Module 2 (Algebra and Calculus) Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. In the marking scheme, marks are classified into the following three categories:

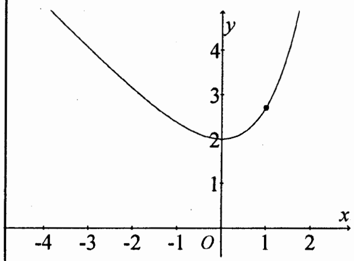
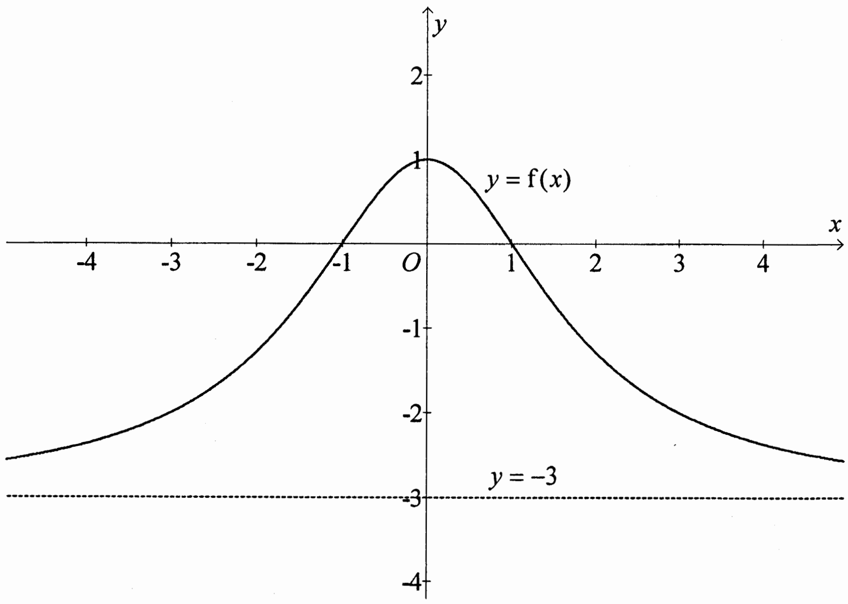
'M' marks	–	awarded for applying correct methods
'A' marks	–	awarded for the accuracy of the answers
Marks without 'M' or 'A'	–	awarded for correctly completing a proof or arriving at an answer given in the question.

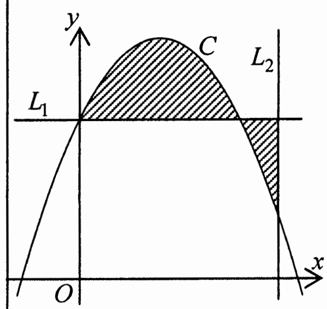
   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.

6. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by $\boxed{\text{solid rectangles}}$.

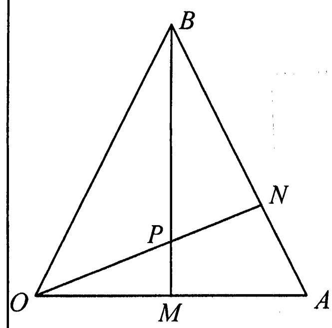
7. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted.

