4. Indefinite Integration

(1979-CE-A MATH 1 #07) (5 marks) (Modified)

7. Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. Hence, find the indefinite integral $\int 4\sin^3\theta \, \mathrm{d}\theta$.

(1980-CE-A MATH 1 #04) (6 marks)

4. Find $\int \frac{2}{\cot\frac{x}{2}+\tan\frac{x}{2}} \, \mathrm{d}x$.

(1980-CE-A MATH 2 #02) (5 marks) (Modified)

2. Find the indefinite integral $\int (x+2)\sqrt{x-1} \, \mathrm{d}x$.

(1981-CE-A MATH 2 #01) (5 marks)

1. Find the indefinite integral $\int (1+\cos\theta)^2 \, \mathrm{d}\theta$.

(1982-CE-A MATH 2 #01) (5 marks) (Modified)

1. Find the indefinite integral $\int \frac{x}{\sqrt{x+9}} \, \mathrm{d}x$.

(1983-CE-A MATH 2 #02) (5 marks) (Modified)

2. Find the indefinite integral $\int x\sin^2(x^2) \, \mathrm{d}x$.

(1986-CE-A MATH 2 #07) (6 marks)

7. Let $y = \frac{\tan^3\theta}{3} - \tan\theta$.

   Find $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ in terms of $\tan\theta$.

   Hence, or otherwise, find $\int \tan^4\theta \, \mathrm{d}\theta$.

(1989-CE-A MATH 2 #04) (5 marks)

4. (a) Find $\int \cos^2 2x \, \mathrm{d}x$.

   (b) Using the result of (a), find $\int \sin^2 2x \, \mathrm{d}x$.

(1990-CE-A MATH 2 #03) (5 marks)

3. Using the substitution $u = \sin^2 x$, find $\int \frac{\sin x \cos x}{\sqrt{9\sin^2 x + 4\cos^2 x}}\,\mathrm{d}x$.

(1990-AL-P MATH 2 #04) (Part) (3 marks)

4. (b) Evaluate $\int \frac{\mathrm{d}x}{\sqrt{x^2 + 4x + 2}}$.

(1992-CE-A MATH 2 #04) (6 marks)

4. The slope of the tangent to a curve C at any point (x, y) on C is $x^2 - 2$. C passes through the point $(3, 4)$.

(a) Find an equation of C.

(b) Find the coordinates of the point on C at which the slope of the tangent is -2.

(1993-AL-P MATH 2 #05) (7 marks)

5. Evaluate $\int e^{2x}(\sin x + \cos x)^2\,\mathrm{d}x$.

(1993-CE-A MATH 2 #06) (7 marks)

6. The slope of a curve C at any point (x, y) on C is $3x^2 - 6x - 1$. C passes through the point $(1, 0)$.

(a) Find the equation of C.

(b) Find the equation of the tangent to C at the point where C cuts the y-axis.

(1994-AL-P MATH 2 #02) (Modified) (6 marks)

1. (a) Evaluate $\int \tan^3 x\,\mathrm{d}x$,

(b) Let $\frac{x^2 - x + 2}{x(x-2)^2} \equiv \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$, where A, B, and C are constants. Find the values of A, B and C, hence find $\int \frac{x^2 - x + 2}{x(x-2)^2}\,\mathrm{d}x$.

(1994-CE-A MATH 2 #01) (4 marks)

1. Find $\int (\sin x - \cos x)^2\,\mathrm{d}x$.

(1994-CE-A MATH 2 #08) (7 marks)

8. The slope at any point (x, y) of a curve C is given by

$$\frac{dy}{dx} = 8 - 10x$$

and C passes through the point $A(1, 13)$.

(a) Find the equation of C .

(b) Find the equation of the normal to C at the point where C cuts the y-axis.

(1995-AL-P MATH 2 #02) (5 marks)

2. (a) Using the substitution $x = \sin^2\theta$ ($0 < \theta < \frac{\pi}{2}$) , prove that

$$\int \frac{f(x)}{\sqrt{x(1-x)}}\,dx = 2\int f(\sin^2\theta)\,d\theta$$

(b) Hence, or otherwise, evaluate $\int \frac{dx}{\sqrt{x(1-x)}}$ and $\int \sqrt{\frac{x}{1-x}}\,dx$.

