4. Product of Vectors

(1991-CE-A MATH 1 #08) (16 marks)

8. A , B and C are three points on a plane such that

$\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j}$,

$\overrightarrow{BC} = 7\mathbf{i} + \mathbf{j}$,

and $\overrightarrow{OC} = x\mathbf{i} + y\mathbf{j}$,

where O is the origin.

(a) Find $\overrightarrow{CA}$, $\overrightarrow{OB}$ and $\overrightarrow{AB}$ in terms of x , y , $\mathbf{i}$ and $\mathbf{j}$.

(b) Given $\overrightarrow{AB} \cdot \overrightarrow{BC} = 4\overrightarrow{BC} \cdot \overrightarrow{CA}$.

(i) Show that $y = 30 - 7x$.

(ii) If $\left|\overrightarrow{BC}\right| = \sqrt{5}\left|\overrightarrow{CA}\right|$ and x , y are positive,

(1) find x and y ,

(2) show that CA is perpendicular to AB ,

(3) show that O lies on AB .

(1992-CE-A MATH 1 #08) (16 marks) (Modified - No figure given)

8. Given ΔOAB where $OA = 2$, $OB = 3$ and $\angle AOB = \dfrac{\pi}{3}$. D is a point on OB such that AD is perpendicular to OB . Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$.

(a) Find $\mathbf{a} \cdot \mathbf{a}$ and $\mathbf{a} \cdot \mathbf{b}$.

(b) Find the length of OD .

Hence express $\overrightarrow{OD}$ in terms of $\mathbf{b}$.

(c) Let H be a point on AD such that $AH : HD = 1 : k$ and $\overrightarrow{OH}$ is perpendicular to $\overrightarrow{AB}$.

(i) Express $\overrightarrow{OH}$ in terms of k , $\mathbf{a}$ and $\mathbf{b}$.

Hence find the value of k .

(ii) OH produced meets AB at a point C . Let $AC : CB = 1 : m$ and $OH : HC = 1 : n$.

(1) Express $\overrightarrow{OC}$ in terms of m , $\mathbf{a}$ and $\mathbf{b}$.

(2) Express $\overrightarrow{OC}$ in terms of n , $\mathbf{a}$ and $\mathbf{b}$.

(3) Hence find m and n .

(1993-CE-A MATH 1 #06) (7 marks)

6. Given $\overrightarrow{OA} = 3\mathbf{i} - 2\mathbf{j}$, $\overrightarrow{OB} = \mathbf{i} + \mathbf{j}$. C is a point such that $\angle ABC$ is a right angle.

(a) Find $\overrightarrow{AB}$.

(b) Find $\overrightarrow{AB} \cdot \overrightarrow{AB}$ and $\overrightarrow{AB} \cdot \overrightarrow{BC}$.

Hence find $\overrightarrow{AB} \cdot \overrightarrow{AC}$.

(1993-CE-A MATH 1 #08) (16 marks)

8.

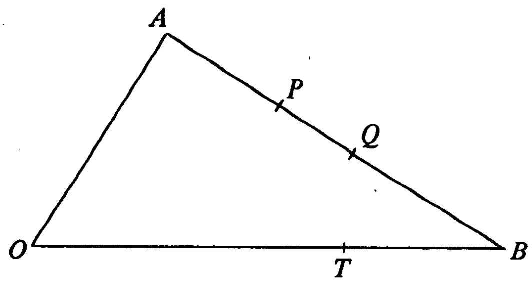

Figure 1

In Figure 1, OAB is a triangle. P , Q are two points on AB such that $AP : PB = PQ : QB = r : 1$, where $r > 0$. T is a point on OB such that $OT : TB = 1 : r$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in terms of r , $\mathbf{a}$ and $\mathbf{b}$.

(b) Express $\overrightarrow{OT}$ in terms of r and $\mathbf{b}$.

Hence show that $\overrightarrow{TQ} = \dfrac{a + (r^2 + r - 1)b}{(r+1)^2}$.

(c) Find the value(s) of r such that $\overrightarrow{OA}$ is parallel to $\overrightarrow{TQ}$.

(d) Suppose $OA = 2$, $OB = 16$ and $\angle AOB = \dfrac{\pi}{3}$.

(i) Find $\mathbf{a} \cdot \mathbf{a}$ and $\mathbf{a} \cdot \mathbf{b}$.

(ii) Find the value(s) of r such that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{TQ}$.

(1994-CE-A MATH 1 #03) (6 marks)

3. P , Q and R are points on a plane such that $\overrightarrow{OP} = \mathbf{i} + 2\mathbf{j}$, $\overrightarrow{OQ} = 3\mathbf{i} + \mathbf{j}$ and $\overrightarrow{PR} = -3\mathbf{i} - 2\mathbf{j}$, where O is the origin.

(a) Find $\overrightarrow{PQ}$ and $\left|\overrightarrow{PQ}\right|$.

(b) Find the value of $\cos\angle QPR$.

(1994-CE-A MATH 1 #10) (16 marks)

10.