Solution	Marks	Remarks
1. $\frac{d}{dx}(\sin 2x) = \lim_{h\to 0}\frac{\sin 2(x+h)-\sin 2x}{h}$	1M	
$= \lim_{h\to 0}\left(\frac{2}{h}\cos\frac{2x+2h+2x}{2}\sin\frac{2x+2h-2x}{2}\right)$	1M	
$= \lim_{h\to 0}\left[2\cos(2x+h)\frac{\sin h}{h}\right]$		
$= 2\lim_{h\to 0}\cos(2x+h)\cdot\lim_{h\to 0}\frac{\sin h}{h}$	1M	
Alternative Solution		
$= \lim_{h\to 0}\frac{\sin 2h\cos 2x+\cos 2h\sin 2x-\sin 2x}{h}$	1M	
$= \lim_{h\to 0}\frac{\sin 2h\cos 2x-\sin 2x\cdot 2\sin^2 h}{h}$		
$= 2\cos 2x\cdot\lim_{h\to 0}\frac{\sin 2h}{2h}-2\sin 2x\cdot\lim_{h\to 0}\sin h\cdot\lim_{h\to 0}\frac{\sin h}{h}$	1M	
$= 2\cos 2x$	1A	
	(4)	
2. $(1+ax)^n = 1+C_1^n ax + C_2^n (ax)^2+\cdots$		OR general term $= C_r^n (ax)^r$
$\begin{cases} na=-20 & \text{-----------------------(1)} \\ \frac{n(n-1)}{2}a^2=180 & \text{-------------------(2)}\end{cases}$	1M	
$(2)\div(1)^2$:		
$\frac{n-1}{2n}=\frac{180}{400}$	1M	
$n=10$	1A	
$\therefore\ a=-2$	1A	
	(4)	
3. For $n=1$,		
L.H.S. $1+\frac{1}{1\times 4}=\frac{5}{4}$ and R.H.S. $=\frac{4(1)+1}{3(1)+1}=\frac{5}{4}$		
$\therefore$ L.H.S. = R.H.S. and the statement is true for $n=1$.	1	
Assume $1+\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3k-2)(3k+1)}=\frac{4k+1}{3k+1}$, where k is a positive integer.	1	
$1+\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3k-2)(3k+1)}+\frac{1}{[3(k+1)-2][3(k+1)+1]}$		
$=\frac{4k+1}{3k+1}+\frac{1}{(3k+1)(3k+4)}$ by the assumption	1	
$=\frac{(12k^2+19k+4)+1}{(3k+1)(3k+4)}$		
$=\frac{(3k+1)(4k+5)}{(3k+1)(3k+4)}$		
$=\frac{4(k+1)+1}{3(k+1)+1}$	1	
Hence the statement is true for $n=k+1$.		
By the principle of mathematical induction, the statement is true for all positive integers n .	1	Follow through
	(5)	

Solution	Marks	Remarks
4. (a) $\dfrac{dy}{dx}=e^x-1$		
$y=\int(e^x-1)dx$		
$=e^x-x+C$	1A	
Since the curve passes through the point $(1,e)$, $e=e^1-1+C$.	1M	
i.e. $C=1$		
$\therefore\ y=e^x-x+1$	1A	
(b) The curve cuts the y-axis at $(0,2)$.		
When $x=0$, $\dfrac{dy}{dx}=0$.	1M	
Hence the equation of tangent to the curve at $(0,2)$ is		
$y-2=0(x-0)$		
$y=2$	1A	
	(5)	
5. (a) $f(x)=\dfrac{3-3x^2}{3+x^2}$		
$\therefore\ f(0)=1,\ f(1)=0$ and $f(-1)=0$		
$\therefore$ maximum point is $(0,1)$,	1A	
and points of inflexion are $(1,0)$ and $(-1,0)$.	1A	For both
(b) Since $x^2+3>0$, there is no vertical asymptote.		
$f(x)=-3+\dfrac{12}{x^2+3}$	1M	OR $f(x)=\dfrac{\frac{3}{x^2}-3}{\frac{3}{x^2}+1}$
When $x\to\pm\infty$, $y\to-3$.		
Hence $y=-3$ is a horizontal asymptote.	1A	
(c) 	1A 1A	For shape of $y=f(x)$ For all correct
	(6)	

