$		
$\sim \left(\begin{array}{ccc\|c} 1 & 1 & 1 & 100 \\ 0 & 5 & 9 & 100 \end{array}\right)$	1M	
Let $z = t$, where t is a real number.		
$\therefore\ y = 20 - \dfrac{9t}{5}$ and $x = 80 + \dfrac{4t}{5}$	1A+1A	
Alternative Solution Let $y = t$, where t is a real number. $\therefore\ z = \dfrac{100-5t}{9}$ and $x = \dfrac{800-4t}{9}$	1A+1A	OR let $x = t$ and so $y = 200 - \dfrac{9t}{4}$, $z = \dfrac{5t}{4} - 100$
(b) $\begin{cases} m + n + k = 100 \\ 0.5m + 3n + 5k = 100 \end{cases}$	1A	
$\therefore\ \begin{cases} m + n + k = 100 \\ m + 6n + 10k = 200 \end{cases}$		
By (a), if both $20 - \dfrac{9t}{5}$ and $80 + \dfrac{4t}{5}$ are integers, then t is a multiple of 5 . $m \geq 0$ gives $t \geq -100$ $n \geq 0$ gives $t \leq \dfrac{100}{9}$ $k \geq 0$ gives $t \geq 0$ Combining all the conditions above, we have $t = 0$, 5 or 10 .	1M	
Alternative Solution (1) By trying out different values of t , we see that m , n and k are all non-negative when $t = 0$, 5 or 10 (OR any two of these).	1M	
Alternative Solution (2) By trying out different values of t , we see that (m, n, k) can be $(80, 20, 0)$, $(84, 11, 5)$ or $(88, 2, 10)$ (OR any two of these).	1M	
Hence there are more than one set of combination of m , n and k and so Aubrey cannot be agreed with.	1	
	(6)	
10. (a) $HK = HB + BK$		
$= \left(\dfrac{24}{\cos\theta} + \dfrac{192}{\sin\theta}\right)$ cm	1A	OR $(24\sec\theta + 192\csc\theta)$ cm
	(1)	

Solution	Marks	Remarks

(b) $\dfrac{dHK}{d\theta} = -24(\cos\theta)^{-2}(-\sin\theta) - 192(\sin\theta)^{-2}\cos\theta$ — 1M — OR $24\sec\theta\tan\theta - 192\csc\theta\cot\theta$

$\dfrac{dHK}{d\theta} = 0$ when $\dfrac{24\sin\theta}{\cos^2\theta} = \dfrac{192\cos\theta}{\sin^2\theta}$ — 1M



$\tan^3\theta = 8$

$\tan\theta = 2$

$\theta = \tan^{-1}2$ — 1A

θ	$0<\theta<\tan^{-1}2$	$\theta=\tan^{-1}2$	$\tan^{-1}2<\theta<\dfrac{\pi}{2}$
$\dfrac{dHK}{d\theta}$	-ve	0	+ve

— 1M

When $\theta = \tan^{-1}2$, HK is minimum.

By (a), the shortest length of the ladder $= 24\cdot\dfrac{\sqrt{5}}{1} + 192\cdot\dfrac{\sqrt{5}}{2}$

$= 120\sqrt{5}$ cm — 1

(5)

(c) (i) $x + HK\cos\theta = AB + CK$

$x = -270\cos\theta + 24 + 192\cot\theta$ — 1M

$\dfrac{dx}{dt} = 270\sin\theta\cdot\dfrac{d\theta}{dt} - 192\csc^2\theta\cdot\dfrac{d\theta}{dt}$ -------------------------------- (*) — 1M

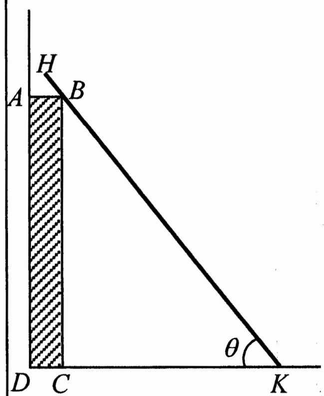
When $CK = 160$ cm, $\tan\theta = \dfrac{192}{160} = \dfrac{6}{5}$

$\therefore\ \sin\theta = \dfrac{6}{\sqrt{61}}$ — OR $\theta = 0.87605805$

$\dfrac{dx}{dt} = 270\left(\dfrac{6}{\sqrt{61}}\right)(-0.1) - 192\left(\dfrac{\sqrt{61}}{6}\right)^2(-0.1)$ — OR $270\sin 0.87605805\cdot(-0.1) - 192\csc^2 0.87605805\cdot(-0.1)$

≈ 11.79 — 1A

i.e. the rate of change of x is 11.79 cm s^{-1} .