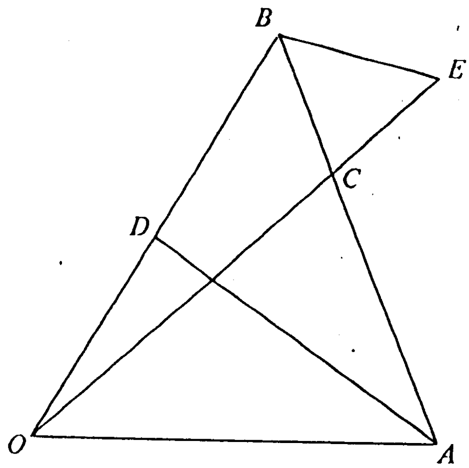

Figure 2

In Figure 2, D is the mid-point of OB and C is a point on AB such that $AC : CB = 2 : 1$. OC is produced to a point E such that $OC : CE = 1 : k$. Let $\overrightarrow{OQ} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OC}$ and $\overrightarrow{DA}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ **.**

(b) Show that $\overrightarrow{BE} = \dfrac{k+1}{3}\mathbf{a} + \dfrac{2k-1}{3}\mathbf{b}$.

(c) Find the value of k such that $\overrightarrow{BE}$ is parallel to $\overrightarrow{DA}$.

(d) Given $|\mathbf{a}| = 1$, $|\mathbf{b}| = 2$, $\angle BOA = \dfrac{\pi}{3}$.

(i) Find $\mathbf{a} \cdot \mathbf{b}$.

(ii) Find the value of k such that $\overrightarrow{BE}$ is perpendicular to $\overrightarrow{OE}$.
Hence find the distance of B from OC .

(1995-CE-A MATH 1 #07) (8 marks)

7. Let $\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OQ} = -6\mathbf{i} + 4\mathbf{j}$. Let R be a point on PQ such that $PR : RQ = k : 1$, where $k > 0$.

(a) Express $\overrightarrow{OR}$ in terms of k , $\mathbf{i}$ and $\mathbf{j}$.

(b) Express $\overrightarrow{OP} \cdot \overrightarrow{OR}$ and $\overrightarrow{OQ} \cdot \overrightarrow{OR}$ in terms of k .

(c) Find the value of k such that OR bisects $\angle POQ$.

(1995-CE-A MATH 1 #08) (16 marks)

8.

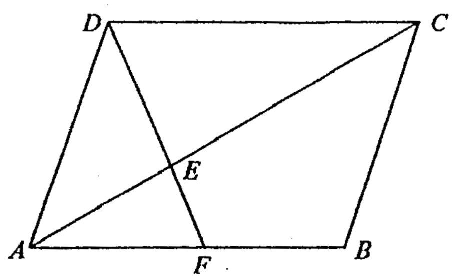

Figure 1

In Figure 1, $ABCD$ is a parallelogram and F is a point on AB . DF meets AC at a point E such that $DE : EF = \lambda : 1$, where λ is a positive number. Let $\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AD} = \mathbf{q}$ and $\overrightarrow{AE} = h\overrightarrow{AC}$, $\overrightarrow{AF} = k\overrightarrow{AB}$, where h , k are positive numbers.

(a) (i) Express $\overrightarrow{AE}$ in terms of h , $\mathbf{p}$ and $\mathbf{q}$.

(ii) Express $\overrightarrow{AE}$ in terms of λ , k , $\mathbf{p}$ and $\mathbf{q}$. Hence show that $\lambda = \dfrac{1}{k}$.

(b) It is given that $|\mathbf{p}| = 3$, $|\mathbf{q}| = 2$, $\angle DAB = \dfrac{\pi}{3}$.

(i) Find $\mathbf{p} \cdot \mathbf{q}$.

(ii) Suppose DF is perpendicular to AC .

(1) By expressing $\overrightarrow{DF}$ in terms of k , $\mathbf{p}$ and $\mathbf{q}$, find the value of k .

(2) Using (a), or otherwise, find the length of AE .

(1996-CE-A MATH 1 #07) (6 marks)

7. Given $\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j}$ and C is a point on OA such that $\left|\overrightarrow{OC}\right| = \frac{16}{5}$.

(a) Find the unit vector in the direction of $\overrightarrow{OA}$.

Hence find $\overrightarrow{OC}$.

(b) If $\overrightarrow{OB} = \mathbf{i} + 4\mathbf{j}$, show that BC is perpendicular to OA .

(1996-CE-A MATH 1 #10) (16 marks)

10.

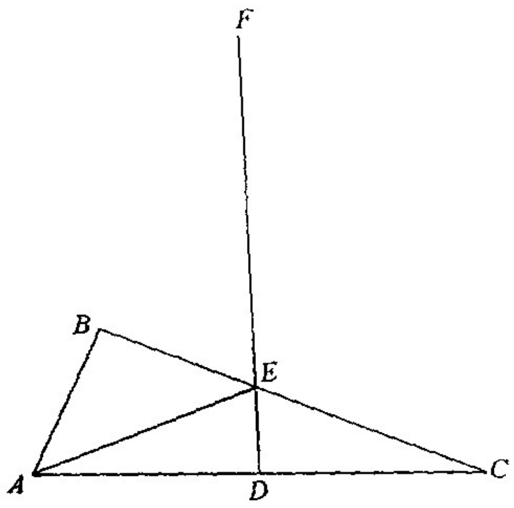

Figure 2

In Figure 2, D is the mid-point of AC and E is a point on BC such that $BE : EC = 1 : t$, where $t > 0$. DE is produced to a point F such that $DE : EF = 1 : 7$. Let $\overrightarrow{AD} = \mathbf{a}$ and $\overrightarrow{AB} = \mathbf{b}$.