Solution	Marks	Remarks
6. (a) Area $=\int_0^4\left[\left(\frac{-x^2}{2}+2x+4\right)-4\right]dx+\int_4^5\left[4-\left(\frac{-x^2}{2}+2x+4\right)\right]dx$	1M	
$=\int_0^4\left(\frac{-x^2}{2}+2x\right)dx+\int_4^5\left(\frac{x^2}{2}-2x\right)dx$		
$=\left[\frac{-x^3}{6}+x^2\right]_0^4+\left[\frac{x^3}{6}-x^2\right]_4^5$	1M	
$=\frac{13}{2}$	1A	
(b) Volume $=\pi\int_0^5\left(\frac{-x^2}{2}+2x+4-4\right)^2 dx$	1M+1A	
$=\pi\int_0^5\left(\frac{x^4}{4}-2x^3+4x^2\right)dx$		
$=\pi\left[\frac{x^5}{20}-\frac{x^4}{2}+\frac{4x^3}{3}\right]_0^5$		
$=\frac{125\pi}{12}$	1A	
	(6)	
7. (a) R.H.S. $=\frac{\sin 2x}{1+\cos 2x}$		
$=\frac{2\sin x\cos x}{1+2\cos^2 x-1}$	1M	For either formula
$=\frac{\sin x}{\cos x}$		
$=\tan x$	1	
$=$ L.H.S.		
(b) R.H.S. $=\frac{\sin 8y\cos 4y\cos 2y}{(1+\cos 8y)(1+\cos 4y)(1+\cos 2y)}$		
$=\tan 4y\cdot\frac{\cos 4y\cos 2y}{(1+\cos 4y)(1+\cos 2y)}$ by (a)	1M	
$=\frac{\sin 4y\cos 2y}{(1+\cos 4y)(1+\cos 2y)}$	1M	
$=\tan 2y\cdot\frac{\cos 2y}{1+\cos 2y}$ by (a)		
$=\frac{\sin 2y}{1+\cos 2y}$		
$=\tan y$ by (a)	1	
Alternative Solution		
$=\frac{\sin 8y\cos 4y\cos 2y}{\left(\frac{\sin 8y}{\tan 4y}\right)\left(\frac{\sin 4y}{\tan 2y}\right)\left(\frac{\sin 2y}{\tan y}\right)}$ by (a)	1M	
$=\frac{\sin 8y}{\sin 8y}\cdot\frac{\tan 4y\cos 4y}{\sin 4y}\cdot\frac{\tan 2y\cos 2y}{\sin 2y}\cdot\tan y$		
$=\tan y$	1M + 1	1M for $\tan x\cos x=\sin x$
$=$ L.H.S.		
	(5)	

Solution	Marks	Remarks
8. (a) $\begin{pmatrix}1&k&0\\0&1&1\\k&0&0\end{pmatrix}^{-1}=\dfrac{1}{\begin{vmatrix}1&k&0\\0&1&1\\k&0&0\end{vmatrix}}\begin{pmatrix}0&k&-k\\0&0&k^2\\k&-1&1\end{pmatrix}^T$	1M+1A	1M for minors
$=\dfrac{1}{k^2}\begin{pmatrix}0&0&k\\k&0&-1\\-k&k^2&1\end{pmatrix}$	1A	
(b) $\begin{pmatrix}1&k&0\\0&1&1\\k&0&0\end{pmatrix}\begin{pmatrix}x\\1\\z\end{pmatrix}=\begin{pmatrix}2\\2\\1\end{pmatrix}$		
$\begin{pmatrix}x\\1\\z\end{pmatrix}=\dfrac{1}{k^2}\begin{pmatrix}0&0&k\\k&0&-1\\-k&k^2&1\end{pmatrix}\begin{pmatrix}2\\2\\1\end{pmatrix}$ by (a)	1M	
$=\dfrac{1}{k^2}\begin{pmatrix}k\\2k-1\\2k^2-2k+1\end{pmatrix}$		
From the second row, we have $\dfrac{2k-1}{k^2}=1$.		
Alternative Solution $\begin{pmatrix}x+k\\1+z\\kx\end{pmatrix}=\begin{pmatrix}2\\2\\1\end{pmatrix}$ From the first and third rows, we have $x+k=2$ and $x=\dfrac{1}{k}$. $\therefore\ \dfrac{1}{k}+k=2$.	1M	
i.e. $k^2-2k+1=0$ $k=1$	1A	
	(5)	
9. (a) The augmented matrix is $\left(\begin{array}{ccc\|c}1&-a&1&2\\2&1-2a&2-b&a+4\\3&1-3a&3-ab&4\end{array}\right)$		
$\sim\left(\begin{array}{ccc\|c}1&-a&1&2\\0&1&-b&a\\0&1&-ab&-2\end{array}\right)$	1M	
$\sim\left(\begin{array}{ccc\|c}1&-a&1&2\\0&1&-b&a\\0&0&ab-b&a+2\end{array}\right)$		
Hence the system has infinitely many solutions when $\begin{cases}b(a-1)=0\\a+2=0\end{cases}$	1M	
i.e. $a=-2$ and $b=0$	1A	For both

