2. The Binomial Theorem

(1992-CE-A MATH 2 #02) (5 marks)

2. In the expansion of $(1+3x)^2(1+x)^n$, where n is a positive integer, the coefficient of x is 10.

   (a) Find the value of n.

   (b) Find the coefficient of x^2.

(1994-CE-A MATH 2 #03) (5 marks)

3. (a) Expand $(1-2x)^3$ and $\left(1+\frac{1}{x}\right)^5$.

   (b) Find, in the expansion of $(1-2x)^3\left(1+\frac{1}{x}\right)^5$,

   (1) the constant term, and
   (2) the coefficient of x.

(1995-CE-A MATH 2 #04) (6 marks)

4. Given $\left(x^2+\frac{1}{x}\right)^5-\left(x^2-\frac{1}{x}\right)^5=ax^7+bx+\frac{c}{x^5}$, find the values of a, b and c.

   Hence evaluate $\left(2+\frac{1}{\sqrt{2}}\right)^5-\left(2-\frac{1}{\sqrt{2}}\right)^5$.

(1997-CE-A MATH 2 #08) (7 marks)

8. Expand $(1+x)^n(1-2x)^4$ is ascending powers of x up to the term x^2, where n is a positive integer.
   If the coefficient of x^2 is 54, find the coefficient of x.

(1998-CE-A MATH 2 #01) (4 marks)

1. Find the coefficient of x^2 in the expansion of $\left(x-\frac{2}{x}\right)^6$.

(1999-CE-A MATH 2 #07) (6 marks)

7. (a) Expand $(1+2x)^n$ in ascending powers of x up to the term x^3, where n is a positive integer.

   (b) In the expansion of $\left(x-\frac{3}{x}\right)^2(1+2x)^n$, the constant term is 210. Find the value of n.

(2000-CE-A MATH 2 #02) (5 marks)

2. Expand $(1+2x)^7(2-x)^2$ in ascending powers of x up to the term x^2.

(2001-CE-A MATH #04) (4 marks)

4. Find the constant term in the expansion of $\left(2x^3+\frac{1}{x}\right)^8$.

(2002-CE-A MATH #01) (4 marks)

1. If n is a positive integer and the coefficient of x^2 in the expansion of $(1+x)^n+(1+2x)^n$ is 75, find the value(s) of n.

(2003-CE-A MATH #12) (6 marks)

12. Determine whether the expansion of $\left(2x^2+\frac{1}{x}\right)^9$ consists of

(a) a constant term,

(b) an x^2 term.

In each part, find the term if it exists.

(2004-CE-A MATH #02) (4 marks)

2. (a) Expand $(1+2x)^6$ in ascending powers of x up to the term x^3.

(b) Find the constant term in the expansion of $\left(1-\frac{1}{x}+\frac{1}{x^2}\right)(1+2x)^6$.

(2005-CE-A MATH #02) (4 marks)

2. (a) Expand $(1+y)^5$.

(b) Using (a), or otherwise, expand $(1+x+2x^2)^5$ in ascending powers of x up to the term x^2.

(2008-CE-A MATH #02) (4 marks)

2. (a) Expand $\left(2x+\frac{1}{x}\right)^3$.

(b) Find the coefficient of x in the expansion of $\left(3x^2-x-5\right)\left(2x+\frac{1}{x}\right)^3$.

(2009-CE-A MATH #11) (6 marks)

11. In the expansion of the binomial $\left(x^2+\frac{1}{x}\right)^{20}$, find

(a) the coefficient of x^{16},

(b) the constant term.

(2010-CE-A MATH #05) (5 marks)

5. The sum of the coefficients of x and x^2 in the expansion of $(1+4x)^n$ is 180, where n is a positive integer. Find the value of n and the coefficient of x^3.

(PP-DSE-MATH-EP(M2) #01) (4 marks)

1. Find the coefficient of x^5 in the expansion of $(2-x)^9$.

(2012-DSE-MATH-EP(M2) #02) (5 marks)

2. It is given that

$(1+ax)^n = 1+6x+16x^2+$ terms involving higher powers of x,

where n is a positive integer and a is a constant. Find the values of a and n.