(a) (i) Express $\overrightarrow{AE}$ in terms of t , $\mathbf{a}$ and $\mathbf{b}$.

(ii) Express $\overrightarrow{AE}$ in terms of $\mathbf{a}$ and $\overrightarrow{AF}$.

Hence, or otherwise, show that $\overrightarrow{AF} = \frac{9-7t}{1+t}\mathbf{a} + \frac{8t}{1+t}\mathbf{b}$.

(b) Suppose that A , B and F are collinear.

(i) Find the value of t .

(ii) It is given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$ and $\cos\angle BAC = \frac{1}{3}$.

(1) Find $\mathbf{a} \cdot \mathbf{b}$.

(2) Find $\overrightarrow{AB} \cdot \overrightarrow{BC}$ and $\overrightarrow{AD} \cdot \overrightarrow{DE}$.

(3) Does the circle passing through points B , C and D also pass through point F ?
Explain your answer.

(1997-CE-A MATH 1 #07) (7 marks)

7. Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors such that $\mathbf{a} = 2\mathbf{i} + 4\mathbf{j}$, $|\mathbf{b}| = \sqrt{5}$ and $\cos\theta = \frac{4}{5}$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

(a) Find $|\mathbf{a}|$.

(b) Find $\mathbf{a} \cdot \mathbf{b}$.

(c) If $\mathbf{b} = m\mathbf{i} + n\mathbf{j}$, find the values of m and n .

(1997-CE-A MATH 1 #09) (16 marks)

9.

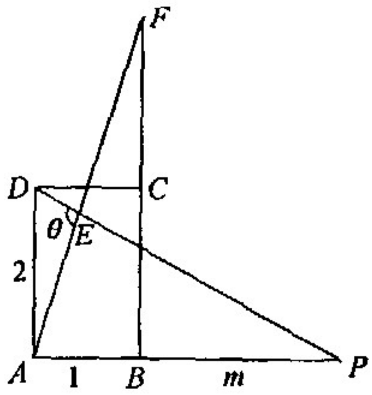

Figure 2

In Figure 2, $ABCD$ is a rectangle wilt $AB = 1$ and $AD = 2$. F is a point on BC produced with $BC = CF$. P is a variable point on AB produced such that $BP = m$. AF and DP intersect at a point E . Let $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AD} = \mathbf{b}$ and $\angle AED = \theta$.

(a) (i) Express $\overrightarrow{AF}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Express $\overrightarrow{DP}$ in terms of m , $\mathbf{a}$ and $\mathbf{b}$.

(b) Suppose $\theta = \frac{\pi}{2}$.

(i) Show that $m = 7$.

(ii) Let $AE : EF = 1 : r$ and $DE : EP = 1 : k$.

(1) Express $\overrightarrow{AE}$ in terms of r , $\mathbf{a}$ and $\mathbf{b}$.

(2) Express $\overrightarrow{AE}$ in terms of k , $\mathbf{a}$ and $\mathbf{b}$.
Hence find the values of r and k .

(c) As m tends to infinity, θ approaches a certain value θ_1 . Find θ_1 correct to the nearest degree.

(1998-CE-A MATH 1 #05) (6 marks)

5.

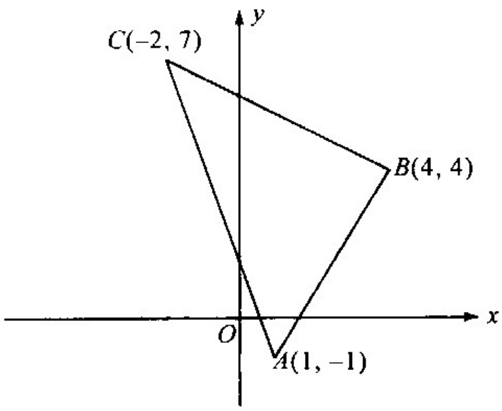

Figure 1

Figure 1 shows the points A , B and C whose position vectors are $\mathbf{i}-\mathbf{j}$, $4\mathbf{i}+4\mathbf{j}$ and $-2\mathbf{i}+7\mathbf{j}$ respectively.

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(b) By considering $\overrightarrow{AB}\cdot\overrightarrow{AC}$, find $\angle BAC$ to the nearest degree.

(1998-CE-A MATH 1 #09) (16 marks)

9.

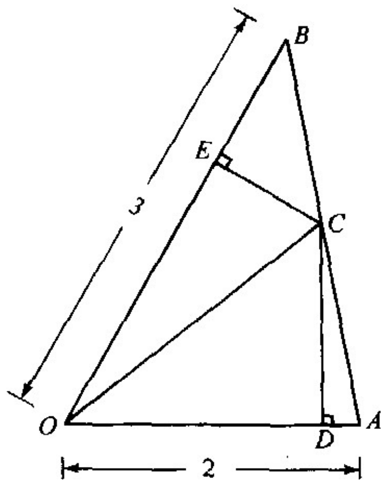

Figure 2

In Figure 2, OAB is a triangle with $OA = 2$, $OB = 3$ and $\angle AOB = \frac{\pi}{3}$. C is a point on AB such that $AC : CB = t : 1 - t$, where $0 < t < 1$. D and E are respectively the feet of perpendicular from C to OA and OB . Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) (i) Find $\mathbf{a} \cdot \mathbf{b}$,

(ii) Express $\overrightarrow{OC}$ in terms of t , $\mathbf{a}$ and $\mathbf{b}$.