(1995-CE-A MATH 2 #01) (5 marks)

1. The slope at any point (x, y) of a curve C is given by

$$\frac{dy}{dx} = 2x\sqrt{x^2+1}$$

and C cuts the y-axis at the point $(0, 1)$. Find the equation of C .

(Hint : Let $u = x^2 + 1$.)

(1996-CE-A MATH 2 #06) (6 marks)

6. The slope at any point (x, y) of a curve is given by $\frac{dy}{dx} = \tan^3 x \sec x$. If the curve passes through the origin, find its equation.

(Hint : Let $u = \sec x$.)

(1997-AL-P MATH 2 #01) (5 marks)

1. (a) Show that $\frac{d}{dx}\tan\frac{x}{2} = \frac{1}{1+\cos x}$.

(b) Using (a), or otherwise, find $\int \frac{x + \sin x}{1+\cos x}\,dx$.

(1997-CE-A MATH 2 #02) (4 marks)

2. Find $\int x\sqrt{x-1}\,dx$. (Hint : Let $u = x - 1$.)

(1997-CE-A MATH 2 #05) (5 marks)

5. The slope at any point (x, y) of a curve is given by

$$\frac{dy}{dx} = 6x + \frac{1}{x^2} ,$$

where $x > 0$. If the curve cuts the x-axis at the point $(1, 0)$, find its equation.

(1998-CE-A MATH 2 #04) (5 marks)

4. The slope at any point (x, y) of a curve is given by

$$\frac{dy}{dx} = \cos^2 x .$$

If the curve passes through the point $\left(\frac{\pi}{2}, \pi\right)$, find its equation.

(1999-CE-A MATH 2 #02) (4 marks)

2. Evaluate $\int x(x + 2)^{99} \, dx$.

(1999-CE-A MATH 2 #06) (6 marks)

6. The slope at any point (x, y) of a curve is given by

$$\frac{dy}{dx} = 3x^2 - 2x + k .$$

If the curve touches the x-axis at the point $(2, 0)$, find

(a) the value of k ,

(b) the equation of the curve.

(2000-CE-A MATH 2 #01) (4 marks)

1. Find $\int \sqrt{2x + 1} \, dx$.

(2000-CE-A MATH 2 #06) (7 marks)

6.

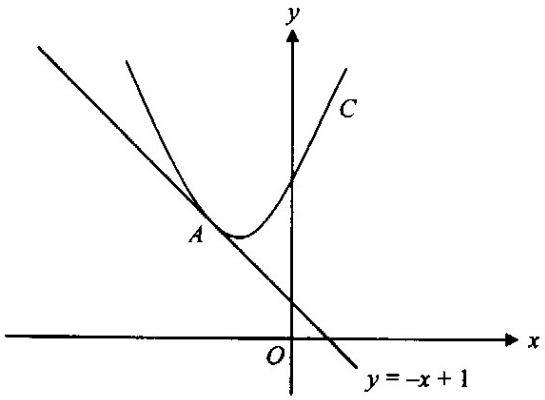

Figure 3

The slope at any point (x, y) of a curve C is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 3$. The line $y = -x + 1$ is a tangent to the curve at point A . (See Figure 3.) Find

(a) the coordinates of A ,

(b) the equation of C .

(2001-AL-P MATH 2 #02) (5 marks)

2. Evaluate

(a) $\int \frac{x^3}{1+x^2}\,\mathrm{d}x$.

(b) $\int x^2 \tan^{-1} x\,\mathrm{d}x$.

(2001-AL-P MATH 2 #12) (Part / Modified) (6 marks)

12. (a) (i) Evaluate $\int \frac{1}{x^2 - x + 1}\,\mathrm{d}x$,

(ii) Let $\frac{x^2 + 1}{(x^2 - x + 1)(x^2 + x + 1)} \equiv \frac{Ax + B}{x^2 - x + 1} + \frac{Cx + D}{x^2 + x + 1}$, where A , B , C and D are constants. Find A , B , C and D . Hence, evaluate $\int \frac{x^2 + 1}{(x^2 - x + 1)(x^2 + x + 1)}\,\mathrm{d}x$.