Solution	Marks	Remarks
(b) The system becomes $\begin{cases} x + 2y + z = 2 \\ 2x + 5y + 2z = 2 \\ 3x + 7y + 3z = 4 \end{cases}$	1M	
i.e. $\begin{cases} x+z=6 \\ y=-2 \end{cases}$		
$(x,\,y,\,z)=(6-t,\,-2,\,t)$ for any real number t	1A	OR $(t,-2,6-t)$
	(5)	
10. (a) $\overrightarrow{ON}=\dfrac{k\overrightarrow{OA}+\overrightarrow{OB}}{k+1}$	1M	
$=\dfrac{k(2\mathbf{i})+(\mathbf{i}+2\mathbf{j})}{k+1}$		
$=\dfrac{(2k+1)\mathbf{i}+2\mathbf{j}}{k+1}$	1A	
(b) $\because$ $\overrightarrow{MB}=2\mathbf{j}$, $\therefore$ $BM \perp OA$	1M	
Since A , N , P and M are concyclic, $ON \perp AB$.		
$\therefore$ $\overrightarrow{ON}\cdot\overrightarrow{AB}=0$	1M	
$\dfrac{(2k+1)\mathbf{i}+2\mathbf{j}}{k+1}\cdot(\mathbf{i}+2\mathbf{j}-2\mathbf{i})=0$		
$-(2k+1)+2\cdot 2=0$		
$k=\dfrac{3}{2}$	1A	
	(5)	
11. (a) $\dfrac{\mathrm{d}}{\mathrm{d}\theta}\ln(\sec\theta+\tan\theta)=\dfrac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}$	1M	
$=\sec\theta$		
Hence $\displaystyle\int\sec\theta\,\mathrm{d}\theta=\int\frac{\mathrm{d}}{\mathrm{d}\theta}\ln(\sec\theta+\tan\theta)\,\mathrm{d}\theta$		
$=\ln(\sec\theta+\tan\theta)+C$	1	
<u>Alternative Solution</u>		
$\displaystyle\int\sec\theta\,\mathrm{d}\theta=\int\frac{\sec\theta(\sec\theta+\tan\theta)}{\sec\theta+\tan\theta}\,\mathrm{d}\theta$	1M	
Let $u=\sec\theta+\tan\theta$ which gives $\mathrm{d}u=(\sec\theta\tan\theta+\sec^2\theta)\mathrm{d}\theta$.		
$\therefore$ $\displaystyle\int\sec\theta\,\mathrm{d}\theta=\int\frac{\mathrm{d}u}{u}$		
$=\ln\lvert u\rvert+C$		
$=\ln(\sec\theta+\tan\theta)+C$ since $\sec\theta+\tan\theta>0$ for $0<\theta<\dfrac{\pi}{2}$	1	
	(2)	

Solution	Marks	Remarks
(b) (i) Let $u=\sec\theta$, where $0<\theta<\frac{\pi}{2}$.	1M	
$\therefore\ \mathrm{d}u=\sec\theta\tan\theta\,\mathrm{d}\theta$		
$\int\frac{\mathrm{d}u}{\sqrt{u^2-1}}=\int\frac{\sec\theta\tan\theta\,\mathrm{d}\theta}{\sqrt{\sec^2\theta-1}}$		
$=\int\sec\theta\,\mathrm{d}\theta$ since $\tan\theta>0$ for $0<\theta<\frac{\pi}{2}$		
$=\ln(\sec\theta+\tan\theta)+C$ by (a)		
$=\ln(\sec\theta+\sqrt{\sec^2\theta-1})+C$ since $\tan\theta>0$ for $0<\theta<\frac{\pi}{2}$		
$=\ln(u+\sqrt{u^2-1})+C$	1	
(ii) $\int_0^1\frac{2x}{\sqrt{x^4+4x^2+3}}\mathrm{d}x=\int_0^1\frac{2x}{\sqrt{(x^2+2)^2-1}}\mathrm{d}x$		
Let $u=x^2+2$ which gives $\mathrm{d}u=2x\,\mathrm{d}x$.	1M	
When $x=0$, $u=2$; when $x=1$, $u=3$.		
$\therefore\ \int_0^1\frac{2x}{\sqrt{x^4+4x^2+3}}\mathrm{d}x=\int_2^3\frac{\mathrm{d}u}{\sqrt{u^2-1}}$		
$=\left[\ln(u+\sqrt{u^2-1})\right]_2^3$ by (i)	1M	For primitive function
$=\ln(3+\sqrt{8})-\ln(2+\sqrt{3})$		
$=\ln\left(\frac{3+2\sqrt{2}}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)$		
$=\ln(6+4\sqrt{2}-3\sqrt{3}-2\sqrt{6})$	1	
	(5)	
(c) $t=\tan\phi$		
$\frac{\mathrm{d}t}{\mathrm{d}\phi}=\sec^2\phi$		
$=1+\tan^2\phi$		
$\therefore\ \frac{\mathrm{d}\phi}{\mathrm{d}t}=\frac{1}{1+t^2}$	1	
$\cos^2\phi=\frac{1}{\sec^2\phi}$		
$=\frac{1}{1+t^2}$	1A	
$\int_0^{\frac{\pi}{4}}\frac{\tan\phi}{\sqrt{1+2\cos^2\phi}}\mathrm{d}\phi=\int_0^1\frac{t}{\sqrt{1+\frac{2}{1+t^2}}}\cdot\frac{1}{1+t^2}\mathrm{d}t$ where $t=\tan\phi$	1M	
$=\int_0^1\frac{t}{\sqrt{(3+t^2)(1+t^2)}}\mathrm{d}t$	1A	
$=\frac{1}{2}\int_0^1\frac{2t}{\sqrt{t^4+4t^2+3}}\mathrm{d}t$		
$=\frac{1}{2}\ln(6+4\sqrt{2}-3\sqrt{3}-2\sqrt{6})$	1A	OR $\ln\sqrt{6+4\sqrt{2}-3\sqrt{3}-2\sqrt{6}}$
	(5)	