(iii) Express $\mathbf{a} \cdot \overrightarrow{OC}$ and $\mathbf{b} \cdot \overrightarrow{OC}$ in terms of t .

(b) (i) Using (a) (iii), show that $\mathbf{a} \cdot \overrightarrow{OD} = 4 - t$ and $\mathbf{b} \cdot \overrightarrow{OE} = 3 + 6t$.

(ii) If $\overrightarrow{OD} = k\mathbf{a}$ and $\overrightarrow{OE} = s\mathbf{b}$, express k and s in terms of t .

(c) Find the value of t such that $\overrightarrow{DE}$ is parallel to $\overrightarrow{AB}$.

(1999-CE-A MATH 1 #07) (6 marks)

7. Let $\mathbf{a}$, $\mathbf{b}$ be two vectors such that $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$ and $|\mathbf{b}| = 4$. The angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{3}$.

(a) Find $|\mathbf{a}|$.

(b) Find $\mathbf{a} \cdot \mathbf{b}$.

(c) If the vector $(m\mathbf{a} + \mathbf{b})$ is perpendicular to $\mathbf{b}$, find the value of m .

(1999-CE-A MATH 1 #10) (16 marks)

10.

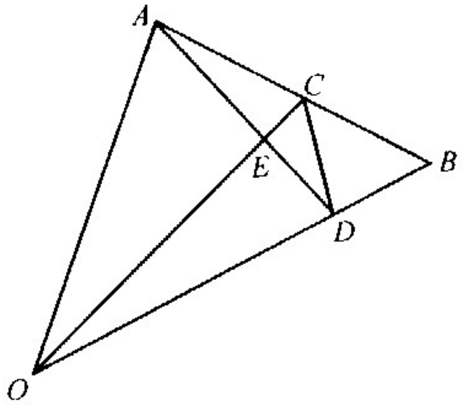

Figure 3

In Figure 3, OAB is a triangle. C and D are points on AB and OB respectively such that $AC : CB = 8 : 7$ and $OD : DB = 16 : 5$. OC and AD intersect at a point E . Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OC}$ and $\overrightarrow{AD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Let $\overrightarrow{OE} = r\overrightarrow{OC}$ and $\overrightarrow{AE} = k\overrightarrow{AD}$.

(i) Express $\overrightarrow{OE}$ in terms of r , $\mathbf{a}$ and $\mathbf{b}$.

(ii) Express $\overrightarrow{OE}$ in terms of k , $\mathbf{a}$ and $\mathbf{b}$.

Hence show that $r = \frac{6}{7}$ and $k = \frac{3}{5}$.

(c) It is given that $EC : ED = 1 : 2$.

(i) Using (b), or otherwise, find $EA : EO$.

(ii) Explain why $OACD$ is a cyclic quadrilateral.

(2000-CE-A MATH 1 #08) (7 marks)

8.

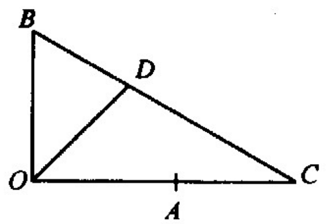

Figure 1

In Figure 1, $\overrightarrow{OA} = \mathbf{i}$, $\overrightarrow{OB} = \mathbf{j}$. C is a point on OA produced such that $AC = k$, where $k > 0$. D is a point on BC such that $BD : DC = 1 : 2$.

(a) Show that $\overrightarrow{OD} = \dfrac{1+k}{3}\mathbf{i} + \dfrac{2}{3}\mathbf{j}$.

(b) If $\overrightarrow{OD}$ is a unit vector, find

(i) k ,

(ii) $\angle BOD$, giving your answer correct to the nearest degree.

(2000-CE-A MATH 1 #09) (16 marks)

9.

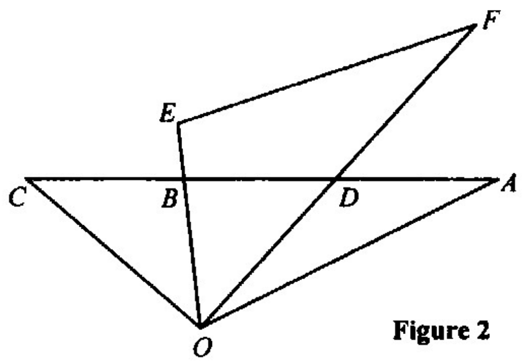

Figure 2

In Figure 2, OAC is a triangle. B and D are points on AC such that $AD = DB = BC$. F is a point on OD produced such that $OD = DF$. E is a point on OB produced such that $OE = k(OB)$, where $k > 1$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) (i) Express $\overrightarrow{OD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Show that $\overrightarrow{OC} = \dfrac{-1}{2}\mathbf{a} + \dfrac{3}{2}\mathbf{b}$.

(iii) Express $\overrightarrow{EF}$ in terms of k , $\mathbf{a}$ and $\mathbf{b}$.

(b) It is given that $OA = 3$, $OB = 2$ and $\angle AOB = \dfrac{\pi}{3}$.

(i) Find $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{b} \cdot \mathbf{b}$.

(ii) Suppose that $\angle OEF = \dfrac{\pi}{2}$.

(1) Find the value of k .

