6. Applications of Definite Integration

6. (a) Areas of Plane Figures

(1986-CE-A MATH 2 #11) (20 marks)

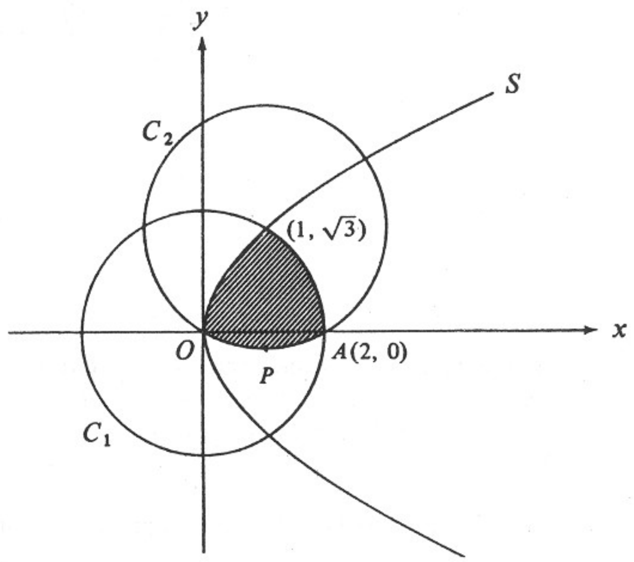
11. (a) (i) Using the substitution $x = 2\sin\theta$, evaluate

$$\int_1^2 \sqrt{4-x^2}\,\mathrm{d}x$$

(ii) Express $3+2x-x^2$ in the form $a^2-(x-b)^2$ where a and b are constants.

Using the substitution $x-b = a\sin\theta$, evaluate $\int_0^1 \sqrt{3+2x-x^2}\,\mathrm{d}x$.

(b)



In Figure 1, the shaded region is bounded by the two circles

C_1 : $x^2+y^2 = 4$,

C_2 : $(x-1)^2+(y-\sqrt{3})^2 = 4$

and the parabola

S : $y^2 = 3x$.

(i) $P(x, y)$ is a point on the minor arc OA of C_2 . Express y in terms of x .

(ii) Find the area of the shaded region.

(1991-CE-A MATH 2 #05) (7 marks)

5. The slope at any point (x, y) of a curve C is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 4-2x$ and C passes through the point $(1, 0)$.

(a) Find an equation of C .

(b) Find the area of the finite region bounded by C and the x-axis.

(1992-CE-A MATH 2 #06) (6 marks)

6.

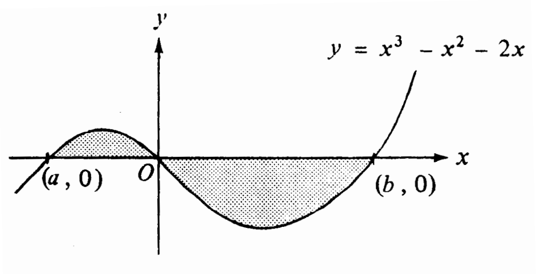


Figure 1

The curve $y = x^3 - x^2 - 2x$ cuts the x-axis at the origin and the points $(a, 0)$ and $(b, 0)$, as shown in Figure 1.

(a) Find the values of a and b .

(b) Find the total area of the shaded parts.

(1993-CE-A MATH 2 #05) (6 marks)

5.

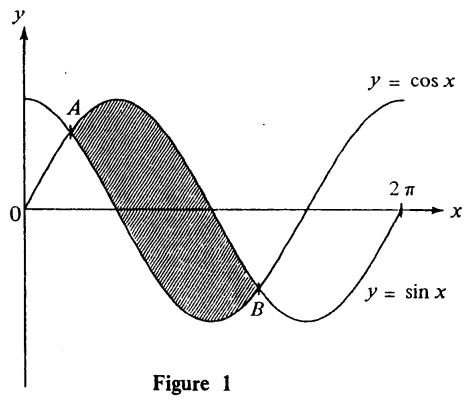


Figure 1

Figure 1 shows the curves of $y = \sin x$ and $y = \cos x$, where $0 \le x \le 2\pi$, intersecting at points A and B .

(a) Find the coordinates of A and B .

(b) Find the area of the shaded region as shown in Figure 1.

(1994-CE-A MATH 2 #07) (6 marks)

7.

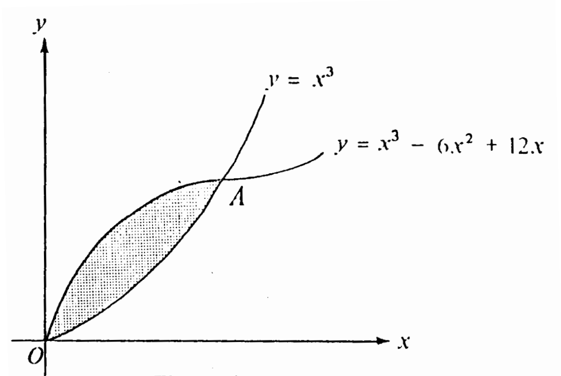


Figure 1

Figure 1 shows two curves $y = x^3$ and $y = x^3 - 6x^2 + 12x$ intersecting at the origin and a point A .

(a) Find the coordinates of A .

(b) Find the area of the shaded region in Figure 1.

(1995-CE-A MATH 2 #05) (6 marks)

5.

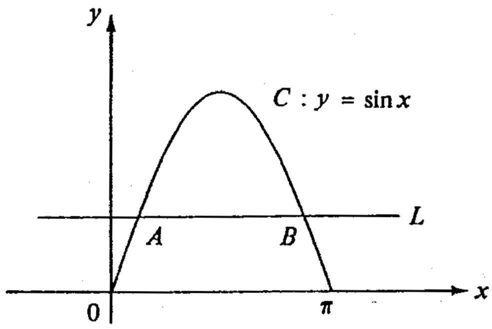


Figure 1

Figure 1 shows the curve $C : y = \sin x$ for $0 \leq x \leq \pi$.

(a) Find the area of the finite region bounded by the curve C and the x-axis.

(b) A horizontal line L cuts C at two points A and B . A is the point $\left(\frac{\pi}{6}, \frac{1}{2}\right)$.

(i) Write down the coordinates of B .

(ii) Find the area of the finite region bounded by C and L .

(1996-CE-A MATH 2 #05) (6 marks)

5.

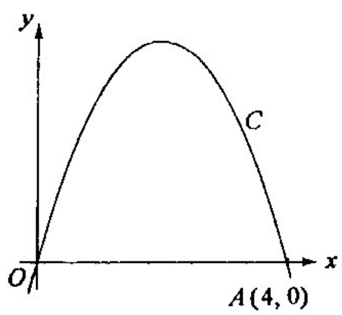


Figure 1(a)

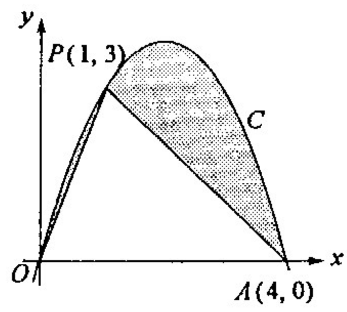


Figure 1(b)

The curve $C : y = 4x - x^2$ cuts the x-axis at the origin O and the point $A(4, 0)$ as shown in Figure 1(a).

(a) Find the area of the region bounded by C and the line segment OA .

(b) In Figure 1(b), the shaded region is enclosed by the curve C and the line segments OP and PA , where P is the point $(1, 3)$. Using (a), find the total area of the shaded region.

(1998-CE-A MATH 2 #08) (Modified)

8.

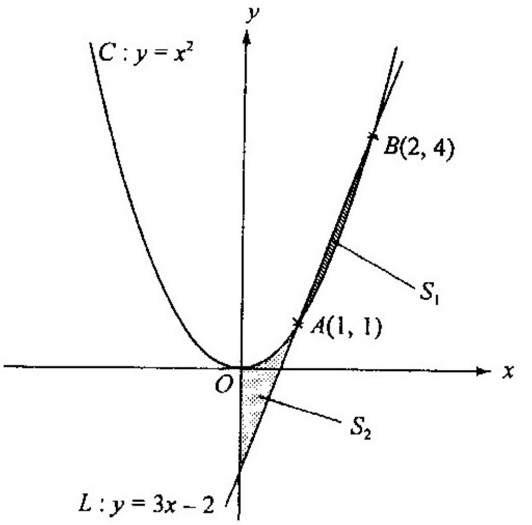


Figure 1

In Figure 1, the line $L : y = 3x - 2$ and the curve $C : y = x^2$ interest at two points $A(1, 1)$ and $B(2, 4)$. Denote the area of the region bounded by C and line segment AB by S_1 . Also, denote the area bounded by C , L and the y-axis by S_2 . Find $S_1 + S_2$.

(1999-CE-A MATH 2 #04) (5 marks)

1.

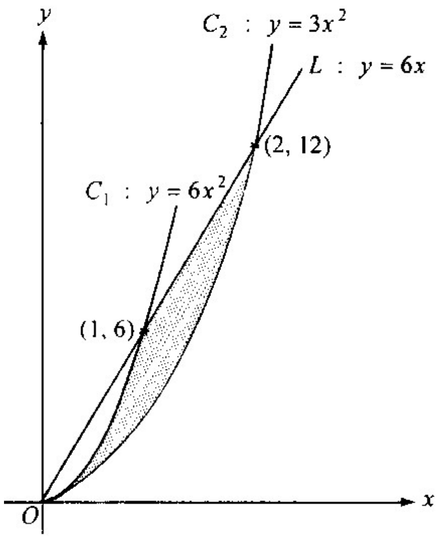


Figure 2

In Figure 2, the line $L : y = 6x$ and the curves $C_1 : y = 6x^2$ and $C_2 : y = 3x^2$ all pass through the origin, L also intersects C_1 and C_2 at the points $(1, 6)$ and $(2, 12)$ respectively. Find the area of the shaded region.

(2000-AS-M & S #03) (5 marks)

3.

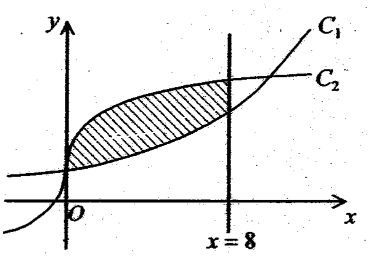


Figure 1

Figure 1 shows the graph of the two curves

$C_1 : y = e^{\frac{x}{8}}$ and

$C_2 : y = 1 + x^{\frac{1}{3}}$.

Find the area of the shaded region.

(2000-CE-A MATH 2 #08) (16 marks)

8. (a) Find $\int \cos 3x \cos x \, dx$.

(b) Show that $\dfrac{\sin 5x - \sin x}{\sin x} = 4 \cos 3x \cos x$.

Hence, or otherwise, find $\int \dfrac{\sin 5x}{\sin x} dx$.

(c) Using a suitable substitution, show that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin 5x}{\sin x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos 5x}{\cos x} dx$$

(d)

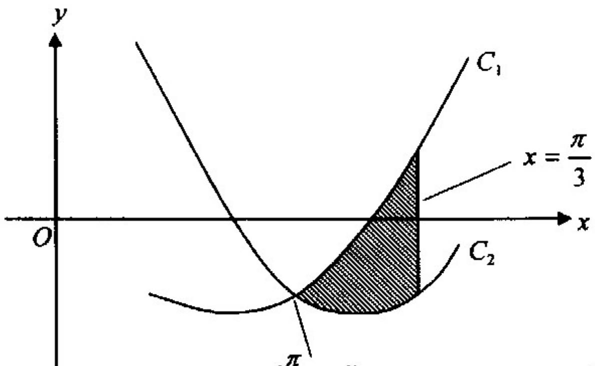


Figure 4

In Figure 4, the curves $C_1 : y = \dfrac{\cos 5x}{\cos x}$ and $C_2 : y = \dfrac{\sin 5x}{\sin x}$ intersect at the point $\left(\dfrac{\pi}{4}, -1\right)$. Find the area of the shaded region bounded by C_1 , C_2 and the line $x = \dfrac{\pi}{3}$.

(2002-CE-A MATH #06) (5 marks)

6.

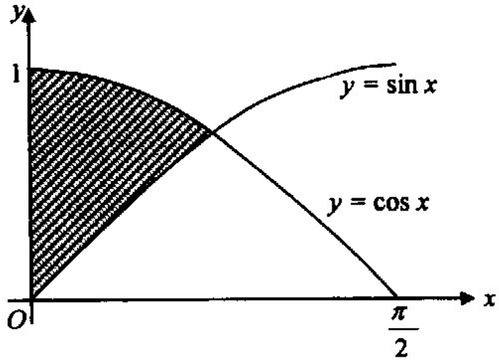


Figure 1

Figure 1 shows the curves $y = \sin x$ and $y = \cos x$. Find the area of the shaded region.

(2003-CE-A MATH #09) (5 marks)

9.

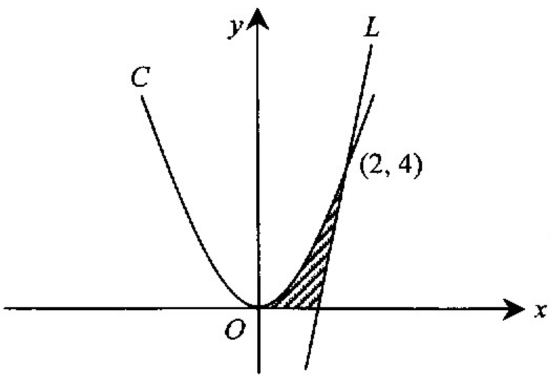


Figure 2

In Figure 2, the curve $C : y = x^2$ and line $L : y = 4x - 4$ intersect at the point $(2, 4)$. Find the area of the shaded region bounded by C , L and the x-axis.

(2005-CE-A MATH #13) (6 marks)

13. (a) Find $\int \sin \pi x \, dx$.

(b)

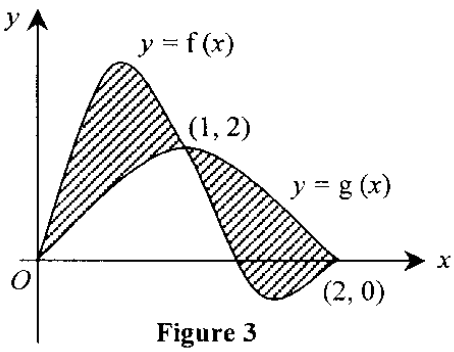


Figure 3

Figure 3 shows two curves $y = f(x)$ and $y = g(x)$ intersecting at three points $(0, 0)$, $(1, 2)$ and $(2, 0)$ for $0 \le x \le 2$. It is given that $f(x) - g(x) = 2 \sin \pi x$. Find the area of the shaded region as shown in Figure 3.

(2007-CE-A MATH #06) (5 marks)

6.

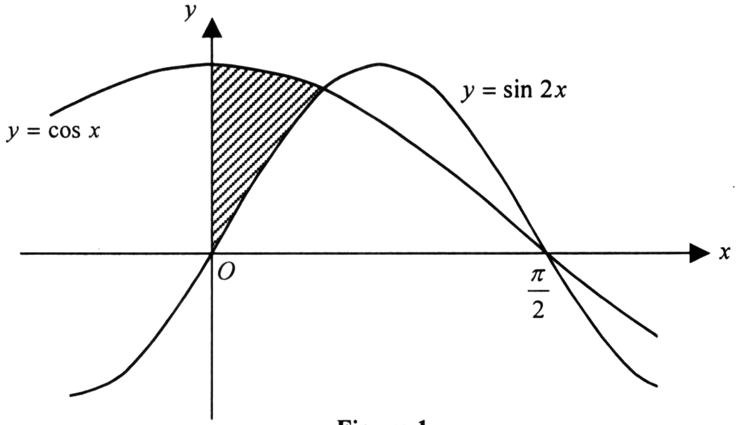


Figure 1

Figure 1 shows the graphs of $y = \sin 2x$ and $y = \cos x$. Find the area of the shaded region.