(2001-CE-A MATH #02) (4 marks)

2. Find $\int \frac{x}{\sqrt{3x^2 + 1}}\,\mathrm{d}x$. (Hint : Let $u = 3x^2 + 1$.)

(2003-CE-A MATH #01) (3 marks)

1. Find $\int \cos^2\theta\,\mathrm{d}\theta$.

(2004-CE-A MATH #01) (4 marks)

1. Find

(a) $\int \cos(3x + 1)\,\mathrm{d}x$,

(b) $\int (2 - x)^{2004}\,\mathrm{d}x$.

(2004-CE-A MATH #03) (4 marks)

3. The slope at any point (x, y) of a curve C is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 1$. If the x-intercept of C is 1 , find the equation of C .

(2005-CE-A MATH #01) (2 marks)

1. Find $\int (2x - 3)^7\,\mathrm{d}x$.

(2005-CE-A MATH #10) (6 marks)

10. (a) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left[x(x + 1)^n\right] = (x + 1)^{n-1}\left[(n + 1)x + 1\right]$, where n is a rational number.

(b) The slope at any point (x, y) of a curve C is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = (x + 1)^{2004}(2006x + 1)$. If C passes through the point $(-1, 1)$, find the equation of C .

(2006-AL-P MATH 2 #04) (Part)

4. (a) Using the substitution $t = \sqrt{1+x^2}$, find $\int \frac{x^3}{\sqrt{1+x^2}}\,\mathrm{d}x$.

(2006-CE-A MATH #10) (5 marks)

10. The slope at any point (x, y) of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 3 + 2\cos 2x$. If the curve passes through the point $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$, find its equation.

(2007-AL-P MATH 2 #04) (Part)

4. (a) Using integration by parts, find $\int e^x \sin x\,\mathrm{d}x$.

(2007-CE-A MATH #01) (3 marks)

1. Find $\int \frac{x^4+1}{x^2}\,\mathrm{d}x$.

(2008-CE-A MATH #01) (2 marks)

1. Find $\int (8x+5)^{250}\,\mathrm{d}x$.

(2009-CE-A MATH #01) (5 marks)

1. Find

(a) $\int (4x+1)^2\,\mathrm{d}x$,

(b) $\int \sin 3\theta \cos\theta\,\mathrm{d}\theta$.

(2011-CE-A MATH #05) (5 marks)

5. (a) Find $\int (2x+1)^2\,\mathrm{d}x$.

(b) The slope at any point (x, y) of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = (2x+1)^2$. If the curve passes through the point $(-1,0)$, find its equation.

(SP-DSE-MATH-EP(M2) #03) (4 marks)

3. The slope at any point (x, y) of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\ln(x^2+1)$. It is given that the curve passes through the point $(0,1)$. Find the equation of the curve.

(SP-DSE-MATH-EP(M2) #04) (4 marks)

4. Find $\int \left(x^2 - \frac{1}{x}\right)^4 \mathrm{d}x$.

(PP-DSE-MATH-EP(M2) #08) (5 marks)

8. (a) Using integration by substitution, find $\int \frac{\mathrm{d}x}{\sqrt{4 - x^2}}$.

(b) Using integration by parts, find $\int \ln x \, \mathrm{d}x$.

(2012-DSE-MATH-EP(M2) #04) (5 marks)

4. (a) Find $\int \frac{x + 1}{x} \mathrm{d}x$.

(b) Using the substitution $u = x^2 - 1$, find $\int \frac{x^3}{x^2 - 1} \mathrm{d}x$.

(2013-DSE-MATH-EP(M2) #04) (5 marks)

4. The slope at any point (x, y) of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = e^x - 1$. It is given that the curve passes through the point $(1, e)$.

(a) Find the equation of the curve.

(b) Find the equation of tangent to the curve at the point where the curve cuts the y-axis.

(2014-DSE-MATH-EP(M2) #05) (6 marks)

5. (a) Find $\int \frac{\mathrm{d}x}{\sqrt{9 - x}}$, where $x < 9$.

(b) Using integration by substitution, find $\int \frac{\mathrm{d}x}{\sqrt{9 - x^2}}$, where $-3 < x < 3$.

(2015-DSE-MATH-EP(M2) #04) (7 marks)

4. (a) Using integration by parts, find $\int x^2 \ln x \, \mathrm{d}x$.

(b) At any point (x, y) on the curve Γ , the slope of the tangent to Γ is $9x^2 \ln x$. It is given that Γ passes through the point (1, 4). Find the equation of Γ .