(2) A student states that points C , E and F are collinear. Explain whether the student is correct.

(2001-AL-P MATH 1 #04) (5 marks)

4. A , B , C are the points $(a,0,0)$, $(0,b,0)$, $(0,0,c)$ respectively and O is the origin.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

(b) Let $S_{\Delta XYZ}$ denote the area of the triangle with vertices X , Y and Z . Prove that

$$S_{\Delta ABC}{}^2 = S_{\Delta OAB}{}^2 + S_{\Delta OBC}{}^2 + S_{\Delta OCA}{}^2 .$$

(2001-CE-A MATH #08) (6 marks)

8. Let $\mathbf{a}$, $\mathbf{b}$ be two vectors such that $|\mathbf{a}| = 4$, $|\mathbf{b}| = 3$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{3}$.

(a) Find $\mathbf{a} \cdot \mathbf{b}$.

(b) Find the value of k if the vectors $(\mathbf{a} + k\mathbf{b})$ and $(\mathbf{a} - 2\mathbf{b})$ are perpendicular to each other.

(2001-CE-A MATH #14) (12 marks)

14. (a)

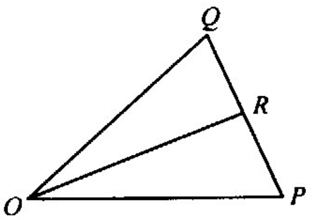

Figure 1(a)

In Figure 1 (a), OPQ is a triangle. R is a point on PQ such that $PR : RQ = r : s$.

Express $\overrightarrow{OR}$ in terms of r , s , $\overrightarrow{OP}$ and $\overrightarrow{OQ}$. Hence show that if $\overrightarrow{OR} = m\overrightarrow{OP} + n\overrightarrow{OQ}$, then $m + n = 1$.

(b)

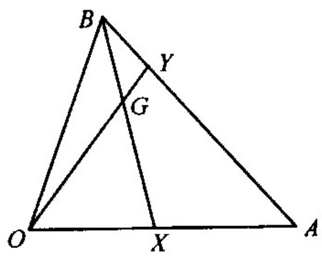

Figure 1(b)

In Figure 1 (b), OAB is a triangle. X is the mid-point of OA and Y is a point on AB . BX and OY intersect at point G where $BG : GX = 1 : 3$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(i) Express $\overrightarrow{OG}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Using (a), express $\overrightarrow{OY}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(Hint: Put $\overrightarrow{OY} = k\overrightarrow{OG}$.)

(iii) Moreover, AG is produced to a point Z on OB . Let $\overrightarrow{OZ} = h\overrightarrow{OB}$.

(1) Find the value of h .

(2) Explain whether ZY is parallel to OA or not.

(2002-AL-P MATH 1 #04) (6 marks)

4. Let $\mathbf{i} = (1,0,0)$, $\mathbf{j} = (0,1,0)$, $\mathbf{k} = (0,0,1)$ and $\mathbf{a} = \mathbf{i}$, $\mathbf{b} = 1 + \mathbf{j}$, $\mathbf{c} = \mathbf{j} + \mathbf{k}$.

(a) Prove that $\mathbf{a}$ is not perpendicular to $\mathbf{b} \times \mathbf{c}$.

(b) Find all unit vectors which are perpendicular to both $\mathbf{a}$ and $\mathbf{b} \times \mathbf{c}$.

(c) If $\theta \in [0,\pi]$ is the angle between $\mathbf{a}$ and $\mathbf{b} \times \mathbf{c}$, prove that $\frac{\pi}{4} < \theta < \frac{\pi}{3}$.

(2002-CE-A MATH #10) (6 marks)

10.

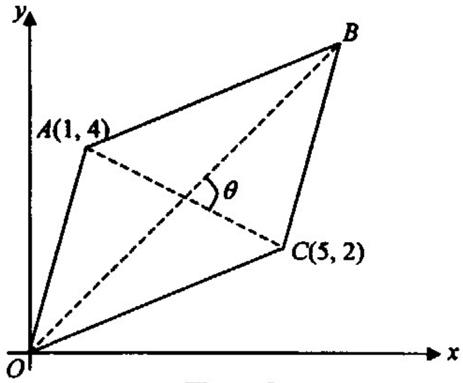

Figure 2

Figure 2 shows a parallelogram $OABC$. The position vectors of the points A and C are $\mathbf{i} + 4\mathbf{j}$ and $5\mathbf{i} + 2\mathbf{j}$ respectively.

(a) Find $\overrightarrow{OB}$ and $\overrightarrow{AC}$.

(b) Let θ be the acute angle between OB and AC . Find θ correct to the nearest degree.

(2002-CE-A MATH #13) (12 marks)

13.

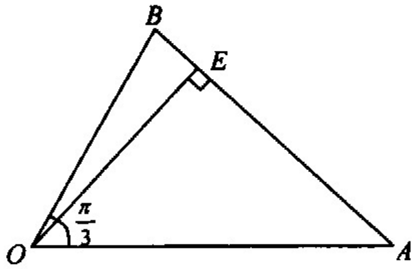

Figure 4

In Figure 4, OAB is a triangle. Point E is the foot of perpendicular from O to AB. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. It is given that $OA = 3$, $OB = 2$ and $\angle AOB = \dfrac{\pi}{3}$.

(a) Find $\mathbf{a} \cdot \mathbf{b}$.