(2008-CE-A MATH #10) (5 marks)

10.

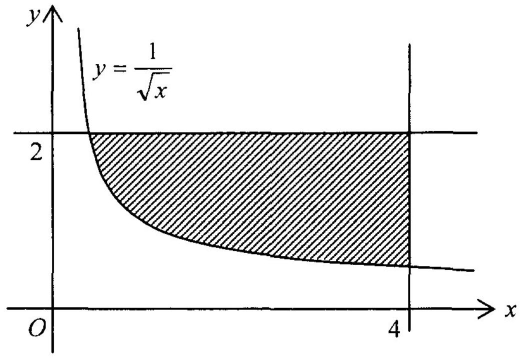


Figure 2

Find the area of the shaded region bounded by the curve $y = \dfrac{1}{\sqrt{x}}$ and the straight lines $y = 2$, $x = 4$.

(2009-CE-A MATH #10) (5 marks)

10. (a) Solve $8\cos x = \sec^2 x$ for $0 < x < \dfrac{\pi}{2}$.

(b)

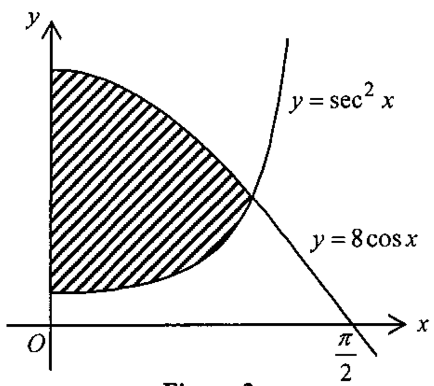


Figure 2

Figure 2 shows the graphs of $y = 8\cos x$ and $y = \sec^2 x$. Find the area of the shaded region.

(2010-CE-A MATH #03) (4 marks)

3. Let $I = \int_{-2}^{1} x(1-x)(x+2)\,\mathrm{d}x$.

(a) Find the value of I .

(b)

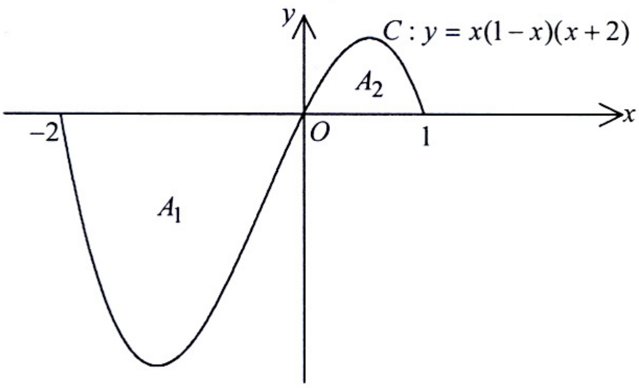


Figure 1

Figure 1 shows the graph of $C : y = x(1-x)(x+2)$ for $-2 \le x \le 1$. Let A_1 denote the area of the region bounded by C and the x-axis when $x < 0$, and A_2 denote the area of the region bounded by C and the x-axis when $x \ge 0$. Without finding the values of A_1 and A_2 , express I in terms of A_1 and A_2 .

(2011-CE-A MATH #11) (7 marks)

11.

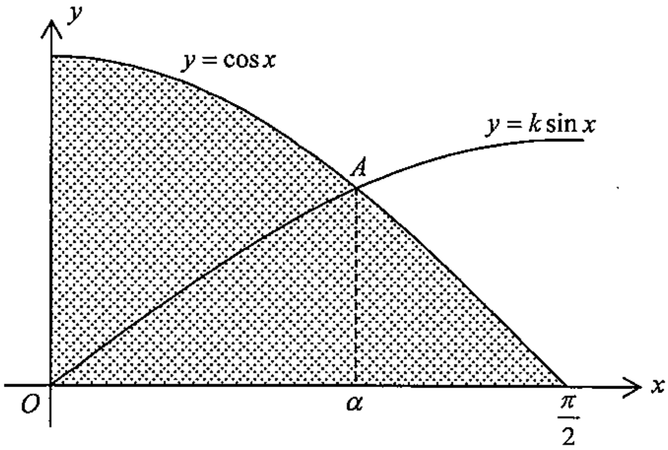


Figure 1

In Figure 1, the curves $y = \cos x$ and $y = k\sin x$, where $k > 0$, intersect at the point A . It is given that the x-coordinate of A is α , where $0 < \alpha < \frac{\pi}{2}$.

(a) Show that $\tan\alpha = \frac{1}{k}$.

(b) If the curve $y = k\sin x$ bisects the shaded region bounded by the curves $y = \cos x$, x-axis and y-axis, find the value of k .

(2012-DSE-MATH-EP(M2) #14) (12 marks)

14. Consider the curve $\Gamma : y = kx^p$, where $k > 0$, $p > 0$. In Figure 7, the tangent to Γ at $A\left(a, ka^p\right)$ cuts the x-axis at $B(-a, 0)$, where $a > 0$.

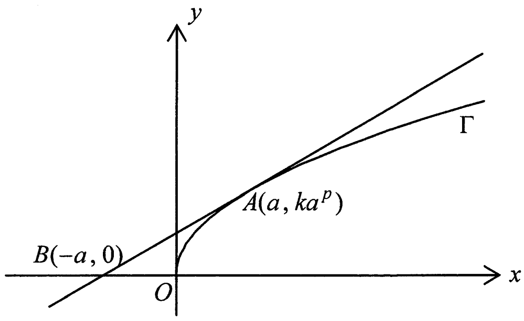


Figure 7

(a) Show that $p = \dfrac{1}{2}$.

(b) Suppose that $a = 1$. As shown in Figure 8, the circle C , with radius 2 and centre on the y-axis, touches Γ at point A .

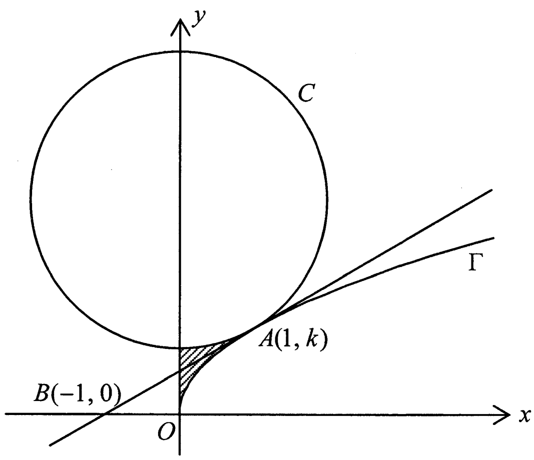


Figure 8

(i) Show that $k = \dfrac{2\sqrt{3}}{3}$.

(ii) Find the area of the shaded region bounded by Γ , C and the y-axis.

(2014-DSE-MATH-EP(M2) #06) (6 marks)

6. (a) Find $\int xe^{-x}\,dx$.

(b)

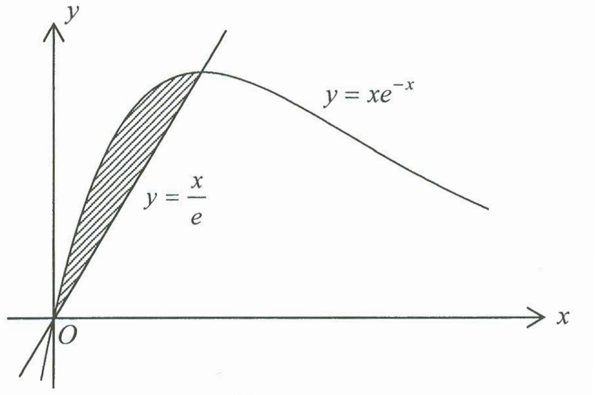


Figure 1

Figure 1 shows the shaded region bounded by curve $y = xe^{-x}$ and the straight line $y = \frac{x}{e}$. Find the area of the shaded region.

(2017-DSE-MATH-EP(M2) #04) (6 marks)

4. (a) Using integration by parts, find $\int x^2 e^{-x}\,dx$.

(b) Find the area of the region bounded by the graph of $y = x^2 e^{-x}$, the x-axis and the straight line $x = 6$.

(2018-DSE-MATH-EP(M2) #04) (6 marks)

4. (a) Using integration by parts, find $\int u\left(5^u\right)du$.

(b) Define $f(x) = x\left(5^{2x}\right)$ for all real numbers x. Find the area of the region bounded by the graph of $y = f(x)$, the straight line $x = 1$ and the x-axis.

(2019-DSE-MATH-EP(M2) #09) (12 marks)

9. Consider the curve $\Gamma : y = \frac{1}{3}\sqrt{12 - x^2}$, where $0 < x < 2\sqrt{3}$. Denote the tangent to Γ at $x = 3$ by L .

(a) Find the equation of L .

(b) Let C be the curve $y = \sqrt{4 - x^2}$, where $0 < x < 2$. It is given that L is a tangent to C . Find

(i) the point(s) of contact of L and C ;

(ii) the point(s) of intersection of C and Γ ;

(iii) the area of the region bounded by L , C and Γ .

ANSWERS

(1986-CE-A MATH 2 #11) (20 marks)

11. (a) (i) $2\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$

(ii) $3+2x-x^2=2^2-(x-1)^2$

$\int_0^1 \sqrt{3+2x-x^2}\,\mathrm{d}x=\frac{\pi}{3}+\frac{\sqrt{3}}{2}$

(b) (i) $y=-\sqrt{4-(x-1)^2}+\sqrt{3}$

(ii) $\left(\frac{4\pi}{3}-\frac{5\sqrt{3}}{6}\right)$

(1991-CE-A MATH 2 #05) (7 marks)

5. (a) $y=-x^2+4x-3$

(b) $\frac{4}{3}$

(1992-CE-A MATH 2 #06) (6 marks)

6. (a) $a=-1$, $b=2$

(b) $\frac{37}{12}$

(1993-CE-A MATH 2 #05) (6 marks)

5. (a) $A=\left(\frac{\pi}{4},\frac{\sqrt{2}}{2}\right)$, $B=\left(\frac{5\pi}{4},\frac{-\sqrt{2}}{2}\right)$

(b) $2\sqrt{2}$

(1994-CE-A MATH 2 #07) (6 marks)

7. (a) $A=(2,8)$

(b) 8

(1995-CE-A MATH 2 #05) (6 marks)

5. (a) 2

(b) (i) $B=\left(\frac{5\pi}{6},\frac{1}{2}\right)$

(ii) $\sqrt{3}-\frac{\pi}{3}$

(1996-CE-A MATH 2 #05) (6 marks)

5. (a) $\frac{32}{3}$

(b) $\frac{14}{3}$

(1998-CE-A MATH 2 #08) (6 marks)

8. 1

(1999-CE-A MATH 2 #04) (5 marks)

1. $\frac{\pi}{4}$

(2000-AS-M & S #03) (5 marks)

3. $28-8e$

(2000-CE-A MATH 2 #08) (16 marks)

8. (a) $\frac{1}{8}\sin 4x+\frac{1}{4}\sin 2x+\text{constant}$

(b) $\int \frac{\sin 5x}{\sin x}\mathrm{d}x=x+\frac{1}{2}\sin 4x+\sin 2x+\text{constant}$

(d) $2-\sqrt{3}$

(2002-CE-A MATH #06) (5 marks)

6. $\sqrt{2}-1$

(2003-CE-A MATH #09) (5 marks)

9. $\frac{2}{3}$

(2005-CE-A MATH #13) (6 marks)

13. (a) $-\frac{1}{\pi}\cos \pi x+\text{constant}$

(b) $\frac{8}{\pi}$

(2007-CE-A MATH #06) (5 marks)

6. $\frac{1}{4}$

(2008-CE-A MATH #10) (5 marks)

10. $\frac{9}{2}$

(2009-CE-A MATH #10) (5 marks)

10. (a) $x = \dfrac{\pi}{3}$

(b) $3\sqrt{3}$

(2010-CE-A MATH #03) (4 marks)

3. (a) $\dfrac{-9}{4}$

(b) $A_2 - A_1$

(2011-CE-A MATH #11) (7 marks)

11. (b) $k = \dfrac{3}{4}$

(2012-DSE-MATH-EP(M2) #14) (12 marks)

14. (b) (ii) $\dfrac{13\sqrt{3}}{18} - \dfrac{\pi}{3}$

(2014-DSE-MATH-EP(M2) #06) (6 marks)

6. (a) $-xe^{-x} - e^{-x}$ + constant

(b) $1 - \dfrac{5}{2e}$

(2017-DSE-MATH-EP(M2) #04) (6 marks)

4. (a) $-e^{-x}\left(x^2 + 2x + 2\right)$ + constant

(b) $2 - \dfrac{50}{e^6}$

(2018-DSE-MATH-EP(M2) #04) (6 marks)

4. (a) $\dfrac{5^u(u \ln 5 - 1)}{(\ln 5)^2}$ + constant

(b) $\dfrac{25 \ln 5 - 12}{2(\ln 5)^2}$

(2019-DSE-MATH-EP(M2) #09) (12 marks)

9. (a) $x + \sqrt{3}y - 4 = 0$

(b) (i) $\left(1,\sqrt{3}\right)$

(ii) $\left(\sqrt{3},1\right)$

(iii) $\dfrac{4}{\sqrt{3}} - \dfrac{2\pi}{3}$

6. (b) Curve Sketching and Area

(2001-AS-M & S #10) (15 marks)

10. Let $f(x) = \dfrac{5x+45}{x+3}$ for $x \neq -3$ and $g(x) = ka^{\frac{-1}{9}x}$ where k and a are positive constants. Figure 3 shows a sketch of $y = g(x)$. It is known that $f(0) = g(0)$ and $f(9) = g(9)$.

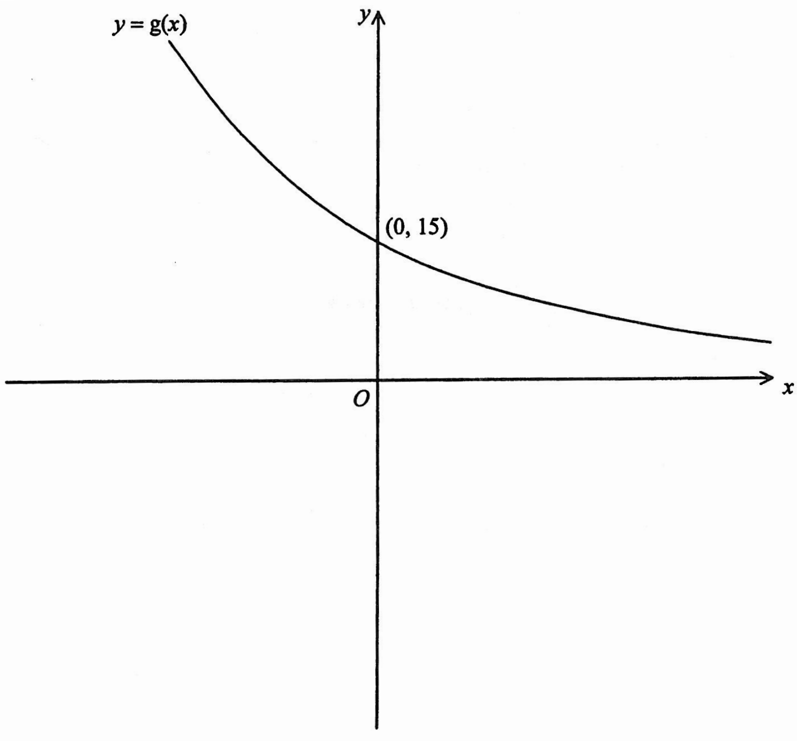


Figure 3

(a) Determine the values of k and a.