(2017-DSE-MATH-EP(M2) #08) (8 marks)

8. Let $\mathrm{f}(x)$ be a continuous function defined on $\mathbf{R}^+$, where $\mathbf{R}^+$ is the set of positive real numbers. Denote the curve $y = \mathrm{f}(x)$ by Γ. It is given that Γ passes through the point $P(e^3, 7)$ and $\mathrm{f}'(x) = \frac{1}{x}\ln x^2$ for all $x > 0$. Find

(a) the equation of the tangent to Γ at P,

(b) the equation of Γ,

(c) the point(s) of inflexion of Γ.

(2018-DSE-MATH-EP(M2) #05) (7 marks)

5. (a) Using integration by substitution, find $\int x^3\sqrt{1+x^2}\,\mathrm{d}x$.

(b) At any point (x, y) on the curve Γ, the slope of the tangent to Γ is $15x^3\sqrt{1+x^2}$. The y-intercept of Γ is 2. Find the equation of Γ.

(2019-DSE-MATH-EP(M2) #03) (6 marks)

3. A researcher performs an experiment to study the rate of change of the volume of liquid X in a vessel. The experiment lasts for 24 hours. At the start of the experiment, the vessel contains $580\ \mathrm{cm}^3$ of liquid X. The researcher finds that during the experiment, $\frac{\mathrm{d}V}{\mathrm{d}t} = -2t$, where $V\ \mathrm{cm}^3$ is the volume of liquid X in the vessel and t is the number of hours elapsed since the start of the experiment.

(a) The researcher claims that the vessel contains some liquid X at the end of the experiment. Is the claim correct? Explain your answer.

(b) It is given that $V = h^2 + 24h$, where h cm is the depth of liquid X in the vessel. Find the value of $\frac{\mathrm{d}h}{\mathrm{d}t}$ when $t = 18$.

(2019-DSE-MATH-EP(M2) #08) (8 marks)

8. Let $\mathrm{h}(x)$ be a continuous function defined on $\mathbf{R}^+$, where $\mathbf{R}^+$ is the set of positive real numbers. It is given that $\mathrm{h}'(x) = \frac{2x^2 - 7x + 8}{x}$ for all $x > 0$.

(a) Is $\mathrm{h}(x)$ an increasing function? Explain your answer.

(b) Denote the curve $y = \mathrm{h}(x)$ by H. It is given that H passes through the point $(1,3)$. Find

(i) the equation of H,

(ii) the point(s) of inflexion of H.

(2020-DSE-MATH-EP(M2) #07) (8 marks)

7. Let $\mathrm{f}(x)$ be a continuous function defined on $\mathbf{R}$. Denote the curve $y = \mathrm{f}(x)$ be Γ. It is given that Γ passes through the point $(1, 2)$ and $\mathrm{f}'(x) = -2x + 8$ for all $x \in \mathbf{R}$.

(a) Find the equation of Γ.

(b) Let L be a tangent to Γ such that L passes through the point $(5, 14)$ and the slope of L is negative. Denote the point of contact of Γ and L by P. Find

(i) the coordinates of P,

(ii) the equation of the normal to Γ at P.

ANSWERS

(1979-CE-A MATH 1 #07) (5 marks) (Modified)

7. $-3\cos\theta + \frac{1}{3}\cos 3\theta + \text{constant}$

(1980-CE-A MATH 1 #04) (6 marks)

4. $-\cos x + \text{constant}$

(1980-CE-A MATH 2 #02) (5 marks) (Modified)

2. $\frac{2}{5}(x-1)^{\frac{5}{2}} + 2(x-1)^{\frac{3}{2}} + \text{constant}$

(1981-CE-A MATH 2 #01) (5 marks)

1. $\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta + \text{constant}$

(1982-CE-A MATH 2 #01) (5 marks) (Modified)

1. $\frac{2}{3}(x+9)^{\frac{3}{2}} - 18(x+9)^{\frac{1}{2}} + \text{constant}$

(1983-CE-A MATH 2 #02) (5 marks) (Modified)

2. $\frac{x^2}{4} - \frac{1}{8}\sin 2x^2 + \text{constant}$

(1986-CE-A MATH 2 #07) (6 marks)

7. $\frac{dy}{d\theta} = \tan^4\theta - 1$

$$\int \tan^4\theta \, d\theta = \frac{\tan^3\theta}{3} - \tan\theta + \theta + \text{constant}$$

(1989-CE-A MATH 2 #04) (5 marks)