(b) Find $\overrightarrow{OE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
(Hint : Let $BE : EA = t : (1 - t)$.)

(c) F is a variable point on OE. A student says that $\overrightarrow{BA} \cdot \overrightarrow{BF}$ is always a constant. Explain whether the student is correct or not.
If you agree with the student, find the value of that constant.
If you do not agree with the student, find two possible values of $\overrightarrow{BA} \cdot \overrightarrow{BF}$.

(2003-AL-P MATH 1 #05) (6 marks)

5. Let $\mathbf{m}$ and $\mathbf{n}$ be vectors in $\mathbf{R}^3$ and $\lambda \in \mathbf{R}$. It is given that

$$\begin{cases} \mathbf{u} = \lambda\mathbf{n} + (1-\lambda)\mathbf{m} \\ \mathbf{v} = 2(1-\lambda)\mathbf{n} - \lambda\mathbf{m} \end{cases}$$

(a) Prove that $\mathbf{u} \times \mathbf{v} = (3\lambda^2 - 4\lambda + 2)\mathbf{m} \times \mathbf{n}$.

(b) Suppose $|\mathbf{m}| = 4$, $|\mathbf{n}| = 3$ and the angle between $\mathbf{m}$ and $\mathbf{n}$ is $\dfrac{\pi}{6}$.

(i) Evaluate $|\mathbf{m} \times \mathbf{n}|$.

(ii) Find the smallest area of the parallelogram with adjacent sides $\mathbf{u}$ and $\mathbf{v}$ as λ varies.

(2003-CE-A MATH #06) (5 marks)

6.

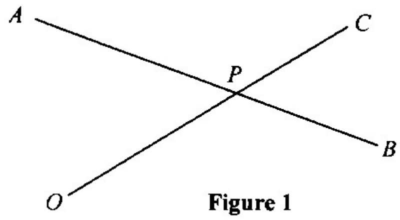

Figure 1

In Figure 1, point P divides both line segments AB and OC in the same ratio $3:1$. Let $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$.

(a) Express $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Express $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Hence show that OA is parallel to BC .

(2003-CE-A MATH #14) (12 marks)

14.

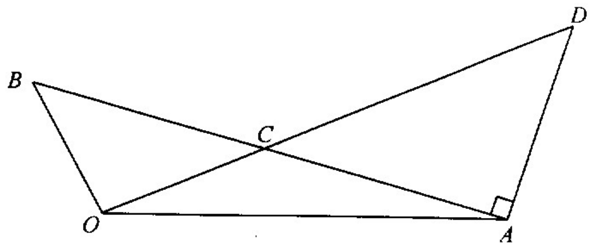

Figure 3

In Figure 3, OAB is a triangle such that $OA=3$, $OB=1$ and $\angle AOB=\frac{2\pi}{3}$. C is a point on AB such that $AC:CB=3:2$. D is a point on OC produced such that $\overrightarrow{OD}=k\overrightarrow{OC}$ and AB is perpendicular to AD . Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

(a) Find $\mathbf{a}\cdot\mathbf{b}$.

(b) Show that $\overrightarrow{AD}=\left(\frac{2k}{5}-1\right)\mathbf{a}+\frac{3k}{5}\mathbf{b}$.
Hence find the value of k .

(c) Determine whether the triangles OCB and ACD are similar.

(2004-CE-A MATH #06) (5 marks)

6.

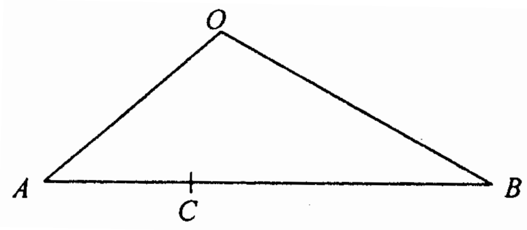

Figure 2

In Figure 2, OAB is a triangle. C is a point on AB such that $AC : CB = 1 : 2$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) If $|\mathbf{a}| = 1$, $|\mathbf{b}| = 2$ and $\angle AOB = \dfrac{2\pi}{3}$, find $\left|\overrightarrow{OC}\right|$.

(2004-CE-A MATH #13) (12 marks)

13.

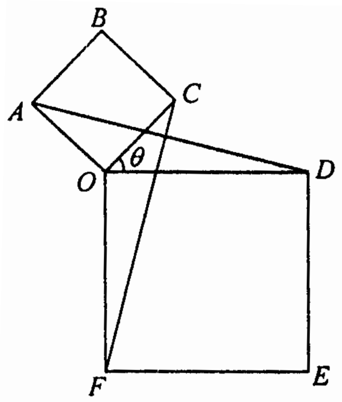

Figure 6

In Figure 6, $OABC$ and $ODEF$ are two squares such that $OA = 1$, $OF = 2$ and $\angle COD = \theta$, where $0 < \theta < \dfrac{\pi}{2}$.

Let $\overrightarrow{OD} = 2\mathbf{i}$ and $\overrightarrow{OF} = -2\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are two perpendicular unit vectors.

(a) (i) Express $\overrightarrow{OC}$ and $\overrightarrow{OA}$ in terms of θ , $\mathbf{i}$ and $\mathbf{j}$.

(ii) Show that $\overrightarrow{AD} = (2 + \sin\theta)\mathbf{i} - \cos\theta\,\mathbf{j}$.