(ii) $y - x = 270\cos\theta$ — 1M

$\dfrac{dy}{dt} - \dfrac{dx}{dt} = -270\sin\theta\cdot\dfrac{d\theta}{dt}$

Alternative Solution

$y = 24 + 192\cot\theta$ — 1M

$\dfrac{dy}{dt} = -192\csc^2\theta\cdot\dfrac{d\theta}{dt}$

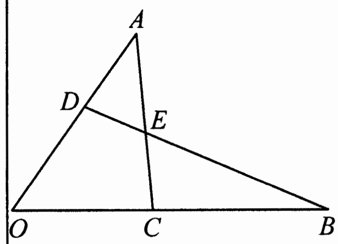
By (*), $\dfrac{dx}{dt} - \dfrac{dy}{dt} = 270\sin\theta\cdot\dfrac{d\theta}{dt}$

$\because\ \sin\theta > 0$ and $\dfrac{d\theta}{dt} < 0$ — 1M

$\therefore\ \dfrac{dy}{dt} > \dfrac{dx}{dt}$

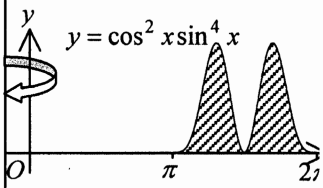
Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with. — 1

(6)

Solution	Marks	Remarks
11. (a) (i) $\overrightarrow{OC} = t\mathbf{b}$ $\therefore \quad \overrightarrow{OE} = \dfrac{\mathbf{a} + mt\mathbf{b}}{1+m}$	1M+1A	
(ii) $\overrightarrow{OD} = (1-t)\mathbf{a}$ $\therefore \quad \overrightarrow{OE} = \dfrac{n(1-t)\mathbf{a} + \mathbf{b}}{1+n}$	1A	
(iii) Comparing (i) and (ii), we have $\begin{cases} \dfrac{1}{1+m} = \dfrac{n(1-t)}{1+n} \text{------------------(1)} \\ \dfrac{mt}{1+m} = \dfrac{1}{1+n} \text{------------------(2)} \end{cases}$	1M	
$(2) \div (1)$: $mt = \dfrac{1}{n(1-t)}$ --------------------(3)	1M	
By (1), $\dfrac{1}{1+\dfrac{1}{nt(1-t)}} = \dfrac{n(1-t)}{1+n}$ $t(1+n) = nt(1-t) + 1$ $t = -nt^2 + 1$		
$n = \dfrac{1-t}{t^2}$	1	
By (3), $mt = \dfrac{1}{\dfrac{1-t}{t^2}(1-t)}$ $m = \dfrac{t}{(1-t)^2}$	1	
(iv) If $m = n$, then $\dfrac{t}{(1-t)^2} = \dfrac{1-t}{t^2}$.	1A	
$t^3 = (1-t)^3$ $t = \dfrac{1}{2}$ Hence C and D are the mid-points of OB and OA respectively. Therefore, E is the centroid of ΔOAB and Chris is agreed with.	1A	
	(9)	
(b) $\overrightarrow{AC} = t\mathbf{b} - \mathbf{a}$ $\overrightarrow{AC} \cdot \overrightarrow{OB} = (t\mathbf{b} - \mathbf{a}) \cdot \mathbf{b}$ $= 4t - \mathbf{a} \cdot \mathbf{b}$	1A	
When $AC \perp OB$, $\overrightarrow{AC} \cdot \overrightarrow{OB} = 0$ which gives $\mathbf{a} \cdot \mathbf{b} = 4t$ ------------------- (4)	1M	
$\overrightarrow{BD} = (1-t)\mathbf{a} - \mathbf{b}$ $\overrightarrow{BD} \cdot \overrightarrow{OA} = [(1-t)\mathbf{a} - \mathbf{b}] \cdot \mathbf{a}$ $= (1-t) - \mathbf{a} \cdot \mathbf{b}$	1A	
By (4), $\overrightarrow{BD} \cdot \overrightarrow{OA} = 1 - 5t$. So, $\overrightarrow{BD} \cdot \overrightarrow{OA} \neq 0$ in general. i.e. BD is not always perpendicular to OA and Francis is not agreed with.	1A	
	(4)	