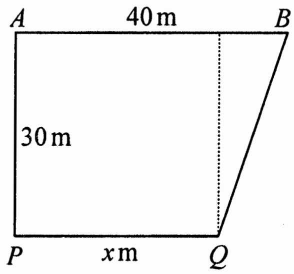
Solution	Marks	Remarks

12. (a) (i) $T=\dfrac{PQ}{7}+\dfrac{QB}{1.4}$ — 1M

$=\dfrac{x}{7}+\dfrac{5\sqrt{30^2+(40-x)^2}}{7}$

$=\dfrac{x+5\sqrt{x^2-80x+2500}}{7}$ — 1A — OR $\dfrac{x}{7}+\dfrac{\sqrt{x^2-80x+2500}}{1.4}$

(ii) When T is minimum, $\dfrac{\mathrm{d}T}{\mathrm{d}x}=0$.



$\dfrac{1}{7}\left[1+\dfrac{5(2x-80)}{2\sqrt{x^2-80x+2500}}\right]=0$ — 1M

$5(x-40)=-\sqrt{x^2-80x+2500}$

$25x^2-2000x+40000=x^2-80x+2500$

$2x^2-160x+3125=0$ — 1

$\therefore\ x=40-\dfrac{5\sqrt{6}}{2}$ or $40+\dfrac{5\sqrt{6}}{2}$ (rejected by checking)

x	$0<x<40-\dfrac{5\sqrt{6}}{2}$	$x=40-\dfrac{5\sqrt{6}}{2}$	$x>40-\dfrac{5\sqrt{6}}{2}$
$\dfrac{\mathrm{d}T}{\mathrm{d}x}$	–	0	+

So, when T is minimum, $x=40-\dfrac{5\sqrt{6}}{2}$. — 1M

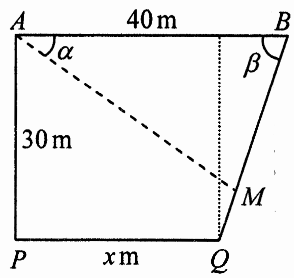
$QB=\sqrt{30^2+\left[40-\left(40-\dfrac{5\sqrt{6}}{2}\right)\right]^2}$

$=\dfrac{25\sqrt{6}}{2}$ m — 1

(6)

(b) (i) $\sin\beta=\dfrac{30}{\frac{25\sqrt{6}}{2}}=\dfrac{2\sqrt{6}}{5}$ — 1A

$\cos\beta=\dfrac{40-\left(40-\frac{5\sqrt{6}}{2}\right)}{\frac{25\sqrt{6}}{2}}=\dfrac{1}{5}$ — 1A



In ΔMAB , $\dfrac{MB}{\sin\alpha}=\dfrac{AB}{\sin(\pi-\alpha-\beta)}$. — 1M

$MB=\dfrac{40\sin\alpha}{\sin(\alpha+\beta)}$

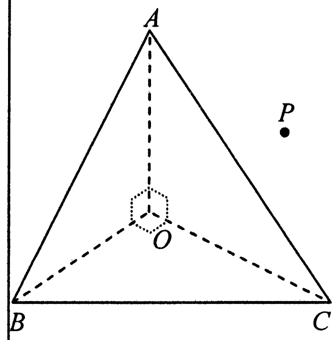
$=\dfrac{40\sin\alpha}{\sin\alpha\cos\beta+\cos\alpha\sin\beta}$ — 1M