4. (a) $\frac{1}{2}x + \frac{1}{8}\sin 4x + \text{constant}$

(b) $\frac{1}{2}x - \frac{1}{8}\sin 4x + \text{constant}$

(1990-CE-A MATH 2 #03) (5 marks)

3. $\frac{1}{5}\sqrt{5\sin^2 x + 4} + \text{constant}$

(1990-AL-P MATH 2 #04) (Part) (3 marks)

4. (b) $\ln\left(x + 2 + \sqrt{x^2 + 4x + 2}\right) + \text{constant}$

(1992-CE-A MATH 2 #04) (6 marks)

4. (a) $y = \frac{1}{3}x^3 - 2x + 1$

(b) (0,1)

(1993-AL-P MATH 2 #05) (7 marks)

5. $\frac{1}{2}e^{2x} + \frac{1}{4}e^{2x}(\sin 2x - \cos 2x) + \text{constant}$

(1993-CE-A MATH 2 #06) (7 marks)

6. (a) $y = x^3 - 3x^2 - x + 3$

(b) $y = -x + 3$

(1994-AL-P MATH 2 #02) (Modified) (6 marks)

1. (a) $\frac{1}{2}\tan^2 x + \ln|\cos x| + \text{constant}$

(b) $\frac{1}{2}\ln|x| + \frac{1}{2}\ln|x-2| - \frac{2}{x-2} + \text{constant}$

(1994-CE-A MATH 2 #01) (4 marks)

1. $x - \sin^2 x + \text{constant}$

(1994-CE-A MATH 2 #08) (7 marks)

8. (a) $y = 8x - 5x^2 + 10$

(b) $y = \frac{-1}{8}x + 10$

(1995-AL-P MATH 2 #02) (5 marks)

2. (b) $\int \frac{dx}{\sqrt{x(1-x)}} = 2\sin^{-1}\sqrt{x} + \text{constant}$

$$\int \sqrt{\frac{x}{1-x}}\, dx = \sin^{-1}\sqrt{x} - \sqrt{x(1-x)} + \text{constant}$$

(1995-CE-A MATH 2 #01) (5 marks)

1. $y = \frac{2}{3}(x^2+1)^{\frac{3}{2}} + \frac{1}{3}$

(1996-CE-A MATH 2 #06) (6 marks)

6. $y = \frac{1}{3}\sec^3 x - \sec x + \frac{2}{3}$

(1997-AL-P MATH 2 #01) (5 marks)

1. (b) $x \tan \frac{x}{2}+\text{constant}$

(1997-CE-A MATH 2 #02) (4 marks)

2. $\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}+\text{constant}$

(1997-CE-A MATH 2 #05) (5 marks)

5. $y=3x^2-\frac{1}{x}-2$

(1998-CE-A MATH 2 #04) (5 marks)

4. $y=\frac{x}{2}+\frac{\sin 2x}{4}+\frac{3\pi}{4}$

(1999-CE-A MATH 2 #02) (4 marks)

2. $\frac{(x+2)^{101}}{101}-\frac{(x+2)^{100}}{50}+\text{constant}$

(1999-CE-A MATH 2 #06) (6 marks)

6. (a) $k=-8$

(b) $y=x^3-x^2-8x+12$

(2000-CE-A MATH 2 #01) (4 marks)

1. $\frac{1}{3}(2x+1)^{\frac{3}{2}}+\text{constant}$

(2000-CE-A MATH 2 #06) (7 marks)

6. (a) $A=(-2,3)$

(b) $y=x^2+3x+5$

(2001-AL-P MATH 2 #02) (5 marks) (Modified)

2. (a) (i) $\frac{x^2}{2}-\frac{1}{2}\ln(1+x^2)+\text{constant}$

(ii) $\tan^{-1}x+\text{constant}$

(b) $\frac{x^3}{3}\tan^{-1}x-\frac{x^2}{6}+\frac{1}{6}\ln(1+x^2)+\text{constant}$

(2001-AL-P MATH 2 #12) (Part / Modified) (6 marks)

12. (a) (i) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+\text{constant}$

(ii) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\text{constant}$

(2001-CE-A MATH #02) (4 marks)

2. $\frac{1}{3}(3x^2+1)^{\frac{1}{2}}+\text{constant}$

(2003-CE-A MATH #01) (3 marks)