Solution	Marks	Remarks
12. (a) (i) $A^{-1}=\begin{pmatrix}1 & p\\ -1 & 1\end{pmatrix}^{-1}$		
$=\dfrac{1}{\begin{vmatrix}1 & p\\ -1 & 1\end{vmatrix}}\begin{pmatrix}1 & 1\\ -p & 1\end{pmatrix}^T$	1M	
$=\dfrac{1}{1+p}\begin{pmatrix}1 & -p\\ 1 & 1\end{pmatrix}$	1A	
(ii) $A^{-1}MA=\dfrac{1}{1+p}\begin{pmatrix}1 & -p\\ 1 & 1\end{pmatrix}\begin{pmatrix}k-1 & k\\ 1 & 0\end{pmatrix}\begin{pmatrix}1 & p\\ -1 & 1\end{pmatrix}$		
$=\dfrac{1}{1+p}\begin{pmatrix}k-p-1 & k\\ k & k\end{pmatrix}\begin{pmatrix}1 & p\\ -1 & 1\end{pmatrix}$	1M+1A	OR $\dfrac{1}{1+p}\begin{pmatrix}1 & -p\\ 1 & 1\end{pmatrix}\begin{pmatrix}-1 & k+kp-p\\ 1 & p\end{pmatrix}$
$=\dfrac{1}{1+p}\begin{pmatrix}-1-p & k+kp-p-p^2\\ 0 & k+kp\end{pmatrix}$		
$=\begin{pmatrix}-1 & k-p\\ 0 & k\end{pmatrix}$	1	
(iii) By (ii), $(A^{-1}MA)^n=\begin{pmatrix}-1 & 0\\ 0 & k\end{pmatrix}^n$ for $p=k$		
$A^{-1}M^nA=\begin{pmatrix}(-1)^n & 0\\ 0 & k^n\end{pmatrix}$	1M	For either side
$M^n=\begin{pmatrix}1 & k\\ -1 & 1\end{pmatrix}\begin{pmatrix}(-1)^n & 0\\ 0 & k^n\end{pmatrix}\cdot\dfrac{1}{1+k}\begin{pmatrix}1 & -k\\ 1 & 1\end{pmatrix}$	1M	
$=\dfrac{1}{1+k}\begin{pmatrix}(-1)^n & k^{n+1}\\ (-1)^{n+1} & k^n\end{pmatrix}\begin{pmatrix}1 & -k\\ 1 & 1\end{pmatrix}$		OR $\dfrac{1}{1+k}\begin{pmatrix}1 & k\\ -1 & 1\end{pmatrix}\begin{pmatrix}(-1)^n & (-1)^{n+1}k\\ k^n & k^n\end{pmatrix}$
$=\dfrac{1}{1+k}\begin{pmatrix}k^{n+1}+(-1)^n & k^{n+1}+(-1)^{n+1}k\\ k^n+(-1)^{n+1} & k^n+(-1)^nk\end{pmatrix}$	1A	
	(8)	
(b) $\begin{pmatrix}x_n\\ x_{n-1}\end{pmatrix}=M\begin{pmatrix}x_{n-1}\\ x_{n-2}\end{pmatrix}$ where $M=\begin{pmatrix}1 & 2\\ 1 & 0\end{pmatrix}$ after substituting $k=2$		
$=M^2\begin{pmatrix}x_{n-2}\\ x_{n-3}\end{pmatrix}$	1M	
$=\cdots$		
$=M^{n-2}\begin{pmatrix}x_2\\ x_1\end{pmatrix}$	1A	
$=\dfrac{1}{1+2}\begin{pmatrix}2^{n-1}+(-1)^{n-2} & 2^{n-1}+(-1)^{n-1}2\\ 2^{n-2}+(-1)^{n-1} & 2^{n-2}+(-1)^{n-2}2\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}$ by (a)(iii)		
$\therefore\ x_n=\dfrac{2^{n-1}+(-1)^{n-2}}{3}$	1A	OR $\dfrac{2^{n-1}+(-1)^n}{3}$
	(3)	