(b) Find the equations of the horizontal and vertical asymptotes of $y = f(x)$.

(c) Sketch $y = f(x)$ and its asymptotes on Figure 3. Indicate the points where the curve cuts the axes and $y = g(x)$.

(d) Let A be the area bounded by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = 9$.

(i) Find the value of A.

(ii) If the area bounded by the curve $y = g(x)$, the x-axis, the lines $x = \alpha$ and $x = \alpha + 9$ is also A, find the value of α. (Give your answer correct to 4 decimal places.)

(2002-AS-M & S #10) (15 marks)

10. Let $f(x) = \dfrac{ax+b}{cx+1}$ and $g(x) = -(x-3)(x+1)^3$, where a , b and c are constants. It is known that

$f(0) = g(0)$, $f(3) = g(3)$ and $f(-2) = g(-2)$.

(a) (i) Find the values of a , b and c .

(ii) Find the horizontal and vertical asymptotes of the graph of $y = f(x)$.

(iii) Sketch the graph of $y = f(x)$ and its asymptotes. Indicate the point(s) where the curve cuts the y-axis.

(b) (i) Find all relative extreme point(s) and point(s) of inflexion of the graph of $y = g(x)$.

(ii) On the diagram in (a) (iii), sketch the graph of $y = g(x)$. Indicate all the relative extreme point(s) and the point(s) of inflexion, the point(s) where the graph cuts the coordinates axes and where it cuts the graph of $y = f(x)$.

(c) Find the area enclosed by the graph of $y = f(x)$ and $y = g(x)$.

(2003-AS-M & S #07) (15 marks)

7. Define $f(x) = \dfrac{20-4x}{7-2x}$ for all $x \neq \dfrac{7}{2}$ and $g(x) = \dfrac{a+bx}{3+cx}$ for all $x \neq \dfrac{-3}{c}$, where a , b and c are positive constants.

Let C_1 and C_2 be the curves $y = f(x)$ and $y = g(x)$ respectively.

It is given that the x-intercept and the y-intercept of C_2 are -3 and 4 respectively. Also, it is known that C_1 and C_2 have a common horizontal asymptote.

(a) Find the equations of the vertical asymptote(s) and horizontal asymptote(s) to C_1 .

(b) Find the values of a , b and c .

(c) Sketch C_1 and C_2 on the same diagram and indicate the asymptote(s), intercept(s) and the point(s) of intersection of the two curves.

(d) If the area enclosed by C_1 , C_2 and the straight line $x = \lambda$, where $0 < \lambda < \dfrac{7}{2}$, is $3\ln 3$ square units, find the exact value(s) of λ .

(2004-AS-M & S #07) (15 marks)

7. Define $f(x) = \dfrac{2x+5}{2x+6}$ and $g(x) = -f(x)$ for all $x \neq -3$.

Let C_1 and C_2 be the curves $y = f(x)$ and $y = g(x)$ respectively.

(a) (i) Find the equations of the vertical asymptote(s) and horizontal asymptote(s) to C_1 .

(ii) Sketch C_1 and indicate its asymptote and its intercept(s).

(b) On the diagram sketched in (a) (ii), sketch C_2 and indicate its asymptote(s), its intercept(s) and the point(s) of intersection of the two curves.

(c) Let $\lambda > 7$. If the area enclosed by C_1 , C_2 and the straight line $x = \lambda$ is $\left(2\lambda + 5 + \ln(\lambda - 7)\right)$ square units, find the exact value(s) of λ .

(2004-CE-A MATH #17) (12 marks)

17. (a) Let $y = (x - \pi)\sin x + \cos x$.

(i) Show that $\frac{dy}{dx} = (x - \pi)\cos x$.

Hence find $\int (x - \pi)\cos x \, dx$.

(ii) Figure 10 shows the graph of $y = (x - \pi)\cos x$ for $0 \le x \le \frac{3\pi}{2}$.

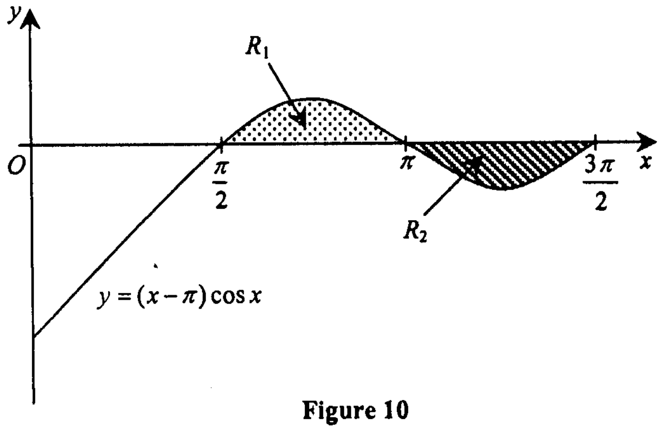


Figure 10

(1) Find the areas of the two shaded region R_1 and R_2 as shown in Figure 10.

(2) Find $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (x - \pi)\cos x \, dx$.

(b)

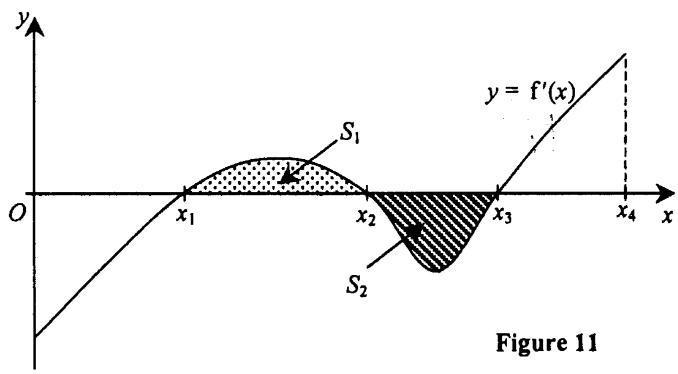


Figure 11

Let $f(x)$ be a continuous function. Figure 11 shows a sketch of the graph of $y = f'(x)$ for $0 \le x \le x_4$. It is known that the areas of the shaded regions S_1 and S_2 as shown in Figure 11 are equal.

(i) Show that $f(x_1) = f(x_3)$.

(ii) Furthermore, $f(0) = f(x_4) = 0$ and $f(x) \ne 0$ for $0 < x < x_4$. In Figure 12, draw a sketch of the graph of $y = f(x)$ for $0 \le x \le x_4$.

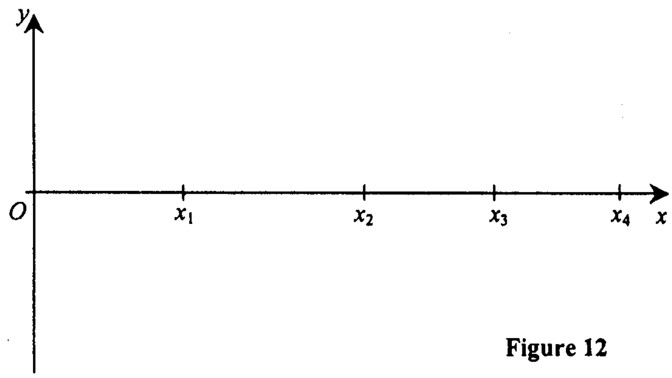


Figure 12

(2005-AS-M & S #07) (15 marks)

7. Define $f(x) = \dfrac{2x+a}{x+2}$ for all $x \neq -2$ and let $g(x) = -x^2 + x + b$, where a and b are constants. Let C_1 and C_2 be the curves $y = f(x)$ and $y = g(x)$ respectively.

It is given that C_1 and C_2 have a common y-intercept and $f(3) = g(3)$.

(a) Find the values of a and b .

(b) Find the equations of the horizontal asymptote(s) and vertical asymptote(s) to C_1 .

(c) Sketch C_1 and C_2 on the same diagram and indicate the asymptote(s) to C_1 . Also indicate the intercept(s) and the points of intersection of C_1 and C_2 .

(d) Find the area enclosed by C_1 and C_2 .

(2006-AS-M & S #07) (15 marks)

7. Define $f(x) = \dfrac{a+bx}{4-x}$ for all $x \neq 4$. Let $g(x) = f(-x)$ for all $x \neq -4$. Let C_1 and C_2 be the curves $y = f(x)$ and $y = g(x)$ respectively. It is given that the y-intercept of C_1 is $-\dfrac{3}{2}$ while the x-intercept of C_2 is -2 .

(a) Find the values of a and b .

(b) (i) Find the equations of the vertical asymptote(s) and the horizontal asymptote(s) to C_1 .

(ii) Sketch C_1 and indicate its asymptote(s) and intercept(s).

(c) On the diagram sketched in (b) (ii), sketch C_2 and indicate its asymptote(s), its intercept(s) and the point(s) of intersection of the two curves.

(d) Find the area enclosed by C_1 , C_2 and the straight line $y = 9$.

(2006-AL-P MATH 2 #07) (15 marks)

7. Let $f(x) = \dfrac{x^2 - x - 6}{x+6}$ ($x \neq -6$).

(a) Find $f'(x)$ and $f''(x)$.

(b) Solve each of the following inequalities:

(i) $f'(x) > 0$

(ii) $f'(x) < 0$

(iii) $f''(x) > 0$

(iv) $f''(x) < 0$

(c) Find the relative extreme point(s) of the graph of $y = f(x)$.

(d) Find the asymptote(s) of the graph of $y = f(x)$.

(e) Sketch the graph of $y = f(x)$.

(f) Find the area of the region bounded by the graph of $y = f(x)$ and the x-axis.

(2006-CE-A MATH #13) (7 marks)

13. Let $f(x)$ be the polynomial. Figure 2 shows a sketch of the curve $y = f'(x)$, where $-2 \le x \le 6$. The curve cuts the x-axis at the origin and $(a, 0)$, where $0 < a < 6$. It is known that the areas of the shaded regions R_1 and R_2 as shown in Figure 2 are 3 and 1 respectively.

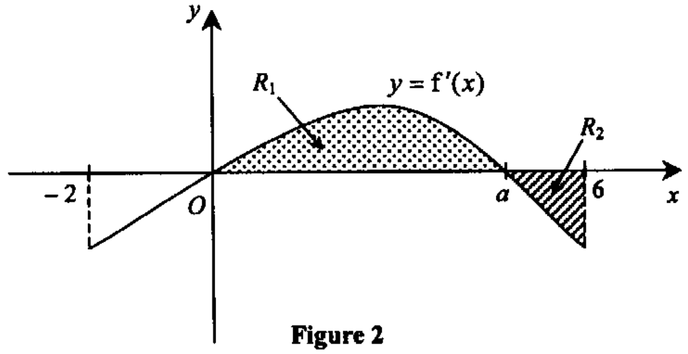


Figure 2

(a) Write down the x-coordinates of the maximum and minimum points of the curve $y = f(x)$ for $-2 < x < 6$.

(b) It is known that $f(-2) = 2$ and $f(0) = 1$.

(i) By considering $\int_0^a f'(x)\,dx$, find the value of $f(a)$.

(ii) In Figure 3, sketch the curve $y = f(x)$ for $-2 \le x \le 6$.

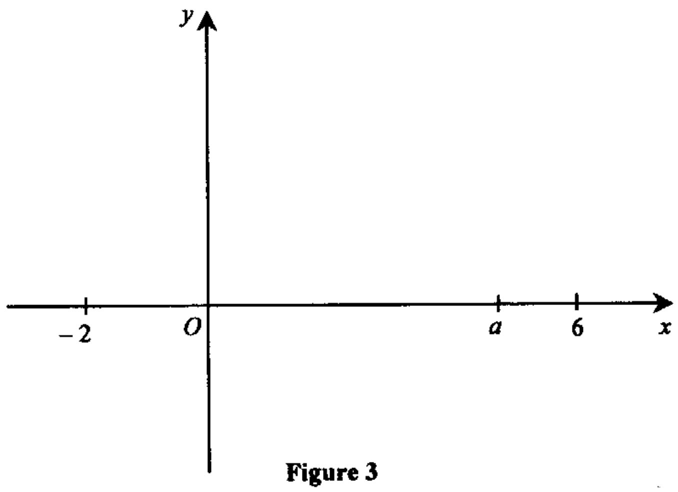


Figure 3

(2007-AS-M & S #07) (15 marks)

7. Define $f(x) = \dfrac{8x - 40}{x + 4}$ for all $x \neq -4$. Let $g(x) = \dfrac{(x+4)^2(x-5)}{8}$.

Let C_1 and C_2 be the curves $y = f(x)$ and $y = g(x)$ respectively.

(a) Sketch C_1 and indicate its asymptote(s) and its intercept(s).

(b) (i) Find the coordinates of the relative extreme point(s) and the point(s) of inflexion of C_2 .

(ii) On the diagram sketched in (a), sketch C_2 and indicate its relative extreme point(s), its intercept(s), its point(s) of inflexion and the point(s) of intersection of the two curves.

(c) Find the area enclosed by C_1 and C_2 .

(2009-AS-M & S #07) (15 marks)

7. Let C_1 be the curve $y = \dfrac{2x-1}{hx-1}$, where h is a non-zero constant and $x \neq \dfrac{1}{h}$.

It is given that C_1 has a vertical asymptote $x = 1$ and a horizontal asymptote $y = k$, where k is a constant.

(a) (i) Find the values of h and k .

(ii) Sketch the graph of C_1 . Indicate its asymptotes and intercepts.

(b) Let C_2 be the curve $y = -x^2 + px + q$, where p and q are constants. Let A be the point where C_1 cuts the x-axis. It is given that C_2 intersects C_1 at A , and the tangents to both curves at A are perpendicular to each other.

(i) Find the values of p and q .

(ii) On the diagram sketched in (a) (ii), sketch the graph of C_2 .

(iii) Find the area of the region bounded by the curves C_1 , C_2 and the y-axis.

(2010-AS-M & S #07) (15 marks)

7. Define $f(x) = \dfrac{x+4}{x+a}$ where a is a constant and $x \neq -a$. Let C_1 be the curve $y = f(x)$ with the vertical asymptote $x = 2$.

(a) (i) Find the value of a .

(ii) Find the equation(s) of the horizontal asymptote(s) of C_1 .

(iii) Find $f'(x)$ and hence find the range of x for which $f(x)$ is decreasing.

(iv) Sketch the curve of C_1 . Indicate its asymptote(s) and intercept(s).

(b) Let C_2 be the curve $y = g(x)$, where $g(x) = f(x-2) + 2$.

(i) Using the result in (a) (iv), or otherwise, sketch the curve C_2 . Indicate its asymptote(s) and intercept(s).

(ii) Let C_3 be the curve $y = [g(x)]^2$. Find the area of the region bounded by the curve C_3 and the axes.

(2011-AS-M & S #07) (15 marks)

7. Let C be the curve $y = \dfrac{ax+b}{cx+d}$ for all $x \neq \dfrac{-d}{c}$, where a , b , c , d are constants, $a \neq 0$ and $c \neq 0$. It is given that C has a vertical asymptote $2x - 1 = 0$ and a horizontal asymptote $y + 2 = 0$, and C passes through the origin. Let D be the curve $y = \dfrac{cx+d}{ax+b}$ for all $x \neq \dfrac{-b}{a}$.

(a) Show that $d = \dfrac{-c}{2}$.

Hence find the equation of C .

(b) Find the coordinates of all the intersecting points of curves C and D .

(c) Sketch the curves C and D on the same diagram, indicating their asymptotes, intercepts and their points of intersection.

(d) Find the exact value of the area of the region bounded by the curves C , D and the positive x-axis.