(b) Show that $\overrightarrow{AD}$ is always perpendicular to $\overrightarrow{FC}$.

(c) Find the value(s) of θ such that points B , C and E are collinear. Give your answer(s) correct to the nearest degree.

(2005-AL-P MATH 1 #12) (15 marks)

12. (a) Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be vectors in $\mathbf{R}^3$.

(i) Prove that $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$,

where $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$.

Hence deduce that $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})$.

(ii) Suppose $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) \neq 0$. Prove that

$$\mathbf{x} = \left(\frac{\mathbf{x}\cdot(\mathbf{b}\times\mathbf{c})}{\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})}\right)\mathbf{a} + \left(\frac{\mathbf{x}\cdot(\mathbf{c}\times\mathbf{a})}{\mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})}\right)\mathbf{b} + \left(\frac{\mathbf{x}\cdot(\mathbf{a}\times\mathbf{b})}{\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})}\right)\mathbf{c}$$

for any vector $\mathbf{x}$ in $\mathbf{R}^3$.

(iii) Suppose $\mathbf{a}\cdot\mathbf{a} = \mathbf{b}\cdot\mathbf{b} = \mathbf{c}\cdot\mathbf{c} = 1$ and $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{c} = \mathbf{c}\cdot\mathbf{a} = 0$.

(1) Prove that $\left|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\right| = 1$.

(2) Using (a) (ii), prove that

$$\mathbf{x} = (\mathbf{x}\cdot\mathbf{a})\mathbf{a} + (\mathbf{x}\cdot\mathbf{b})\mathbf{b} + (\mathbf{x}\cdot\mathbf{c})\mathbf{c}$$

for any vector $\mathbf{x}$ in $\mathbf{R}^3$.

(b) Let $\mathbf{u} = \frac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})$, $\mathbf{v} = \frac{1}{\sqrt{2}}(\mathbf{i}-\mathbf{k})$, $\mathbf{w} = \frac{1}{\sqrt{6}}(\mathbf{i}-2\mathbf{j}+\mathbf{k})$ and $\mathbf{r} = 6\mathbf{i}-\mathbf{j}+10\mathbf{k}$.

Find real numbers α , β and γ such that $\mathbf{r} = \alpha\mathbf{u} + \beta\mathbf{v} + \gamma\mathbf{w}$.

(2005-CE-A MATH #11) (6 marks)

11.

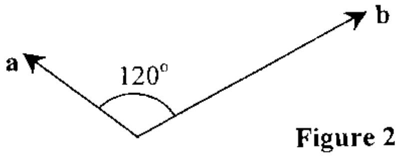

Figure 2

Figure 2 shows two vectors $\mathbf{a}$ and $\mathbf{b}$, where $|\mathbf{a}| = 3$, $|\mathbf{b}| = 5$, and the angle between the two vectors is $\frac{2\pi}{3}$.

(a) Find $\mathbf{a}\cdot\mathbf{b}$.

(b) Let $\mathbf{c}$ be a vector such that $\mathbf{a}+\mathbf{b}+\mathbf{c} = 0$. Find $|\mathbf{c}|$.

(2005-CE-A MATH #14) (12 marks)

14.

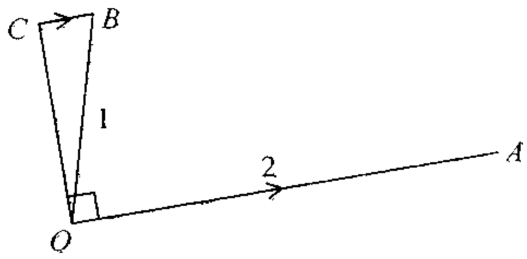

Figure 4

In Figure 4, $OA = 2$, $OB = 1$ and $\cos\angle AOB = \frac{1}{4}$. C is a point such that $CB \,//\, OA$ and $OC \perp OA$. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$.

(a) Find CB in terms of $\mathbf{a}$.

Hence, or otherwise, show that $\mathbf{c} = \mathbf{b} - \frac{1}{8}\mathbf{a}$.

(b)

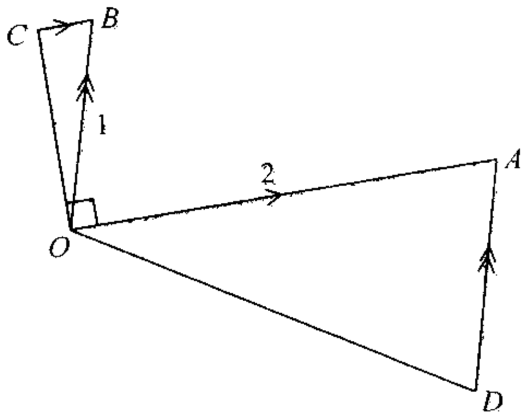

Figure 5

D is a point such that $DA \,//\, OB$ and $OD = OA$ (see Figure 5). Let $\overrightarrow{OD} = \mathbf{d}$.

(i) By finding DA , or otherwise, express $\mathbf{d}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) P is a point on the line segment CD such that $CP : PD = r : 1$. Express $\overrightarrow{OP}$ in terms of r , $\mathbf{a}$ and $\mathbf{b}$.

(iii) If M is the mid-point of AB , find the ratio in which OM divides CD .

(2006-CE-A MATH #07) (5 marks)

7. Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors such that $|\mathbf{a}| = \sqrt{3}$, $|\mathbf{b}| = 2$ and the angle between them is $\frac{5\pi}{6}$.