Solution	Marks	Remarks
13. (a) $1-\cos 4\theta - 2\cos 2\theta \sin^2 2\theta$		
$= 2\sin^2 2\theta - 2\cos 2\theta \sin^2 2\theta$	1M	For $1-\cos 4\theta = 2\sin^2 2\theta$ OR $1-\cos 2\theta = 2\sin^2\theta$
$= 2\sin^2 2\theta (1-\cos 2\theta)$		
$= 2(2\sin\theta\cos\theta)^2(2\sin^2\theta)$		
$= 16\cos^2\theta\sin^4\theta$	1	
	(2)	
(b) $\int_0^{n\pi} \cos^2 x \sin^4 x \, dx$		
$= \int_0^{n\pi} \frac{1-\cos 4x - 2\cos 2x \sin^2 2x}{16} dx$ by (a)		
$= \frac{1}{16}\int_0^{n\pi} (1-\cos 4x)\, dx - \frac{1}{16}\int_0^{n\pi} \sin^2 2x \cdot 2\cos 2x \, dx$	1M	
$= \frac{1}{16}\left[x - \frac{\sin 4x}{4}\right]_0^{n\pi} - \frac{1}{16}\int_{x=0}^{n\pi} \sin^2 2x \, d\sin 2x$	1M	For $d\sin 2x$
$= \frac{1}{16}\left[\left(n\pi - \frac{\sin 4n\pi}{4}\right) - 0\right] - \frac{1}{16}\left[\frac{\sin^3 2x}{3}\right]_0^{n\pi}$	1A	For $\frac{\sin^3 2x}{3}$
Alternative Solution		
$= \frac{1}{16}\int_0^{n\pi} (1-\cos 4x)\, dx - \frac{1}{16}\int_0^{n\pi} \sin 4x \sin 2x \, dx$		
$= \frac{1}{16}\left[x - \frac{\sin 4x}{4}\right]_0^{n\pi} - \frac{1}{16}\int_0^{n\pi} \frac{\cos 2x - \cos 6x}{2} dx$	1M	For $\frac{\cos 2x - \cos 6x}{2}$
$= \frac{1}{16}\left[\left(n\pi - \frac{\sin 4n\pi}{4}\right) - 0\right] - \frac{1}{32}\left[\frac{\sin 2x}{2} - \frac{\sin 6x}{6}\right]_0^{n\pi}$	1A	For $\frac{\sin 2x}{2} - \frac{\sin 6x}{6}$
$= \frac{n\pi}{16}$	1	
	(4)	
(c) Let $x = k-u$.	1M	
$\therefore\ dx = -du$		
When $x=0, u=k$; when $x=k, u=0$.		
$\int_0^k x f(x)\, dx = \int_k^0 (k-u) f(k-u)(-du)$		
$= \int_0^k (k-u) f(u)\, du$	1M+1M	1M for reversing the limits 1M for $f(k-u) = f(u)$
$= k\int_0^k f(u)\, du - \int_0^k u f(u)\, du$		
$= k\int_0^k f(x)\, dx - \int_0^k x f(x)\, dx$		
$\therefore\ 2\int_0^k x f(x)\, dx = k\int_0^k f(x)\, dx$		
i.e. $\int_0^k x f(x)\, dx = \frac{k}{2}\int_0^k f(x)\, dx$	1	
	(4)	

Solution	Marks	Remarks
(d) Let $f(x)=\cos^2 x\sin^4 x$		
$f(\pi-x)=\cos^2(\pi-x)\sin^4(\pi-x)$ $=(-\cos x)^2(\sin x)^4$ $=\cos^2 x\sin^4 x$ $=f(x)$		
$f(2\pi-x)=\cos^2(2\pi-x)\sin^4(2\pi-x)$ $=(\cos x)^2(-\sin x)^4$ $=f(x)$	1M	
The volume of the solid of revolution		
$=2\pi\int_{\pi}^{2\pi} x\cos^2 x\sin^4 x\,dx$	1M	
$=2\pi\left(\int_0^{2\pi} x\cos^2 x\sin^4 x\,dx-\int_0^{\pi} x\cos^2 x\sin^4 x\,dx\right)$		
$=2\pi\cdot\frac{2\pi}{2}\int_0^{2\pi}\cos^2 x\sin^4 x\,dx-2\pi\cdot\frac{\pi}{2}\int_0^{\pi}\cos^2 x\sin^4 x\,dx$ by (c)	1M	
$=2\pi^2\left(\frac{2\pi}{16}\right)-\pi^2\left(\frac{\pi}{16}\right)$ by (b)		
$=\frac{3\pi^3}{16}$	1A	
	(4)	