1. $\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta+\text{constant}$

(2004-CE-A MATH #01) (4 marks)

1. (a) $\frac{1}{3}\sin(3x+1)+\text{constant}$

(b) $\frac{-(2-x)^{2005}}{2005}+\text{constant}$

(2004-CE-A MATH #03) (4 marks)

3. $y=x^3+x-2$

(2005-CE-A MATH #01) (2 marks)

1. $\frac{1}{16}(2x-3)^8+\text{constant}$

(2005-CE-A MATH #10) (6 marks)

10. (b) $y=x(x+1)^{2005}+1$

(2006-AL-P MATH 2 #04) (Part)

4. (a) $\frac{1}{3}(1+x^2)^{\frac{3}{2}}-(1+x^2)^{\frac{1}{2}}+\text{constant}$

(2006-CE-A MATH #10) (5 marks)

10. $y=3x+\sin 2x-1$

(2007-AL-P MATH 2 #04) (Part)

4. (a) $\frac{e^x}{2}(\sin x-\cos x)+\text{constant}$

(2007-CE-A MATH #01) (3 marks)

1. $\frac{x^3}{3} - \frac{1}{x} + \text{constant}$

(2008-CE-A MATH #01) (2 marks)

1. $\frac{(8x+5)^{251}}{2008} + \text{constant}$

(2009-CE-A MATH #01) (5 marks)

1. (a) $\frac{(4x+1)^3}{12} + \text{constant}$

   (b) $\frac{-\cos 4\theta}{6} - \frac{\cos 2\theta}{4} + \text{constant}$

(2011-CE-A MATH #05) (5 marks)

5. (a) $\frac{(2x+1)^3}{6} + \text{constant}$

   (b) $y = \frac{(2x+1)^3 + 1}{6}$

(SP-DSE-MATH-EP(M2) #03) (4 marks)

3. $y = (x^2+1)\ln(x^2+1) - x^2 + 1$

(SP-DSE-MATH-EP(M2) #04) (4 marks)

4. $\frac{x^9}{9} - \frac{2x^6}{3} + 2x^3 - 4\ln|x| - \frac{1}{3x^3} + \text{constant}$

(PP-DSE-MATH-EP(M2) #08) (5 marks)

8. (a) $\sin^{-1}\frac{x}{2} + \text{constant}$

   (b) $x\ln x - x + \text{constant}$

(2012-DSE-MATH-EP(M2) #04) (5 marks)

4. (a) $x + \ln|x| + \text{constant}$

   (b) $\frac{1}{2}(x^2-1) + \frac{1}{2}\ln\left|x^2-1\right| + \text{constant}$

(2013-DSE-MATH-EP(M2) #04) (5 marks)

4. (a) $y = e^x - x + 1$

   (b) $y = 2$

(2014-DSE-MATH-EP(M2) #05) (6 marks)

5. (a) $-2\sqrt{9-x} + \text{constant}$

   (b) $\sin^{-1}\frac{x}{3} + \text{constant}$

(2015-DSE-MATH-EP(M2) #04) (7 marks)

4. (a) $\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + \text{constant}$

   (b) $y = 3x^3\ln x - x^3 + 5$

(2017-DSE-MATH-EP(M2) #08) (8 marks)

8. (a) $6x - e^3y + e^3 = 0$

   (b) $y = (\ln x)^2 - 2$

   (c) $(e, -1)$

(2018-DSE-MATH-EP(M2) #05) (7 marks)

5. (a) $\frac{1}{5}(1+x^2)^{\frac{5}{2}} - \frac{1}{3}(1+x^2)^{\frac{3}{2}} + \text{constant}$

   (b) $y = 3(1+x^2)^{\frac{5}{2}} - 5(1+x^2)^{\frac{3}{2}} + 4$

(2019-DSE-MATH-EP(M2) #03) (6 marks)

3. (a) Correct

   (b) -0.9

(2019-DSE-MATH-EP(M2) #08) (8 marks)

8. (a) h(x) is an increasing function

   (b) (i) $y = x^2 - 7x + 8\ln|x| + 9$

   (ii) $(2, 8\ln 2 - 1)$

(2020-DSE-MATH-EP(M2) #07) (8 marks)

7. (a) $y = -x^2 + 8x - 5$

   (b) (i) $(7, 2)$

   (ii) $x - 6y + 5 = 0$