(SP-DSE-MATH-EP(M2) #12) (14 marks)

12. Let $f(x) = \dfrac{4}{x-1} - \dfrac{4}{x+1} - 1$, where $x \neq \pm 1$.

(a) (i) Find the x- and y- intercept(s) of the graph of $y = f(x)$.

(ii) Find $f'(x)$ and prove that

$$f''(x) = \frac{16\left(3x^2+1\right)}{(x-1)^3(x+1)^3}$$

for $x \neq \pm 1$.

(iii) For the graph of $y = f(x)$, find all the extreme points and show that there are no points of inflexion.

(b) Find all the asymptote(s) of the graph of $y = f(x)$.

(c) Sketch the graph of $y = f(x)$.

(d) Let S be the area bounded by the graph of $y = f(x)$, the straight line $x = 3$, $x = a$ ($a > 3$) and $y = -1$. Find S in terms of a . Deduce that $S < 4\ln 2$.

(2012-AS-M & S #07) (15 marks)

7. Let $f(x) = \dfrac{ax+b}{c-x}$ for all $x \neq c$, where a , b and c are constants, and $g(x) = f(-x)$ for all $x \neq -c$. Let C_1 and C_2 be the curves of $y = f(x)$ and $y = g(x)$ respectively. It is given that the vertical asymptotes of C_2 is $x = -3$, the y-intercept of C_1 is $\dfrac{4}{3}$ and the x-intercept of C_2 is 2.

(a) Find the values of a , b and c .

(b) (i) Find the equations of the vertical and horizontal asymptotes of C_1 .

(ii) Sketch the graphs of C_1 and C_2 on the same diagram. Indicate the asymptotes, intercepts and their intersecting point(s).

(c) If the area of the region bounded by C_1 , C_2 and the line $x = -k$ (where $0 < k < 3$) is $10\ln\dfrac{3}{2}$, find the value of k .

(2013-AS-M & S #07) (15 marks)

7. Let $k \neq 0$ be a constant.

Define $f(x) = \dfrac{x}{kx-1}$ for all $x \neq \dfrac{1}{k}$, and $g(x) = f\left(\dfrac{1}{x}\right)$ for all $x \neq 0$ and $x \neq k$.

Let C_1 be the curve $y = f(x)$ and C_2 be the curve $y = g(x)$. It is given that C_2 has a vertical asymptote $x = 2$.

(a) Find the value of k.

(b) Find the points of intersection of the curves C_1 and C_2 .

(c) Sketch the curves C_1 and C_2 on the same diagram, indicating their asymptotes, intercepts and points of intersection.

(d) Find the exact value of the area enclosed by C_1 , C_2 and the y-axis.

(2015-DSE-MATH-EP(M2) #09) (13 marks)

9. Define $f(x) = \dfrac{x^2+12}{x-2}$ for all $x \neq 2$.

(a) Find $f'(x)$.

(b) Prove that the maximum value and the minimum value of $f(x)$ are -4 and 12 respectively.

(c) Find the asymptote(s) of the graph of $y = f(x)$.

(d) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$.

(2016-DSE-MATH-EP(M2) #09) (13 marks)

9. Let a and b be constants. Define $f(x) = x^3 + ax^2 + bx + 5$ for all real numbers x . Denote the curve $y = f(x)$ by C . It is given that $P(-1, 10)$ is a turning point of C .

(a) Find a and b .

(b) Is P a maximum point of C ? Explain your answer.

(c) Find the minimum value(s) of $f(x)$.

(d) Find the point(s) of inflexion of C .

(e) Let L be the tangent to C at P . Find the area of the region bounded by C and L .

(2020-DSE-MATH-EP(M2) #09) (12 marks)

9. Let $f(x) = \dfrac{(x+4)^3}{(x-4)^2}$ for all real numbers $x \neq 4$. Denote the graph of $y = f(x)$ by H .

(a) Find the asymptote(s) of H .

(b) Find $f''(x)$.

(c) Someone claims that there are two turning points of H . Do you agree? Explain your answer.

(d) Find the point(s) of inflexion of H .

(e) Find the area of the region bounded by H , the x-axis and the y-axis.

ANSWERS

(2001-AS-M & S #10) (15 marks)

10. (a) $k = 15$, $a = 2$

(b) Horizontal asymptotes: $y = 5$

Vertical asymptotes: $x = -3$

(d) (i) $45 + 30\ln 4$

(ii) $\alpha \approx 1.5253$

(2002-AS-M & S #10) (15 marks)

10. (a) (i) $a = -1$, $b = 3$, $c = 1$

(ii) Horizontal asymptotes: $y = -1$

Vertical asymptotes: $x = -1$

(b) (i) Maximum Point: $(2, 27)$

Points of inflexion: $(-1, 0)$, $(1, 16)$

(c) $\dfrac{267}{5} - 8\ln 2$

(2003-AS-M & S #07) (15 marks)

7. (a) Horizontal asymptotes: $y = 2$

Vertical asymptotes: $x = \dfrac{7}{2}$

(b) $a = 12$, $b = 4$, $c = 2$

(d) $\lambda = 1 + \dfrac{5\sqrt{6}}{6}$

(2004-AS-M & S #07) (15 marks)

7. (a) (i) Vertical Asymptote: $x = -3$

Horizontal Asymptote: $y = 1$

(c) $\lambda = 2 + \dfrac{1}{2}\sqrt{102}$

(2004-CE-A MATH #17) (12 marks)

17. (a) (i) $(x - \pi)\sin x + \cos x +$ constant

(ii) (1) Area of $R_1 = \dfrac{\pi}{2} - 1$

Area of $R_2 = \dfrac{\pi}{2} - 1$

(2) $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (x - \pi)\cos x \, dx = 0$

(2005-AS-M & S #07) (15 marks)

7. (a) $a = 24$, $b = 12$

(b) Horizontal asymptotes: $y = 2$

Vertical asymptotes: $x = -2$

(d) $\dfrac{51}{2} - 20\ln\left(\dfrac{5}{2}\right)$

(2006-AS-M & S #07) (15 marks)

7. (a) $a = -6$, $b = 3$

(b) (i) Horizontal asymptotes: $y = -3$

Vertical asymptotes: $x = 4$

(d) $84 - 36\ln 2$

(2006-AL-P MATH 2 #07) (15 marks)

7. (a) $f'(x) = \dfrac{x(x + 12)}{(x + 6)^2}$

$f''(x) = \dfrac{72}{(x + 6)^3}$

(b) (i) $x < -12$ or $x > 0$

(ii) $-12 < x < -6$ or $-6 < x < 0$

(iii) $x > -6$

(iv) $x < -6$

(c) Maximum point: $(-12, -25)$

Minimum point: $(0, -1)$

(d) Vertical Asymptote: $x = -6$

Oblique Asymptote: $y = x - 7$

(f) $\dfrac{65}{2} - 72\ln\dfrac{3}{2}$

(2006-CE-A MATH #13) (7 marks)

13. (a) Maximum: $x = a$

Minimum: $x = 0$

(b) (i) $f(a) = 4$

(2007-AS-M & S #07) (15 marks)

7. (b) (i) Minimum point: $\left(2, \frac{-27}{2}\right)$

Maximum point: $(-4, 0)$

Point of Inflexion: $\left(-1, \frac{-27}{4}\right)$

(c) $\frac{2955}{32} - 144 \ln\left(\frac{3}{2}\right)$

(2009-AS-M & S #07) (15 marks)

7. (a) (i) $h = 1$, $k = 2$

(b) (i) $p = \frac{5}{4}$, $q = \frac{-3}{8}$

(iii) $\frac{103}{96} - \ln 2$

(2010-AS-M & S #07) (15 marks)

7. (a) (i) $a = -2$

(ii) $y = 1$

(iii) $f'(x) = \frac{-6}{(x-2)^2}$

(b) (ii) $27 - 36 \ln 2$

(2011-AS-M & S #07) (15 marks)

7. (a) $y = \frac{-4x}{2x-1}$

(b) $\left(\frac{1}{6}, 1\right)$ and $\left(\frac{-1}{2}, -1\right)$

(d) $\frac{5}{4} \ln 3 - \ln 2 - \frac{1}{2}$

(SP-DSE-MATH-EP(M2) #12) (14 marks)

12. (a) (i) x-intercepts are -3 and 3

y-intercepts are -9

(ii) $f'(x) = \frac{-16x}{(x-1)^2(x+1)^2}$

(iii) Maximum point $(0, -9)$

(b) Vertical Asymptote: $x = -1$ and $x = 1$

Horizontal Asymptote: $y = -1$

(d) $S = 4 \ln\left(\frac{a-1}{a+1}\right) + 4 \ln 2$

(2012-AS-M & S #07) (15 marks)

7. (a) $a = 2$, $b = 4$, $c = 3$

(b) (i) Vertical Asymptote: $x = 3$

Horizontal Asymptote: $y = -2$

(c) $\sqrt{3}$

(2013-AS-M & S #07) (15 marks)

7. (a) $k = 2$

(b) $(1, 1)$ and $\left(-1, \frac{1}{3}\right)$

(d) $\frac{5}{4} \ln 3 - \ln 2 - \frac{1}{2}$

(2015-DSE-MATH-EP(M2) #09) (13 marks)

9. (a) $f'(x) = \frac{x^2 - 4x - 12}{(x-2)^2}$

(c) Vertical Asymptote: $x = 2$

Oblique Asymptote: $y = x + 2$

(d) $30 - 32 \ln 2$

(2016-DSE-MATH-EP(M2) #09) (13 marks)

9. (a) $a = -3$ and $b = -9$

(c) -22

(d) $(1, -6)$

(e) 108

(2020-DSE-MATH-EP(M2) #09) (12 marks)

9. (a) Vertical Asymptote: $x = 4$

Oblique Asymptote: $y = x + 20$

(b) $\frac{384(x+4)}{(x-4)^4}$

(c) ... Disagree

(d) $(-4, 0)$

(e) $136 - 192 \ln 2$

6. (c) Volume of Solids and Rates of Change

(1984-CE-A MATH 2 #10) (20 marks)

10. (a) Use the substitution $x = a \sin \phi$ to show that $\int_{-a}^{a} \sqrt{a^2 - x^2}\, \mathrm{d}x = \frac{\pi a^2}{2}$.

(b) Figure 1 shows two semicircles APB and AQB with a common centre $C(0, b)$ and equal radii a . AB is parallel to the x-axis.

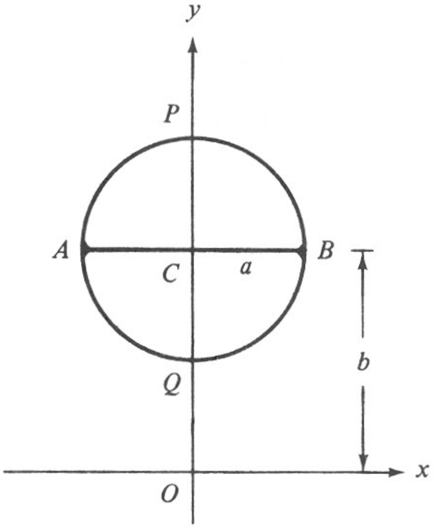


Figure 1

(i) Show that the equation APB is

$$y = b + \sqrt{a^2 - x^2}$$

and that of AQB is

$$y = b - \sqrt{a^2 - x^2} \ .$$

(ii) The region bounded by the two semicircles is revolved about the x-axis to generate a solid (called an anchor-ring). Use the result in (a) to prove that the volume of the anchor-ring is $2\pi^2 a^2 b$.

(c) A sweet has the form of an anchor-ring with $a = 2$ mm and $b = 8$ mm . Write down its volume in terms of π . The sweet is now dropped into water and it dissolves with a rate of change of volume given by

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\ 32\pi^2(2 - t) \text{ mm3/h}$$

where V is the volume in mm3, t is the time in hours.

Find V in terms of t and hence find the time required to dissolve the whole sweet completely.

(1991-CE-A MATH 2 #11) (16 marks)

11.

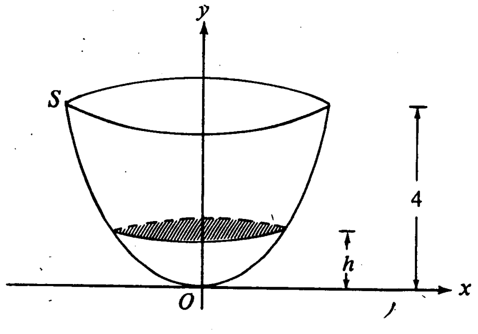


Figure 3(a)

An object S is in the shape of the solid of revolution of the region bounded by the curve $x^2 = 4y$ and the line $y = 4$ revolved about the y-axis, as shown in Figure 3(a).

(a) Find the volume of S .

(b) It is given that if S is cut by a plane parallel to its top surface at a distance h from O (see Figure 3(a)), the mass of the part of S below the plane is given by

$$\int_0^h \pi\left(16y - 3y^2\right) dy \text{ , where } 0 < h \le 4 \text{ .}$$

(i) Find the mass of S .

(ii) If the plane mentioned above cuts S into two parts of equal volumes, find

(1) the value of h ,

(2) the ratio of the mass of the lower part to that of the upper part.

(c)

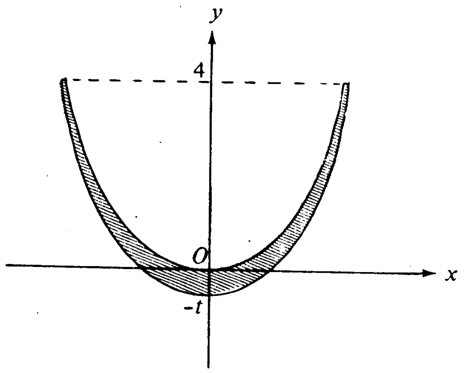


Figure 3(b)

In Figure 3(b), the shaded region is bounded by the curves $x^2 = 4y$, $x^2 = 4(y + t)$ and the line $y = 4$, where t is a small positive number. The solid of revolution of the shaded area revolved about the y-axis represents a layer of paint coated on the curved surface of S . Show that the volume of paint is approximately equal to $16\pi t$.

(1992-CE-A MATH 2 #11) (16 marks)

11. (a)

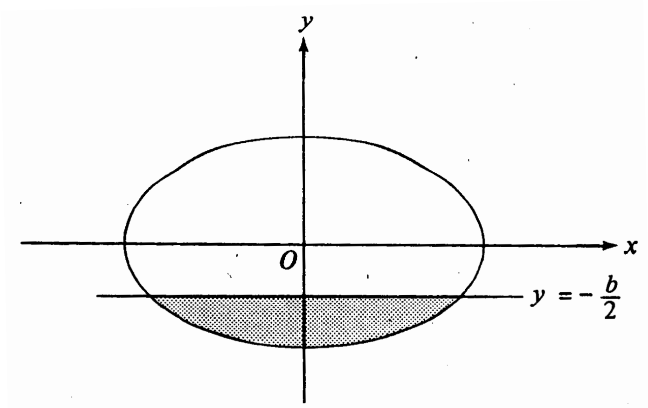


Figure 3 (a)

The shaded region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $y = \frac{-b}{2}$, as shown in Figure 3 (a), is revolved about the y-axis. Show that the volume of the solid of revolution is $\frac{5\pi a^2 b}{24}$.

(b)

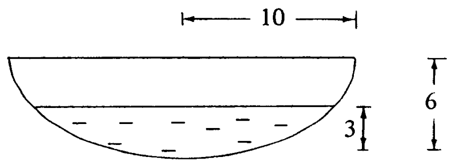


Figure 3 (b)

A bowl is generated by revolving the lower half of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the y-axis. The depth of the bowl is 6 units and the radius of its rim is 10 units. The bowl contains water to a depth of 3 units. (See Figure 3 (b).)

(i) Find the area of the water surface.

(ii) Using the result of (a), find the volume of water.