(2013-DSE-MATH-EP(M2) #02) (4 marks)

2. Suppose the coefficients of x and x^2 in the expansion of $(1+ax)^n$ are -20 and 180 respectively. Find the values of a and n.

(2014-DSE-MATH-EP(M2) #01) (4 marks)

1. In the expansion of $(1-4x)^2(1+x)^n$, the coefficient of x is 1.

(a) Find the value of n.

(b) Find the coefficient of x^2.

(2016-DSE-MATH-EP(M2) #01) (5 marks)

1. Expand $(5+x)^4$. Hence, find the constant term in the expansion of $(5+x)^4\left(1-\frac{2}{x}\right)^3$.

(2017-DSE-MATH-EP(M2) #02) (5 marks)

2. Let $(1+ax)^8 = \sum_{k=0}^{8} \lambda_k x^k$ and $(b+x)^9 = \sum_{k=0}^{9} \mu_k x^k$, where a and b are constants. It is given that $\lambda_2 : \mu_7 = 7:4$ and $\lambda_1 + \mu_8 + 6 = 0$. Find a.

(2018-DSE-MATH-EP(M2) #02) (5 marks)

2. Expand $(x+3)^5$. Hence, find the coefficient of x^3 in the expansion of $(x+3)^5\left(x-\frac{4}{x}\right)^2$.

(2020-DSE-MATH-EP(M2) #01) (4 marks)

1. (a) Expand $(1-x)^4$.

(b) Find the constant k such that the coefficient of x^2 in the expansion of $(1+kx)^9(1-x)^4$ is -3.

(2021-DSE-MATH-EP(M2) #03) (6 marks)

1. The coefficient of x^2 in the expansion of $(1-4x)^n$ is 240, where n is a positive integer. Find

   (a) n,

   (b) the coefficient of x^4 in the expansion of $(1-4x)^n\left(1+\frac{2}{x}\right)^5$.

ANSWERS

(1992-CE-A MATH 2 #02)

2. (a) $n = 4$

(b) 39

(1994-CE-A MATH 2 #03)

3. (a) $(1-2x)^3 = 1 - 6x + 12x^2 - 8x^3$

$$\left(1+\frac{1}{x}\right)^5 = 1 + \frac{5}{x} + \frac{10}{x^2} + \frac{10}{x^3} + \frac{5}{x^4} + \frac{1}{x^5}$$

(b) (1) 11

(2) -26

(1995-CE-A MATH 2 #04)

4. $a = 10$, $b = 20$, $c = 2$

$$\left(2+\frac{1}{\sqrt{2}}\right)^5 - \left(2-\frac{1}{\sqrt{2}}\right)^5 = \frac{401\sqrt{2}}{4}$$

(1997-CE-A MATH 2 #08)

8. $(1+x)^n(1-2x)^4$

$$= 1 + (n-8)x + \left[\frac{n(n-1)}{2} - 8n + 24\right]x^2 + \ldots$$

Coefficient of $x = 12$

(1998-CE-A MATH 2 #01)

1. 60

(1999-CE-A MATH 2 #07)

7. (a) $(1+2x)^n$

$$= 1 + 2nx + 2n(n-1)x^2 + \frac{4}{3}n(n-1)(n-2)x^3 + \ldots$$

(b) $n = 4$

(2000-CE-A MATH 2 #02)

2. $4 + 52x + 281x^2 + \ldots$

(2001-CE-A MATH #04)

4. 112

(2002-CE-A MATH #01)

1. $n = 6$

(2003-CE-A MATH #12)

12. (a) 672

(b) There is no x^2 term

(2004-CE-A MATH #02)

2. (a) $(1+2x)^6$

$$= 1 + 12x + 60x^2 + 160x^3 + \ldots$$

(b) 49

(2005-CE-A MATH #02)

2. (a) $(1+y)^5$

$$= 1 + 5y + 10y^2 + 10y^3 + 5y^4 + y^5$$

(b) $(1 + x + 2x^2)^5$

$$= 1 + 5x + 20x^2 + \ldots$$

(2008-CE-A MATH #02)