$=\dfrac{40\sin\alpha}{\frac{1}{5}\sin\alpha+\frac{2\sqrt{6}}{5}\cos\alpha}$

$=\dfrac{200\tan\alpha}{\tan\alpha+2\sqrt{6}}$ — 1

Solution	Marks	Remarks
(ii) $\dfrac{\mathrm{d}MB}{\mathrm{d}t}=200\cdot\dfrac{(\tan\alpha+2\sqrt{6})\sec^2\alpha-\tan\alpha\sec^2\alpha}{(\tan\alpha+2\sqrt{6})^2}\cdot\dfrac{\mathrm{d}\alpha}{\mathrm{d}t}$	1M	
$=\dfrac{400\sqrt{6}\sec^2\alpha}{(\tan\alpha+2\sqrt{6})^2}\cdot\dfrac{\mathrm{d}\alpha}{\mathrm{d}t}$		
$\therefore\quad -1.4=\dfrac{400\sqrt{6}\sec^2 0.2}{(\tan 0.2+2\sqrt{6})^2}\cdot\dfrac{\mathrm{d}\alpha}{\mathrm{d}t}$		
$\dfrac{\mathrm{d}\alpha}{\mathrm{d}t}\approx -0.0357\ \text{rad s}^{-1}$	1A	
	(7)	
13. (a) (i) $MN=\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}e&f\\g&h\end{pmatrix}$		
$=\begin{pmatrix}ae+bg&af+bh\\ce+dg&cf+dh\end{pmatrix}$		
$\operatorname{tr}(MN)=ae+bg+cf+dh$	1A	Either one
$NM=\begin{pmatrix}e&f\\g&h\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}$		
$=\begin{pmatrix}ea+fc&eb+fd\\ga+hc&gb+hd\end{pmatrix}$		
$\operatorname{tr}(NM)=ea+fc+gb+hd$		
$\therefore\quad \operatorname{tr}(MN)=\operatorname{tr}(NM)$	1	
(ii) $\operatorname{tr}(BAB^{-1})=1+3$		
$\operatorname{tr}(AB^{-1}B)=4$ by (a)(i)	1M	OR $\operatorname{tr}(B^{-1}BA)=4$
$\operatorname{tr}(A)=4$	1	
(iii) $BAB^{-1}=\begin{pmatrix}1&0\\0&3\end{pmatrix}$		
$\lvert BAB^{-1}\rvert=\begin{vmatrix}1&0\\0&3\end{vmatrix}$		
$\lvert B\rvert\cdot\lvert A\rvert\cdot\lvert B^{-1}\rvert=1\cdot3-0\cdot0$	1M	
$\lvert B\rvert\cdot\lvert A\rvert\cdot\lvert B\rvert^{-1}=3$		
$\lvert A\rvert=3$	1A	
	(6)	

Solution	Marks	Remarks
(b) (i) $C\begin{pmatrix}x\\y\end{pmatrix}=\lambda_1\begin{pmatrix}x\\y\end{pmatrix}$		
$\begin{pmatrix}px+qy\\rx+sy\end{pmatrix}=\begin{pmatrix}\lambda_1 x\\\lambda_1 y\end{pmatrix}$		
$\begin{cases}(p-\lambda_1)x+qy=0\\rx+(s-\lambda_1)y=0\end{cases}$	1A	
Since this system of equations has non-zero solutions $\begin{pmatrix}x\\y\end{pmatrix}$, $\begin{vmatrix}p-\lambda_1 & q\\r & s-\lambda_1\end{vmatrix}=0$.	1	
Similarly, $C\begin{pmatrix}x\\y\end{pmatrix}=\lambda_2\begin{pmatrix}x\\y\end{pmatrix}$ gives $\begin{vmatrix}p-\lambda_2 & q\\r & s-\lambda_2\end{vmatrix}=0$.	1	
(ii) By (b)(i), λ_1 and λ_2 are the roots of the equation $\begin{vmatrix}p-\lambda & q\\r & s-\lambda\end{vmatrix}=0$	1M	
$(p-\lambda)(s-\lambda)-qr=0$		
$\lambda^2-(p+s)\lambda+ps-qr=0$		
$\lambda^2-\text{tr}(C)\cdot\lambda+\lvert C\rvert=0$	1	
	(5)	
(c) $A\begin{pmatrix}x\\y\end{pmatrix}=\lambda\begin{pmatrix}x\\y\end{pmatrix}$ for some non-zero matrices $\begin{pmatrix}x\\y\end{pmatrix}$		
$\lambda^2-\text{tr}(A)\cdot\lambda+\lvert A\rvert=0$ by (b)(ii)	1M	For either one
$\lambda^2-4\lambda+3=0$ by (a)(ii) & (a)(iii)		
$\lambda=1$ or 3	1A	
	(2)	