(iii) The water in the bowl is heated. At time t seconds after heating, the volume of water decreases at a rate of $\frac{\pi}{100}(25 + 2t)$ cubic units per second.

(1) Find the volume of water remaining in the bowl after t seconds.

(2) Calculate the time required to dry up the water in the bowl.

(1993-CE-A MATH 2 #12) (16 marks)

12.

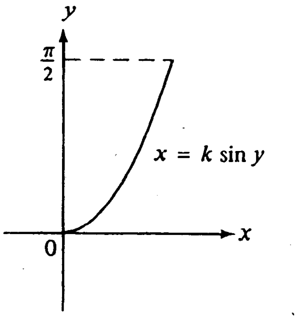


Figure 4 (a)

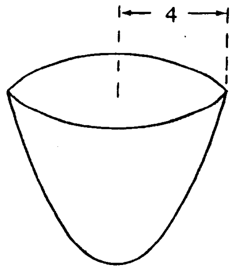


Figure 4 (b)

Figure 4 (a) shows the curve $x = k \sin y$, where $k > 0$ and $0 \le y \le \frac{\pi}{2}$. A bowl is formed by revolving the curve about the y-axis.

(a) Show that the capacity of the bowl is $\frac{1}{4}k^2\pi^2$ cubic units.

(b) Given that the radius of the rim of the bowl is 4 units. (See Figure 4 (b).) The bowl is full of water.

(i) Find the volume of water.

(ii) The water is now pumped out of the bowl at a rate of $(\pi + 2t)$ cubic units per minute, where t is the time in minutes after pumping starts. Find the time taken to pump out half of the water and the time taken to pump out the remaining water in the bowl. Give both answers in terms of π .

(1994-CE-A MATH 2 #13) (16 marks)

13.

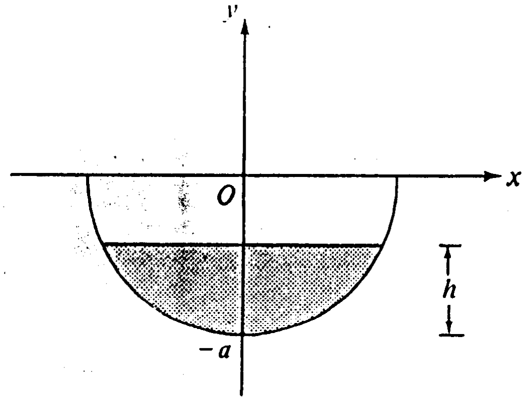


Figure 4

A bowl is generated by revolving the lower half of the circle $x^2 + y^2 = a^2$ about the y-axis. At time t, the bowl contains water to a depth of h, where $0 \le h \le a$ (see Figure 4). Let V be the volume of water in the bowl and A be the area of the water surface.

(a) Show that

(i) $V = \pi h^2 \left(a - \frac{1}{3} h \right)$,

(ii) $A = \pi h(2a - h)$.

(b) Suppose the water evaporates at a rate proportional to the area of the water surface at that instant,

i.e. $\frac{\mathrm{d}v}{\mathrm{d}t} = -kA$,

where k is a positive constant.

(i) Show that $\frac{\mathrm{d}h}{\mathrm{d}t}$ is a constant.

(ii) It is given that the depth of water is $\frac{3}{4}a$ at $t = 0$ and the water will completely evaporate at $t = 30$.

(1) Show that, for $0 \le t \le 30$,

$$h = \frac{a}{40}(30 - t) \ .$$

(2) Find the volume of water in the bowl at $t = 10$.

(1995-CE-A MATH 2 #12) (16 marks)

12. (a) Using the substitution $y - k = \cos\theta$, show that

$$\int_{k-1}^{k+1} \sqrt{1-(y-k)^2}\,dy = \frac{\pi}{2} .$$

Hence show that

(i) $$\int_0^2 \left[2+\sqrt{1-(y-1)^2}\right]^2 dy = \frac{28}{3} + 2\pi ,$$

(ii) $$\int_2^4 \left[2-\sqrt{1-(y-3)^2}\right]^2 dy = \frac{28}{3} - 2\pi .$$

(b)

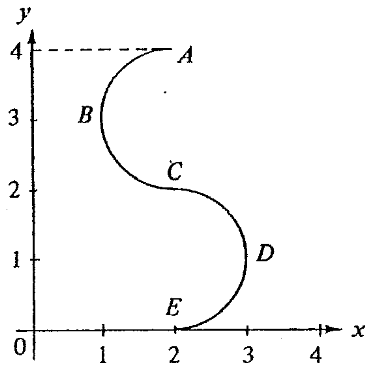


Figure 5(a)

Figure 5(a) shows two semicircles ABC and CDE centred at $(2, 3)$ and $(2, 1)$ respectively. Their radii are both equal to 1.

(i) Show that the equation of the semicircle ABC is

$$x = 2 - \sqrt{1-(y-3)^2} ,$$

and that of the semicircle CDE is

$$x = 2 + \sqrt{1-(y-1)^2} .$$

(ii) A pot is formed by revolving the curve $ABCDE$ and the line segment OE about the y-axis , where O is the origin. Using (a), find the capacity of the pot.

(c)

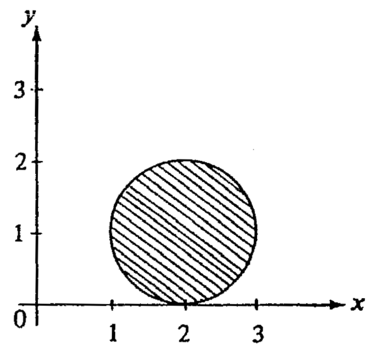


Figure 5(b)

The shaded region enclosed by the circle $(x-2)^2 + (y-1)^2 = 1$, as shown in Figure 5(b), is revolved about the y-axis to form a solid. Using (a) and (b), or otherwise, find the volume of the solid.

(1996-CE-A MATH 2 #11) (16 marks)

11.

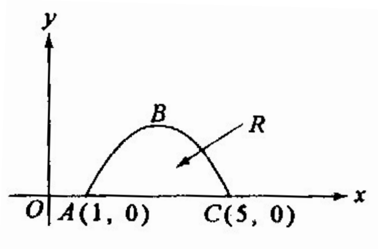


Figure 4 (a)

In Figure 4(a), the region R is enclosed by the parabola $y = 4 - (x - 3)^2$ and the line segment AC, where A and C are the points $(1, 0)$ and $(5, 0)$ respectively. B is the vertex of the parabola.

(a) Write down the coordinates of B.

(b) (i) Show that the equation of the curve AB is

$$x = 3 - \sqrt{4 - y}\ .$$

(ii) Write down the equation of the curve BC.

(c)

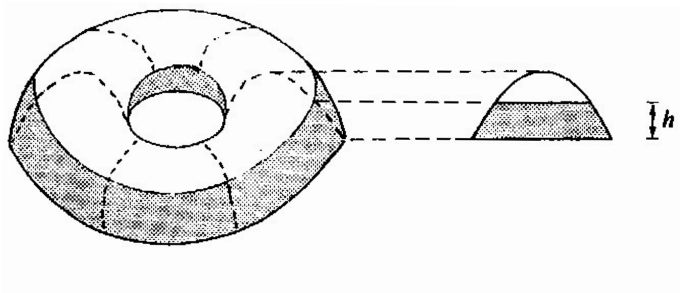


Figure 4(b)

A jelly ring is in the shape of the solid of revolution of the region R in Figure 4(a) about the y-axis. Furthermore, the jelly ring contains two layers. Let h be the height of the lower layer. (See Figure 4(b).)

(i) Show that the volume of the lower layer of the jelly ring is

$$8\pi\left[8 - (4 - h)^{\frac{3}{2}}\right]\ .$$

(ii) If the two layers have equal volumes, find the value of h correct to 3 significant figures.

(d)

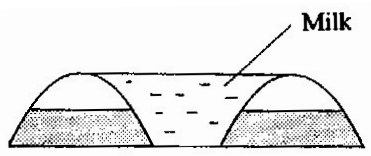


Figure 4(c)

If milk is poured into the centre of the jelly ring in (c) until it is completely filled (see Figure 4(c)), find the volume of milk required.

(1997-CE-A MATH 2 #10) (16 marks)

10.

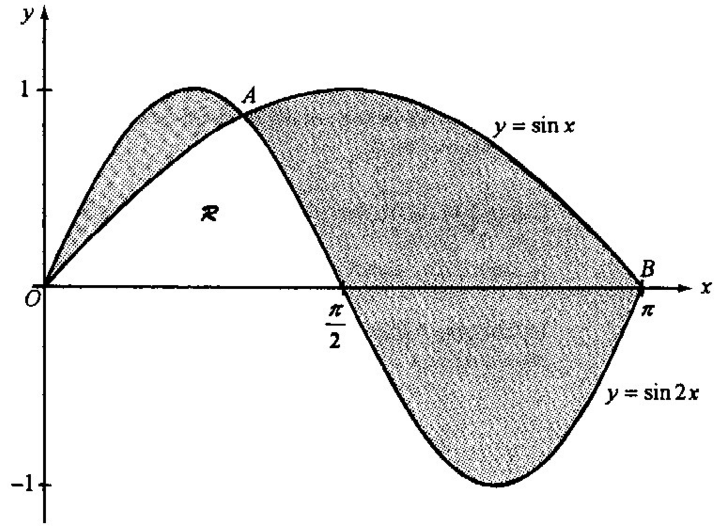


Figure 2(a)

Figure 2(a) shows the curves $y = \sin x$ and $y = \sin 2x$ for $0 \le x \le \pi$. The two curves intersect at the origin O , point A and point $B(\pi, 0)$.

(a) Show that the coordinates of A are $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$.

(b) Find the area of the shaded region in Figure 2(a).

(c) R is the region bounded by the two curves and the x-axis from $x = 0$ to $\frac{\pi}{2}$. (See Figure 2(a).)

If the region R is revolved about the x-axis, find the volume of the solid of revolution generated.

(d) Figure 2(b) shows the curve $y = \sin x$ for $0 \le x \le \pi$.

In Figure 2(b), sketch the curve $y = \left|\sin x\right|$ for $0 \le x \le \pi$.

In the same figure, shade the region whose area is represented by the expression

$$\int_0^{\pi} \left| \left|\sin 2x\right| - \sin x \right| \mathrm{d}x \ .$$

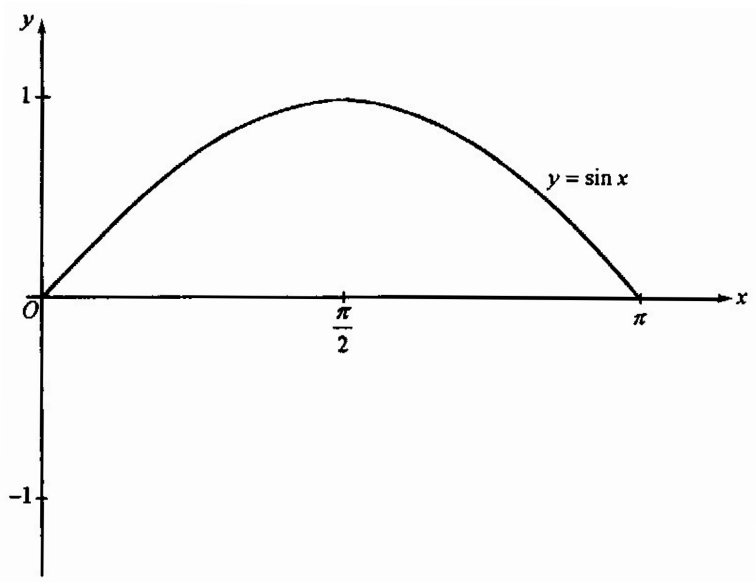


Figure 2(b)

(1998-CE-A MATH 2 #12) (16 marks)

12. (a)

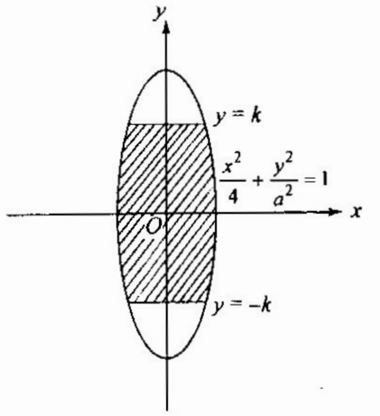


Figure 4(a)

In Figure 4(a), the shaded region enclosed by the ellipse $\frac{x^2}{4}+\frac{y^2}{a^2}=1$, the lines $y=k$ and $y=-k$, where $0 \le k \le a$, is revolved about the y-axis. Show that the volume of the solid of revolution is $8k\left(1-\frac{k^2}{3a^2}\right)\pi$.

(b)

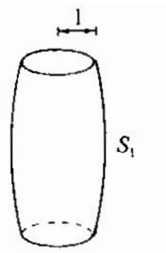


Figure 4(b)

A solid S_1 is in the shape of the solid of revolution described in (a). Furthermore, the radii of the plane circular faces of the solid are both equal to 1 . (See Figure 4(b).)

(i) Show that the height of S_1 is equal to $\sqrt{3}a$.

(ii) Find the volume of S_1 in terms of a and π .

(c)

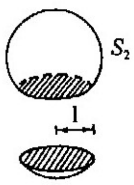


Figure 4(c)

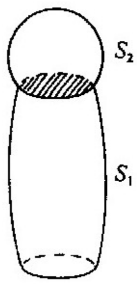


Figure 4(d)

A solid sphere of radius 2 is cut along a plane into two unequal portions as shown in Figure 4 (c). The radius of the plane circular face is equal to 1 . The larger portion S_2 is joined to S_1 in (b) to form a toy as shown in Figure 4 (d).

(i) Show that the height of the toy is $2+(a+1)\sqrt{3}$.

(ii) Using (b), or otherwise, find the volume of the toy in terms of a and π .

(1999-CE-A MATH 2 #13) (16 marks)

13.

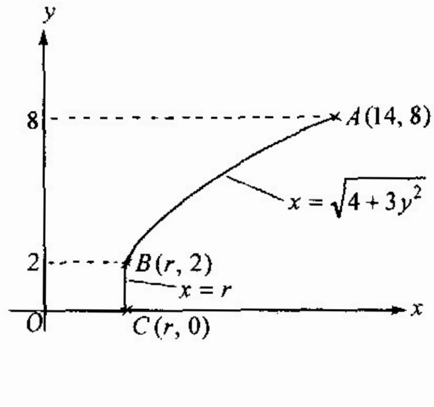


Figure 6(a)

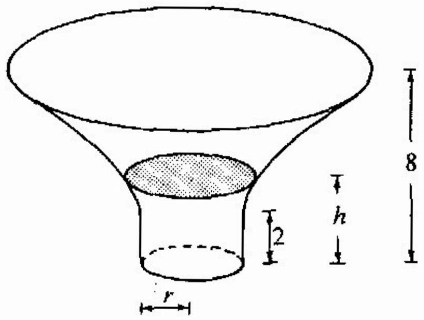


Figure 6(b)

A curve passes through three points $A(14, 8)$, $B(r, 2)$ and $C(r, 0)$ as shown in Figure 6(a). The curve consists of two parts. The equation of the part joining A and B is $x = \sqrt{4 + 3y^2}$ and the part joining B and C is the vertical line $x = r$.

(a) Find the value of r .

(b) A pot, 8 units in height, is formed by revolving the curve and the line segment OC about the y-axis, where O is the origin. (See Figure 6(b).) If the pot contains water to a depth of h units, where $h > 2$, show that the volume of water V in the pot is $\left(h^3 + 4h + 16\right)\pi$ cubic units.

(c) Initially, the pot in (b) contains water to a depth greater than 3 units. The water is now pumped out at a constant rate of 2π cubic units per second. Find the rate of change of the depth of the water in the pot with respect to time when

(i) the depth of the water is 3 units, and

(ii) the depth of the water is 1 unit.