2. (a) $\left(2x+\frac{1}{x}\right)^3 = 8x^3 + 12x + \frac{6}{x} + \frac{1}{x^3}$

(b) -42

(2009-CE-A MATH #11)

11. (a) 125 970

(b) There is no constant term

(2010-CE-A MATH #05)

5. $n = 5$, the coefficient of $x^3 = 640$

(PP-DSE-MATH-EP(M2) #01)

1. -2016

(2012-DSE-MATH-EP(M2) #02)

2. $n = 9$, $a = \frac{2}{3}$

(2013-DSE-MATH-EP(M2) #02)

2. $n = 10$, $a = -2$

(2014-DSE-MATH-EP(M2) #01)

1. (a) $n = 9$

(b) -20

(2016-DSE-MATH-EP(M2) #01)

1. $(5+x)^4 = 625 + 500x + 150x^2 + 20x^3 + x^4$

   Constant term $= -735$

(2017-DSE-MATH-EP(M2) #02)

2. $a = -3$ or $\frac{-3}{7}$

(2018-DSE-MATH-EP(M2) #02)

2. $(x+3)^5$

   $= x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$

   Coefficient of $x^3 = -299$

(2020-DSE-MATH-EP(M2) #01)

1. (a) $1 - 4x + 6x^2 - 4x^3 + x^4$

   (b) $\frac{1}{2}$

(2021-DSE-MATH-EP(M2) #03)

3. (a) 6

   (b) 106 240

OUT-OF-SYLLABUS

(1991-CE-A MATH 2 #01) (5 marks)

1. Given that $(1 + x + ax^2)^8 = 1 + 8x + k_1x^2 + k_2x^3 +$ terms involving higher powers of x .

   (a) Express k_1 and k_2 in terms of a .

   (b) If $k_1 = 4$, find the value of a .
   Hence find the value of k_2 .

(1993-CE-A MATH 2 #03) (6 marks)

3. Given $(1 + 4x + x^2)^n = 1 + ax + bx^2 +$ other terms involving higher powers of x , where n is a positive integer.

   (a) Express a and b in terms of n .

   (b) If $a = 20$, find n and b .

(1996-CE-A MATH 2 #02) (6 marks)

2. It is given that $(1 + x + ax^2)^6 = 1 + 6x + k_1x^2 + k_2x^3 +$ terms involving higher powers of x .

   (a) Express k_1 and k_2 in terms of a .

   (b) If 6 , k_1 and k_2 form an arithmetic sequence, find the value of a .

(2006-CE-A MATH #03) (5 marks)

3. It is given that
   $(1 - 2x + 3x^2)^n = 1 - 10x + kx^2 +$ terms involving higher powers of x ,
   where n is a positive integer and k is a constant. Find the values of n and k .

(2007-CE-A MATH #12) (6 marks)

12. If the coefficient of x^2 in the expansion of $\left(1 - 2x + x^2\right)^n$ is 66 , find the value of n and the coefficient of x^3 .

(2011-CE-A MATH #01) (5 marks)

1. It is given that $(1 + x + kx^2)^3 = 1 + ax + bx^2 +$ terms involving higher powers of x .

   (a) Express b in terms of k .

   (b) If 1 , a , b form a geometric sequence, find the value of k .

(1991-CE-A MATH 2 #01) (5 marks)

1. (a) $k_1 = 8a + 28$, $k_2 = 56a + 56$

   (b) $a = -3$, $k_2 = -112$

(1993-CE-A MATH 2 #03) (6 marks)

3. (a) $a = 4n$, $b = 8n^2 - 7n$

(b) $n = 5$, $b = 165$

(1996-CE-A MATH 2 #02) (6 marks)

2. (a) $k_1 = 6a + 15$, $k_2 = 30a + 20$

(b) $a = \frac{2}{9}$

(2006-CE-A MATH #03) (5 marks)

3. $n = 5$, $k = 55$

(2007-CE-A MATH #12) (6 marks)

12. $n = 6$, The coefficient of $x^3 = -220$

(2011-CE-A MATH #01) (5 marks)

1. (a) $b = 3(k + 1)$

(b) $k = 2$