Solution	Marks	Remarks
14. (a) (i) $\overrightarrow{AP}\cdot\overrightarrow{BP}=(\mathbf{p}-\mathbf{a})\cdot(\mathbf{p}-\mathbf{b})$	1M	For $\mathbf{p}-\mathbf{a}$ or $\mathbf{p}-\mathbf{b}$
$=\mathbf{p}\cdot\mathbf{p}-\mathbf{a}\cdot\mathbf{p}-\mathbf{b}\cdot\mathbf{p}+\mathbf{a}\cdot\mathbf{b}$		
$=\mathbf{p}\cdot\mathbf{p}-(\mathbf{a}+\mathbf{b})\cdot\mathbf{p}$ $(\because\ \mathbf{a}\cdot\mathbf{b}=0)$	1	
(ii) $\overrightarrow{AP}\cdot\overrightarrow{BP}+\overrightarrow{BP}\cdot\overrightarrow{CP}+\overrightarrow{CP}\cdot\overrightarrow{AP}=0$		
By (i) and some similar results, we have		
$\mathbf{p}\cdot\mathbf{p}-(\mathbf{a}+\mathbf{b})\cdot\mathbf{p}+\mathbf{p}\cdot\mathbf{p}-(\mathbf{b}+\mathbf{c})\cdot\mathbf{p}+\mathbf{p}\cdot\mathbf{p}-(\mathbf{c}+\mathbf{a})\cdot\mathbf{p}=0$	1M	
$3\mathbf{p}\cdot\mathbf{p}-2(\mathbf{a}+\mathbf{b}+\mathbf{c})\cdot\mathbf{p}=0$		
$3\mathbf{p}\cdot\mathbf{p}-2(3\mathbf{d})\cdot\mathbf{p}=0$	1M	
$\mathbf{p}\cdot\mathbf{p}=2\mathbf{p}\cdot\mathbf{d}$ ---------------------------------- (*)	1	
(iii) $\lvert\mathbf{p}-\mathbf{d}\rvert^2=(\mathbf{p}-\mathbf{d})\cdot(\mathbf{p}-\mathbf{d})$	1M	
$=\mathbf{p}\cdot\mathbf{p}-2\mathbf{p}\cdot\mathbf{d}+\mathbf{d}\cdot\mathbf{d}$		
$=\mathbf{d}\cdot\mathbf{d}$ by (*)		
$=\lvert\mathbf{d}\rvert^2$		
Hence $\lvert\mathbf{p}-\mathbf{d}\rvert=\lvert\mathbf{d}\rvert$.	1	
$\therefore\ \lvert\overrightarrow{DP}\rvert=\lvert\overrightarrow{OD}\rvert$		
$\therefore\ PD=OD$ ------------------------------ (**)		
Thus, the distance between P and D is a constant and therefore P lies on the sphere centred at D with fixed radius.	1	
	(8)	
(b) (i) Yes. Since O satisfies (**), O lies on the sphere mentioned in (i).	1A	OR "Since OD is equal to the radius of the sphere, ..."
(ii) Yes. Since $\overrightarrow{DP_1}\times\overrightarrow{DP_2}=\overrightarrow{DP_2}\times\overrightarrow{DP_3}$, the normal vector of the plane containing D, P_1 and P_2 equals the normal vector of the plane containing D, P_2 and P_3.	1M	
Thus the plane containing D, P_1 and P_2 is parallel to the plane containing D, P_2 and P_3. Since D and P_2 are common points of the planes, D, P_1, P_2 and P_3 are on the same plane.	1M	
Since D is the centre of the sphere and P_1, P_2 and P_3 lie on the largest circle on the sphere, the radius of the circle equals the radius of the sphere, which is OD.	1A	Follow through
	(4)	