(2000-CE-A MATH 2 #11) (16 marks)

11. (a) In Figure 7(a), the shaded region is bounded by the circle $x^2 + y^2 = r^2$, the x-axis, the y-axis and the line $y = -h$, where $h > 0$. If the shaded region is revolved about the y-axis, show that the volume of the solid generated is $\left(r^2h - \frac{1}{3}h^3\right)\pi$ cubic units.

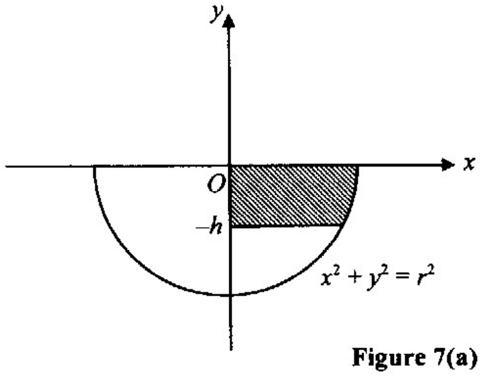


Figure 7(a)

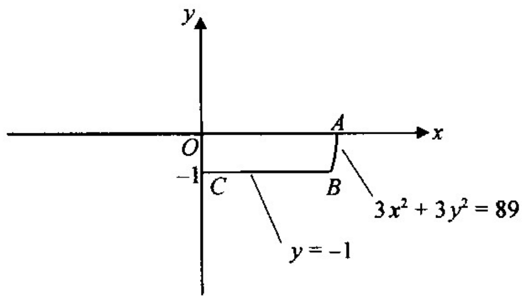


Figure 7(b)

(b) In Figure 7(b), A and C are points on the x-axis and y-axis respectively, AB is an arc of the circle $3x^2 + 3y^2 = 89$ and BC is a segment of the line $y = -1$. A mould is formed by revolving AB and BC about the y-axis. Using (a), or otherwise, show that the capacity of the mould is $\frac{88\pi}{3}$ cubic units.

(c)

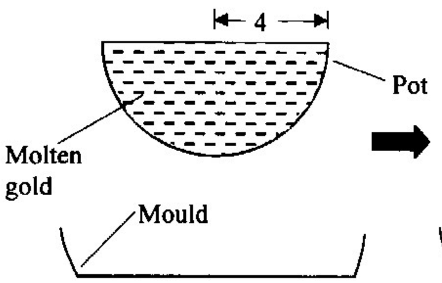


Figure 7(c)

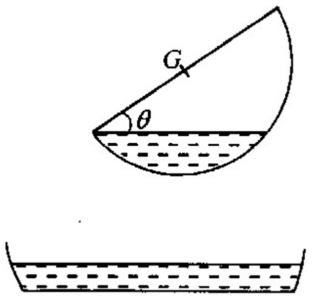


Figure 7(d)

A hemispherical pot of inner radius 4 units is completely filled with molten gold. (See Figure 7(c).) The molten gold is then poured into the mould mentioned in (b) by steadily tilting the pot. Suppose the pot is tilted through an angle θ and G is the centre of the rim of the pot. (See Figure 7(d).)

(i) Find, in terms of θ ,

(1) the distance between G and the surface of the molten gold remaining in the pot.

(2) the volume of gold poured into the mould.

(ii) When the mould is completely filled with molten gold, show that

$$8\sin^3\theta - 24\sin\theta + 11 = 0 \ .$$

Hence find the value of θ .

(2001-CE-A MATH #16) (12 marks)

16.

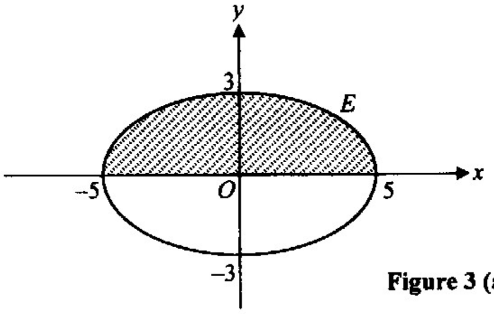


Figure 3 (a)

Figure 3 (a) shows the ellipse $E : \frac{x^2}{25} + \frac{y^2}{9} = 1$. The shaded region in the first and second quadrants is bounded by E and the x-axis.

(a) Using integration and the substitution $x = 5\sin\theta$, find the area of the shaded region.

(b) A piece of chocolate is in the shape of the solid of revolution formed by revolving the shaded region in Figure 3 (a) about the x-axis. Using the integration, find the volume of the chocolate.

(c)

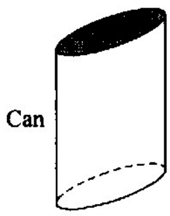


Figure 3 (b)

Four pieces of the chocolate mentioned in (b) are packed in a can which is in the shape of a right non-circular cylinder (see Figure 3 (b)). The chocolates are placed one above the other. Their axes of revolution are parallel to the base of the can and in the same vertical plane. For economic reasons, the can is in a shape such that it can just hold the chocolates. Find the capacity of the can.

(2002-CE-A MATH #16) (12 marks)

16. (a)

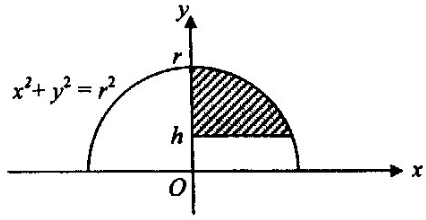


Figure 8

In Figure 8, the shaded region is bounded by the circle $x^2 + y^2 = r^2$, the y-axis and the line $y = h$, where $0 \leq h \leq r$. The shaded region is revolved about the y-axis. Show that the volume of the solid generated is $\frac{\pi}{3}\left(2r^3 - 3r^2h + h^3\right)$.

(b)

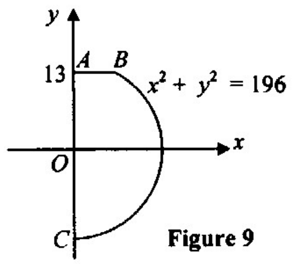


Figure 9

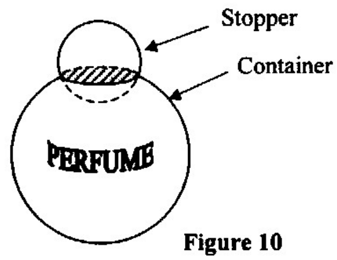


Figure 10

In Figure 9, A and C are points on the y-axis, BC is an arc of the circle $x^2 + y = 196$ and AB is a segment of the line $y = 13$. A pot is formed by revolving BC about the y-axis.

(i) Find the capacity of the pot.

(ii) Figure 10 shows a perfume bottle. The container is in the shape of the pot described above and the stopper is a solid sphere of radius 6 . Find the capacity of the perfume bottle.

(2003-CE-A MATH #19) (12 marks)

19. A water tank is formed by revolving the curve $y = x^2$, where $0 \le x \le 2$, about the y-axis (see Figure 6). Starting from time $t = 0$, water is pumped into the tank at a constant rate of 2π cubic units per minute. Let the volume and the depth of water (measured from the lowest point of the tank) in the tank at time t (in minutes) be V cubic units and h units respectively.

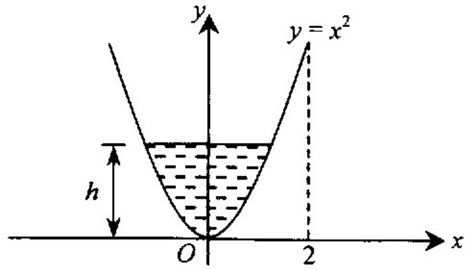


Figure 6

(a) Express V in terms of h .
Hence show that it takes 4 minutes to fill up the tank.

(b) Show that $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{2}{h}$.

(c) Which of the sketches in Figure 7 best describes the relation between h and t ? Explain your answer.

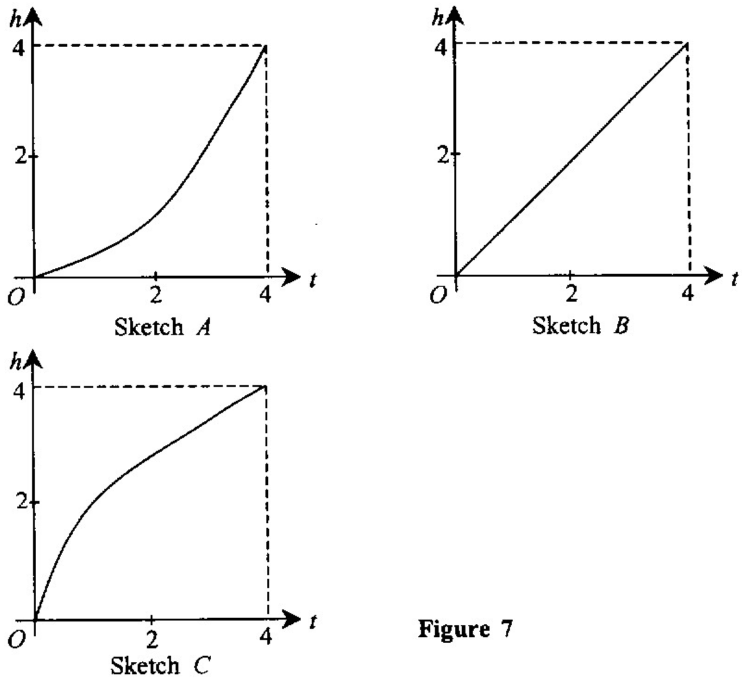


Figure 7

(d)

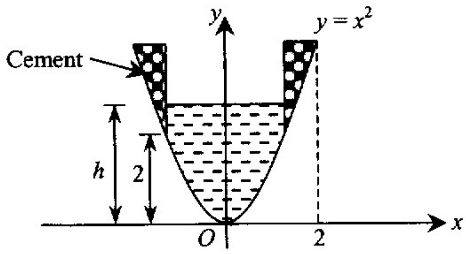


Figure 8

An engineer decides to modify the tank by laying cement on the upper part of its interior wall, so that the interior of the tank becomes cylindrical in shape for $y \ge 2$ as shown in Figure 8. Water is pumped into the empty new tank at a constant rate of 2π cubic units per minute until it is full. Sketch the graph of h against t for the new tank in the same sketch you chose in (c).

(2004-CE-A MATH #04) (4 marks)

4.

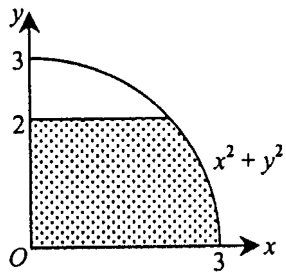


Figure 1

In Figure 1, the shaded region is bounded by the circle $x^2 + y^2 = 9$, the x-axis, the y-axis and the line $y = 2$. Find the volume of the solid generated by revolving the region about the y-axis.

(2005-CE-A MATH #16) (12 marks)

16.

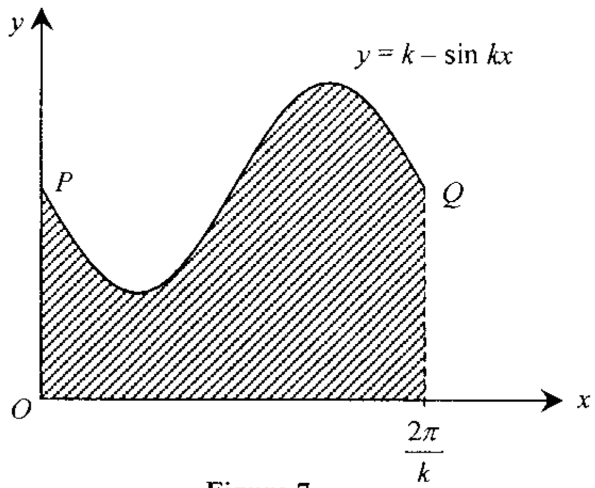


Figure 7

Figure 7 shows the curve $y = k - \sin kx$ for $0 \le x \le \dfrac{2\pi}{k}$, where $k > 1$. P and Q are the end points of the curve.

(a) (i) Find the coordinates of points P and Q .

(ii) Without using integration, find the area of the shaded region as shown in Figure 7.

(b) A solid is formed by revolving the shaded region in Figure 7 about the x-axis. Let V be the volume of the solid.

(i) Show that $V = \pi^2\left(2k + \dfrac{1}{k}\right)$.

(ii) Suppose that $2 \le k \le 3$. Find the greatest value of V as k varies.

(2006-CE-A MATH #16) (12 marks)

16. Let C be the curve $y = \frac{1}{3}x^2 - \frac{4}{3}x + 1$. C_1 is a part of C with $0 \le x \le 1$ and C_2 is a part of C with $3 \le x \le 4$.

(a) (i) Show that the equation fo C_1 is $x = 2 - \sqrt{3y+1}$.

(ii) Write down the equation of C_2 in the form $x = \mathrm{f}(y)$.

(b)

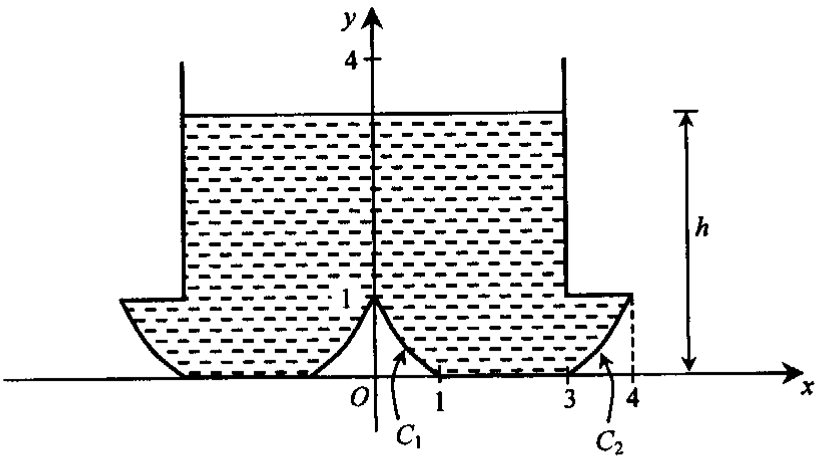


Figure 6

A container is formed by revolving C_1 , the line segment $y = 0$ (for $1 \le x \le 3$), C_2 , the line segment $y = 1$ (for $3 \le x \le 4$) and the line segment $x = 3$ (for $1 \le y \le 4$) about the y-axis (see Figure 6). Starting from time $t = 0$, water is poured into the container at a constant rate of 8π cubic units per minute. Let the volume and depth of water in the container at time t minute be V cubic units and h units respectively.

(i) Consider $0 < h < 1$.

(1) Show that $V = \frac{16\pi}{9}\left[(3h+1)^{\frac{3}{2}} - 1\right]$.

(2) Find $\frac{\mathrm{d}h}{\mathrm{d}t}$ in term of h .

(ii) Consider $1 < h < 4$. Find $\frac{\mathrm{d}h}{\mathrm{d}t}$.

(iii) It is known that $h = 1$ at $t = t_1$ and $h = 4$ at $t = t_2$. Sketch a graph to show how h varies with t for $0 \le t \le t_2$. (You are not required to find the values of t_1 and t_2 .)

(2007-AL-P MATH 2 #05) (7 marks)

5. (a) Find $\int \left(\frac{(x-2)(x-5)}{x}\right)^2 \mathrm{d}x$.

(b) Let D be the region bounded by the curve $y = \frac{x(x-3)}{x+2}$ and the x-axis. Find the volume of the solid of revolution generated by revolving D about the x-axis.

(2007-CE-A MATH #18) (12 marks)

18. (a) It is given that the curve $y = 2\sqrt{x} - x$ has a horizontal tangent at $x = r$. Show that $r = 1$.

(b)

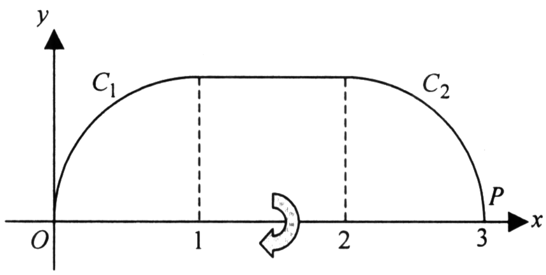


Figure 11

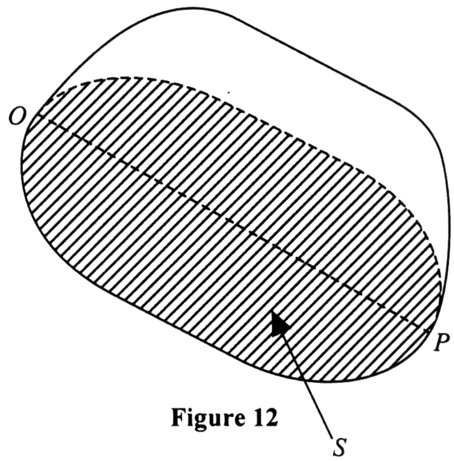


Figure 12

Let O be the origin and P be the point $(3, 0)$. Figure 11 shows a region bounded by:

(1) the curve $C_1 : y = 2\sqrt{x} - x$ (for $0 \le x \le 1$),

(2) the line segment $y = 1$ (for $1 \le x \le 2$),

(3) the curve $C_2 : y = 2\sqrt{3-x} - (3-x)$ (for $2 \le x \le 3$), and

(4) OP.

Figure 12 shows a solid formed by revolving the region about the x-axis by $180°$.

(i) The base of the solid is denoted by S in Figure 12. Find the area of S.

(ii) Show the the volume of the solid is $\frac{37}{30}\pi$.

(iii)

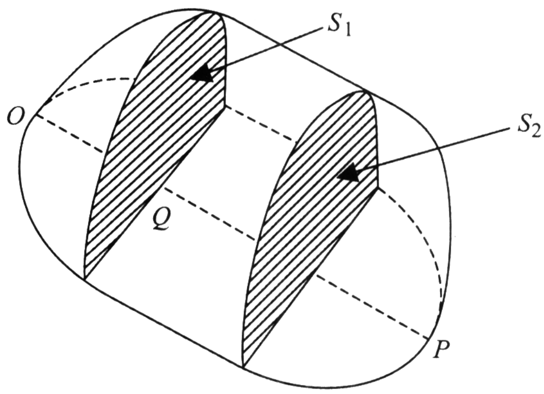


Figure 13

Mrs. Chan has baked a cake which is in the shape of the solid in Figure 12. She cuts the cake into three parts of equal volumes for her three children. The cross-sections formed, S_1 and S_2 , are perpendicular to OP (see Figure 13). Let the intersection of OP and S_1 be Q . Find $OQ : OP$.

(2008-AL-P MATH 2 #05) (6 marks)

5. (a) Find $\int y\sqrt{1-y^2}\,\mathrm{d}y$.

(b) Let D be the region bounded by the curve $x = y^{\frac{1}{2}}\left(1-y^2\right)^{\frac{1}{4}}$ and the y-axis. Find the volume of the solid of revolution generated by revolving D about the y-axis.

(2008-CE-A MATH #17) (12 marks)

17. (a) Let $\mathrm{f}(x) = x \sin x$.

(i) Show that $\int_0^{\pi} \left[f(x)+f(\pi-x)\right] \mathrm{d}x = 2\pi$.

(ii)

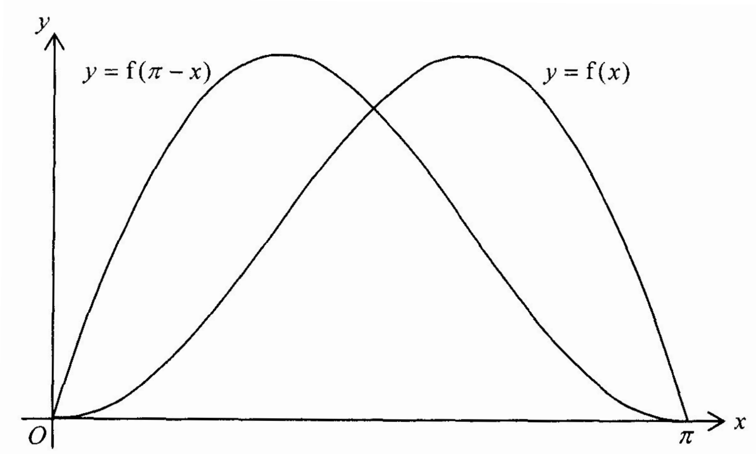


Figure 6

Figure 6 shows the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}(\pi - x)$ for $0 \le x \le \pi$. Using (a)(i), and by considering the symmetry of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}(\pi - x)$, write down the value of $\int_0^{\pi} x \sin x \,\mathrm{d}x$.

(b) (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 \sin x\right)$, and hence evaluate $\int_0^{\pi} x^2 \cos x \,\mathrm{d}x$.

(ii) R denotes the region bounded between $y = x \sin \frac{x}{2}$ and the x-axis for $0 \le x \le \pi$. Find the volume of the solid formed by revolving R about the x-axis.

(2009-CE-A MATH #15) (12 marks)

15. (a)

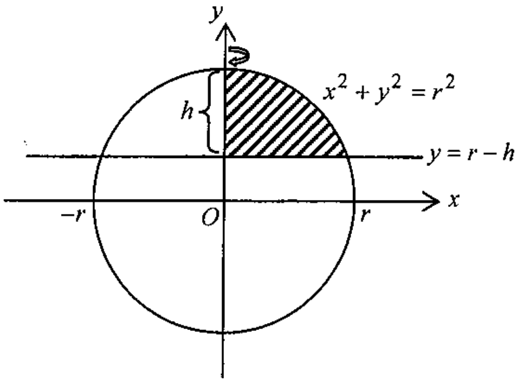
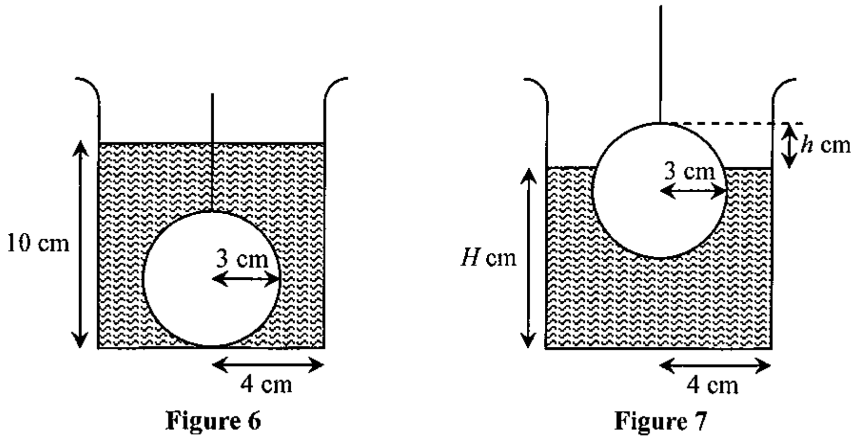


Figure 5

In Figure 5, the shaded region is bounded by the circle $x^2+y^2=r^2$, the y-axis and the straight line $y=r-h$, where $0 \le h \le 2r$. Show that the volume of the solid generated by revolving the shaded region about the y-axis is $\pi r h^2 - \dfrac{\pi h^3}{3}$.

(b)



A metal sphere of radius 3 cm, with a thin string attached, is placed inside a circular cylindrical container of base radius 4 cm. Water is poured into the container until the depth of the water is 10 cm (see Figure 6). The sphere is then being pulled vertically out of the water. Let H cm and h cm be the depth of the water and the distance between the top of the sphere and the water surface respectively (see Figure 7).

(i) Prove that $H=\dfrac{1}{48}\left(h^3-9h^2+480\right)$.

(ii) The sphere is being pulled at a constant speed of $\dfrac{1}{4}$ cm s^{-1}. At the instant when $h=3$, find the rate of change of

(1) the depth of the water,

(2) the distance between the top of the sphere and the water surface.

(2010-AL-P MATH 2 #05) (7 marks)

5. (a) Find $\displaystyle\int (5-x)\sqrt{x-1}\,\mathrm{d}x$.

(b) Let D be the region bounded by the curve $y=(5-x)^{\frac{1}{2}}(x-1)^{\frac{1}{4}}$ and the x-axis. Find the volume of the solid of revolution generated by revolving D about the x-axis.

(2010-CE-A MATH #16) (12 marks)

16. (a)

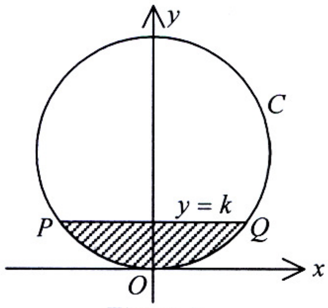


Figure 7

Figure 7 shows the shaded region bounded by the circle $C: x^2 + y^2 - 2ay = 0$ and the line $PQ: y = k$ where $0 \le k \le 2a$ and $a > 0$. Show that the volume of the solid by revolving the shaded region about the y-axis is $\pi\left(ak^2 - \frac{k^3}{3}\right)$.

(b)

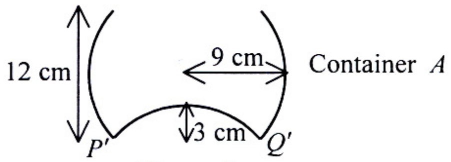


Figure 8

Figure 8 shows a container A in the shape of part of a sphere of radius 9 cm with the bottom part reflected about the horizontal plane passing through P' and Q' . The height of the container is 12 cm and the height of the bottom part of the container is 3 cm .

(i) Find the capacity of container A .

(ii)

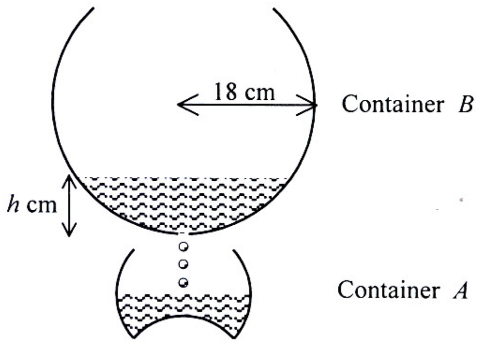


Figure 9

In Figure 9, container B is in the shape of part of the sphere of radius 18 cm . Initially, container A is empty and container B contains water of volume 909π cm³ . Water is leaking through a hole at the bottom of container B into container A . Let h cm be the depth of water in container B . Consider the moment when container A is just full.

(1) Show that $h^3 - 54h^2 + 459 = 0$.

(2) If the volume of water in container A is increasing at the rate 45π cm³ s^{-1} , find the rate of decrease of water level in container B .

(2011-AL-P MATH 2 #05) (7 marks)

5. (a) Using the substitution $x = 5 + 2\sin\theta$, find $\int \sqrt{(x-3)(7-x)}\,\mathrm{d}x$.

(b) Let D be the region bounded by the curve $y = \left[(x-3)(7-x)\right]^{\frac{1}{4}}$ and the x-axis. Find the volume of the solid of revolution generated by revolving D about the x-axis.

(2011-CE-A MATH #14) (12 marks)

14. (a)

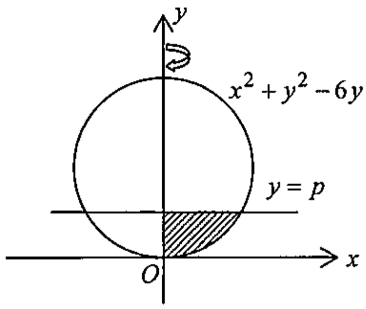


Figure 5

In Figure 5, a shaded region is bounded by the circle $x^2 + y^2 - 6y = 0$, the y-axis and the straight line $y = p$, where $0 \le p \le 6$. Show that the volume of the solid generated by revolving the shaded region about the y-axis is $\dfrac{\pi p^2(9-p)}{3}$.

(b)

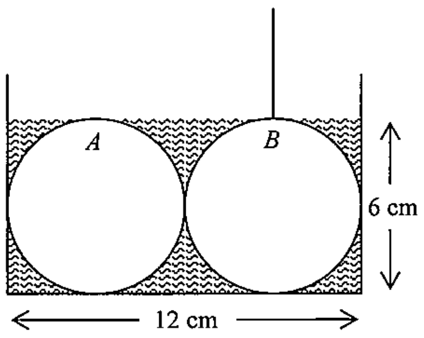


Figure 6

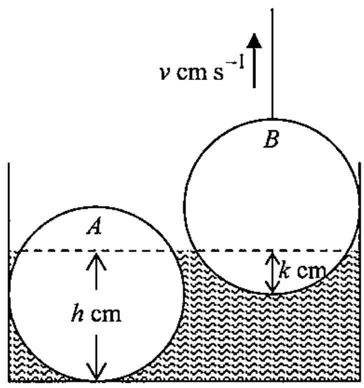


Figure 7

Two metal spheres, A and B , both of diameters 6 cm are placed inside a circular cylindrical container of base diameter 12 cm . The sphere B is attached by a wire. Water is poured into the container until the depth is 6 cm (see Figure 6).

(i) Find the volume of the water.

(ii) By considering the volume of water in Figure 7, or otherwise, prove that

$$k^3 - 9k^2 + h^3 - 9h^2 + 108h - 432 = 0 \ .$$

(iii) Suppose sphere B is being pulled at a uniform speed v cm s^{-1} and the depth of water is decreasing at a rate of 5 cm s^{-1} at the instant when $h = 5$. Find the value of v .

(PP-DSE-MATH-EP(M2) #14) (10 marks)

14. (a)

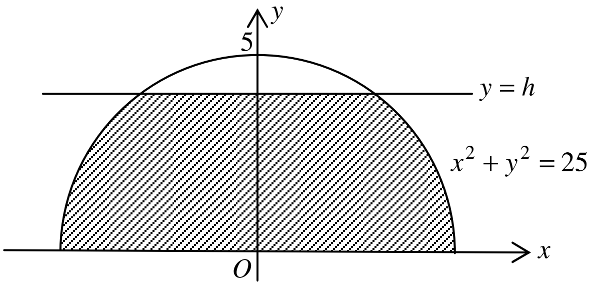


Figure 3

In Figure 3, the shaded region enclosed by the circle $x^2 + y^2 = 25$, the x-axis and the straight line $y = h$ (where $0 \leq h \leq 5$) is revolved about the y-axis. Show that the volume of the solid of revolution is $\left(25h - \dfrac{h^3}{3}\right)\pi$.

(b)

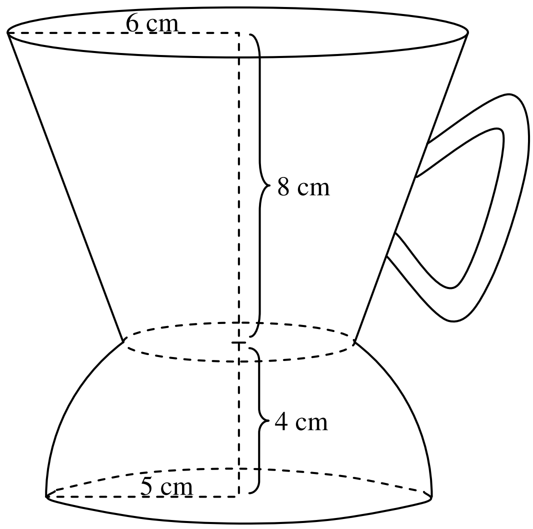


Figure 4

In Figure 4, an empty coffee cup consists of two portions. The lower portion is in the shape of the solid described in (a) with height 4 cm . The upper portion is a frustum of a circular cone. The height of the frustum is 8 cm . The radius of the top of the cup is 6 cm . Hot coffee is poured into the cup to a depth h cm at a rate of 8 cm³ s^{-1} ,where $0 \leq h \leq 12$. Let V cm³ be the volume of coffee in the cup.

(i) Find the rate of increase of depth of coffee when the depth is 3 cm .

(ii) Show that $V = \dfrac{164\pi}{3} + \dfrac{3\pi}{64}(h+4)^3$ for $4 \leq h \leq 12$.

(iii) After the cup is fully filled, suddenly it cracks at the bottom. The coffee leaks at a rate of 2 cm³ s^{-1}. Find the rate of decrease of the depth of coffee after 15 seconds of leaking, giving your answer correct to 3 significant figures.

(2012-AL-P MATH 2 #05) (7 marks)

5. (a) Find the derivative of $\sqrt{x}e^{2\sqrt{x}}$.

(b) Let D be the region bounded by the curve $y = e^{\sqrt{x}}$, the straight line $x = 4$, the straight line $x = 9$ and the x-axis. Find the volume of the solid of revolution generated by revolving D about the x-axis.

(2012-DSE-MATH-EP(M2) #09) (4 marks)

9. (a) Using integration by parts, find $\int x \sin x \, dx$.

(b)

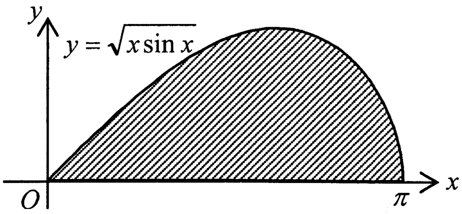


Figure 4

Figure 4 shows the shaded region bounded by the curve $y = \sqrt{x \sin x}$ for $0 \le x \le \pi$ and the x-axis. Find the volume of the solid generated by revolving the region about the x-axis.

(2013-AL-P MATH 2 #05) (6 marks)

5. (a) Find $\int \frac{(x-12)^2(x-6)}{x} dx$.

(b) Define $f(x) = (x-6)\sqrt{\frac{x}{x+6}}$ for all $x \ge 0$.

Let D be the region bounded by the curve $y = f(x)$ and the x-axis. Find the volume of the solid of revolution generated by revolving D about the x-axis.

(2013-DSE-MATH-EP(M2) #05) (6 marks)

6.

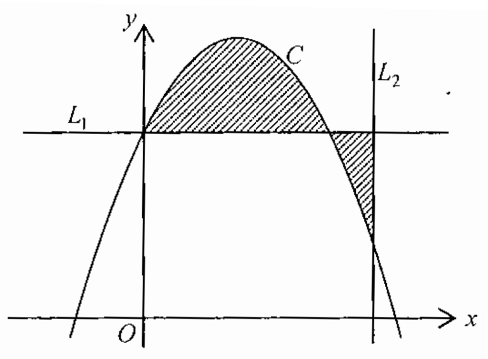


Figure 1

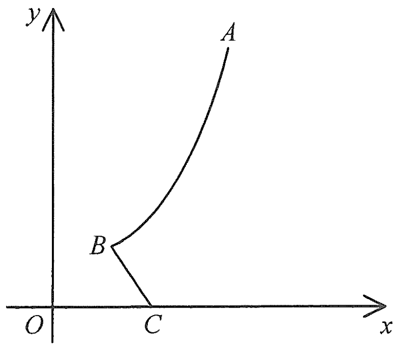
Figure 1 shows the shaded region with boundaries $C : y = \frac{-x^2}{2} + 2x + 4$, $L_1 : y = 4$ and $L_2 : x = 5$. It is given that C intersects L_1 at $(0,4)$ and $(4,4)$.

(a) Find the area of the shaded region.

(b) Find the volume of solid of revolution when the shaded region is revolved about L_1 .

(2015-DSE-MATH-EP(M2) #12) (13 marks)

12. (a) In the figure, the curve Γ consists of the curve AB , the line segments BC and CO , where O is the origin, B lies in the first quadrant and C lies on the x-axis. The equations of AB and BC are $x^2 - 4y + 8 = 0$ and $3x + y - 9 = 0$ respectively.



(i) Find the coordinates of B .

(ii) Let h be the y-coordinate of A , where $h > 3$. A cup is formed by revolving Γ about the y-axis. Prove that the capacity of the cup is $\pi\left(2h^2 - 8h + 25\right)$.

(b) A cup described in (a)(ii) is placed on a horizontal table. The radii of the base and the lip of the cup are 3 cm and 6 cm respectively.

(i) Find the capacity of the cup.

(ii) Water is poured into the cup at a constant rate of 24π cm³/s. Find the rate of change of the depth of water when the volume of water in the cup is 35π cm³ .

(2016-DSE-MATH-EP(M2) #07) (8 marks)

7. (a) Using integration by substitution, find $\int (1 + \sqrt{t+1})^2 \, \mathrm{d}t$.

(b) Consider the curve $\Gamma : y = 4x^2 - 4x$, where $1 \le x \le 4$. Let R be the region bounded by Γ , the straight line $y = 48$ and the two axes. Find the volume of the solid of revolution generated by revolving R about the y-axis.

(2017-DSE-MATH-EP(M2) #09) (13 marks)

9. Define $\mathrm{f}(x) = \dfrac{x^2 - 5x}{x + 4}$ for all $x \neq -4$. Denote the graph of $y = \mathrm{f}(x)$ by G .

(a) Find the asymptote(s) of G .

(b) Find $\mathrm{f}'(x)$.

(c) Find the maximum point(s) and the minimum point(s) of G .

(d) Let R be the region bounded by G and the x-axis. Find the volume of the solid of revolution generated by revolving R about the x-axis.

(2018-DSE-MATH-EP(M2) #10) (13 marks)

10. (a) (i) Prove that $\displaystyle\int \sin^4 x \, dx = \frac{-\cos x \sin^3 x}{4} + \frac{3}{4}\int \sin^2 x \, dx$.

(ii) Evaluate $\displaystyle\int_0^{\pi} \sin^4 x \, dx$.

(b) (i) Let $f(x)$ be a continuous function such that $f(\beta - x) = f(x)$ for all real numbers x , where β is a constant. Prove that $\displaystyle\int_0^{\beta} x f(x) \, dx = \frac{\beta}{2}\int_0^{\beta} f(x) \, dx$.

(ii) Evaluate $\displaystyle\int_0^{\pi} x \sin^4 x \, dx$.

(c) Consider the curve $G : y = \sqrt{x} \sin^2 x$, where $\pi \le x \le 2\pi$. Let R be the region bounded by G and the x-axis. Find the volume of the solid of revolution generated by revolving R about the x-axis.

(2019-DSE-MATH-EP(M2) #04) (6 marks)

4. Define $g(x) = \dfrac{\ln x}{\sqrt{x}}$ for all $x \in (0, 99)$. Denote the graph of $y = g(x)$ by G .

(a) Prove that G has only one maximum point.

(b) Let R be the region bounded by G , the x-axis and the vertical line passing through the maximum point of G . Find the volume of the solid of revolution generated by revolving R about the x-axis.

(2020-DSE-MATH-EP(M2) #04) (6 marks)

4. (a) Find $\displaystyle\int \sin^2 \theta \, d\theta$.

(b) Define $f(x) = 4x(1 - x^2)^{\frac{1}{4}}$ for all $x \in [0, 1]$. Denote the graph of $y = f(x)$ by G . Let R be the region bounded by G and the x-axis. Find the volume of the solid of revolution generated by revolving R about the x-axis.

(2021-DSE-MATH-EP(M2) #07) (7 marks)

7. (a) Using integration by parts, find $\displaystyle\int (\ln x)^2 dx$.

(c) Consider the curve $C : y = \sqrt{x} \ln\left(x^2 + 1\right)$, where $x \ge 0$. Let R be the region bounded by C , the straight line $x = 1$ and the x-axis . Find the volume of the solid of revolution generated by revolving R about the x-axis .

ANSWERS

(1984-CE-A MATH 2 #10) (20 marks)

10. (c) $V = 16\pi^2 t^2 - 64\pi^2 t + C$

$t = 2$, the required time is 2 hours

(1991-CE-A MATH 2 #11) (16 marks)

11. (a) 32π

(b) (i) 64π

(ii) (1) $2\sqrt{2}$

(2) $\left(2\sqrt{2} - 1\right) : 1$

(1992-CE-A MATH 2 #11) (16 marks)

11. (b) (i) 75π

(ii) 125π

(iii) (1) $125\pi - \dfrac{\pi}{100}\left(25t + t^2\right)$

(2) 100 seconds

(1993-CE-A MATH 2 #12) (16 marks)

12. (b) (i) $4\pi^2$

(ii) Half of the water: π min.

Remaining water: $\left(\dfrac{\sqrt{17} - 3}{2}\right)\pi$ min.

(1994-CE-A MATH 2 #13) (16 marks)

13. (b) (ii) (2) $\dfrac{5}{24}\pi a^3$

(1995-CE-A MATH 2 #12) (16 marks)

12. (b) (ii) $\dfrac{56\pi}{3}$

(c) $4\pi^2$

(1996-CE-A MATH 2 #11) (16 marks)

11. (a) $B = (3, 4)$

(b) (ii) $x = 3 + \sqrt{4 - y}$

(c) (ii) 1.48

(d) 12π

(1997-CE-A MATH 2 #10) (16 marks)

10. (b) 2.5

(c) $\dfrac{\pi}{16}\left(4\pi - 3\sqrt{3}\right)$

(1998-CE-A MATH 2 #12) (16 marks)

12. (b) (ii) $3\sqrt{3}a\pi$

(c) (ii) $\dfrac{16\pi}{3} + 3\sqrt{3}\pi$

(1999-CE-A MATH 2 #13) (16 marks)

13. (a) $r = 4$

(c) (i) $\dfrac{dh}{dt} = \dfrac{-2}{31}$ units per sec.

(ii) $\dfrac{dh}{dt} = \dfrac{-1}{8}$ units per sec.

(2000-CE-A MATH 2 #11) (16 marks)

11. (c) (i) (1) $4\sin\theta$

(2) $\pi\left(64\sin\theta - \dfrac{64}{3}\sin^3\theta\right)$

(ii) $\theta = \dfrac{\pi}{6}$

(2001-CE-A MATH #16) (12 marks)

16. (a) $\dfrac{15\pi}{2}$

(b) 60π

(c) 360π

(2002-CE-A MATH #16) (12 marks)

16. (b) (i) 3645π

(ii) 3600π

(2003-CE-A MATH #19) (12 marks)

19. (a) $V = \dfrac{1}{2}\pi h^2$

(c) Sketch C

(2004-CE-A MATH #04) (4 marks)

4. $\dfrac{46\pi}{3}$

(2005-CE-A MATH #16) (12 marks)

16. (a) (i) $P(0,k)$, $Q\left(\frac{2\pi}{k},k\right)$

(ii) 2π

(b) (ii) $\frac{19\pi^2}{3}$

(2006-CE-A MATH #16) (12 marks)

16. (a) (ii) $x=2+\sqrt{3y+1}$

(b) (i) (2) $\frac{dh}{dt}=\frac{1}{(3h+1)^{\frac{1}{2}}}$

(ii) $\frac{dh}{dt}=\frac{8}{9}$

(2007-AL-P MATH 2 #05) (7 marks)

5. (a) $\frac{x^3}{3}-7x^2+69x-140\ln|x|-\frac{100}{x}$ + constant

(b) $\pi\left[129-140\ln\left(\frac{5}{2}\right)\right]$

(2007-CE-A MATH #18) (12 marks)

18. (b) (i) $\frac{16}{3}$

(iii) 49 : 135

(2008-AL-P MATH 2 #05) (6 marks)

5. (a) $\frac{-1}{3}\left(1-y^2\right)^{\frac{3}{2}}$ + constant

(b) $\frac{\pi}{3}$

(2008-CE-A MATH #17) (12 marks)

17. (a) (ii) π

(b) (i) -2π

(ii) $\frac{\pi^4}{6}+\pi^2$

(2009-CE-A MATH #15) (12 marks)

15. (b) (ii) (1) $\frac{-9}{28}$ cm s^{-1}

(2) $\frac{4}{7}$ cm s^{-1}

(2010-AL-P MATH 2 #05) (7 marks)

5. (a) $\frac{8}{3}(x-1)^{\frac{3}{2}}-\frac{2}{5}(x-1)^{\frac{5}{2}}$ + constant

(b) $\frac{128\pi}{15}$

(2010-CE-A MATH #16) (12 marks)

16. (b) (i) 756π cm^3

(ii) (2) $\frac{5}{11}$ cm s^{-1}

(2011-AL-P MATH 2 #05) (7 marks)

5. (a) $2\sin^{-1}\left(\frac{x-5}{2}\right)+\frac{x-5}{2}\sqrt{(x-3)(7-x)}$ + constant

(b) $2\pi^2$

(2011-CE-A MATH #14) (12 marks)

14. (b) (i) 144π cm^3

(iii) 26

(PP-DSE-MATH-EP(M2) #14) (10 marks)

14. (b) (i) $\frac{1}{2\pi}$ cm s^{-1}

(iii) 0.0183 cm s^{-1}

(2012-AL-P MATH 2 #05) (7 marks)

5. (a) $\frac{1}{2\sqrt{x}}e^{2\sqrt{x}}+e^{2\sqrt{x}}$

(b) $\frac{5}{2}\pi e^6-\frac{3}{2}\pi e^4$

(2012-DSE-MATH-EP(M2) #09) (4 marks)

9. (a) $-x\cos x+\sin x$ + constant

(b) π^2

(2013-AL-P MATH 2 #05) (6 marks)

5. (a) $\frac{1}{3}x^3-15x^2+288x-864\ln|x|$ + constant

(b) $\pi(612-864\ln 2)$

(2013-DSE-MATH-EP(M2) #05) (6 marks)

6. (a) $\dfrac{13}{2}$

(b) $\dfrac{125\pi}{12}$

(2015-DSE-MATH-EP(M2) #12) (13 marks)

12. (a) (i) $(2, 3)$

(b) (i) 179π cm³

(ii) 2 cm s^{-1}

(2016-DSE-MATH-EP(M2) #07) (8 marks)

7. (a) $2t + \dfrac{t^2}{2} + \dfrac{4}{3}(t+1)^{\frac{3}{2}} + \text{constant}$

(b) 426π

(2017-DSE-MATH-EP(M2) #09) (13 marks)

9. (a) Vertical asymptote: $x = -4$

Oblique asymptote: $y = x - 9$

(b) $f'(x) = 1 - \dfrac{36}{(x+4)^2}$

(c) Maximum point: $(-10, -25)$

Minimum point: $(2, -1)$

(d) $\left[\dfrac{2285}{3} - 1872\ln\left(\dfrac{3}{2}\right)\right]\pi$

(2018-DSE-MATH-EP(M2) #10) (13 marks)

10. (a) (ii) $\dfrac{3\pi}{8}$

(b) (ii) $\dfrac{3\pi^2}{16}$

(c) $\dfrac{9\pi^3}{16}$

(2019-DSE-MATH-EP(M2) #04) (6 marks)

4. (b) $\dfrac{8\pi}{3}$

(2020-DSE-MATH-EP(M2) #04)

4. (a) $\dfrac{1}{2}\theta - \dfrac{1}{4}\sin 2\theta + \text{constant}$

(b) π^2

(2021-DSE-MATH-EP(M2) #07)

7. (a) $x(\ln x)^2 - 2x\ln x + 2x + \text{constant}$

(b) $\pi\left((\ln 2)^2 - 2\ln 2 + 1\right)$