(a) Find $\mathbf{a} \cdot \mathbf{b}$.

(b) Find $|\mathbf{a} + 2\mathbf{b}|$.

(2006-CE-A MATH #18) (12 marks)

18. Figure 9 shows a triangle OAB . Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OA} = \mathbf{b}$ and M be the mid-point of OA .

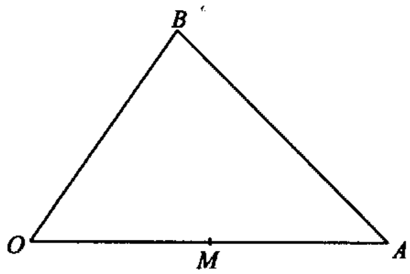

Figure 9

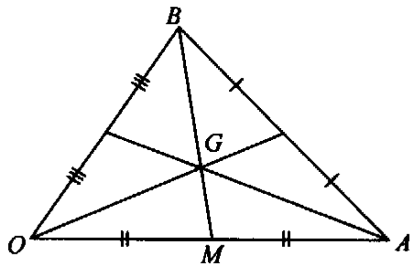

Figure 10

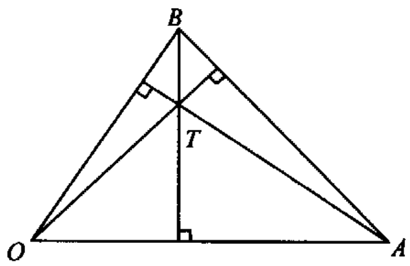

Figure 11

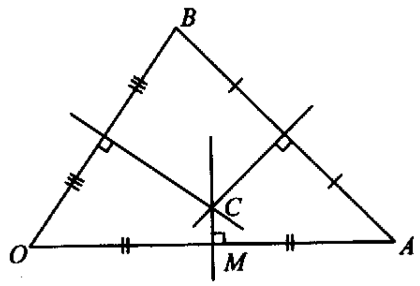

Figure 12

(a) Let G be the centroid of ΔOAB (see Figure 10). It is given that $BG : GM = 2 : 1$. Express $\overrightarrow{OG}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Let T be the orthocentre of ΔOAB (see Figure 11). Show that $\overrightarrow{OT} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{a} = 0$ and write down the value of $\overrightarrow{OT} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{b}$.

(c) Let C be the circumcentre of ΔOAB (see Figure 12). Show that $2\overrightarrow{OC} \cdot \mathbf{a} = |\mathbf{a}|^2$ and find $\overrightarrow{OC} \cdot \mathbf{b}$ in terms of $|\mathbf{b}|$.

(d) Consider the points G , T and C described in (a), (b) and (c) respectively.

(i) Using the above results, find the values of $(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{a}$ and $(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{b}$.

(ii) Show that G , T and C are collinear.

Note: You may use the following property for vectors in the two-dimensional space:
If $\mathbf{w} \cdot \mathbf{u} = \mathbf{w} \cdot \mathbf{v} = 0$, where $\mathbf{u}$ and $\mathbf{v}$ are non-parallel, then $\mathbf{w} = 0$.

(2007-CE-A MATH #08) (5 marks)

8.

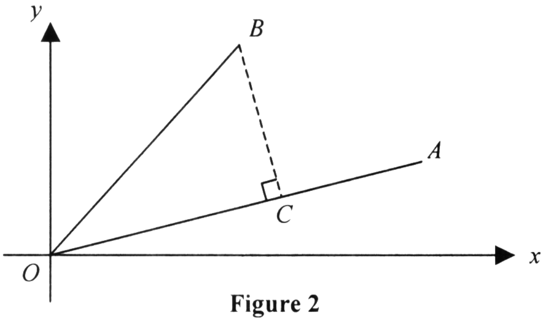

Figure 2

In Figure 2, OCA is a straight line and $BC \perp OA$. It is given that $\overrightarrow{OA} = 6\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} + 6\mathbf{j}$. Let $\overrightarrow{OC} = k\overrightarrow{OA}$.

(a) Express $\overrightarrow{BC}$ in terms of k , $\mathbf{i}$ and $\mathbf{j}$.

(b) Find the value of k .

(2007-CE-A MATH #17) (12 marks)

17.

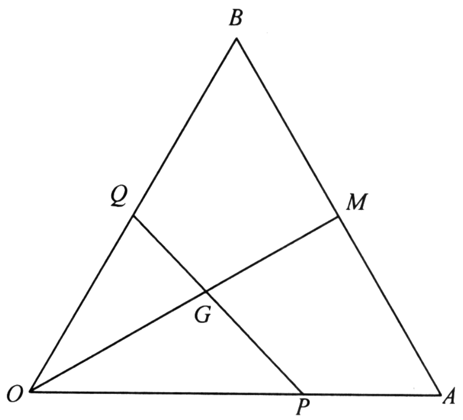

Figure 10

In Figure 10, OAB is an equilateral triangle with $OA = 1$. M is the mid-point of AB and P divides the line segment OA in the ratio $2 : 1$. Q is a point on OB such that PQ intersects OM at G and $PG : GQ = 4 : 3$. Let OA and OB be $\mathbf{a}$ and $\mathbf{b}$ respectively.

(a) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Let $OQ : QB = k : (1-k)$.

(i) Find $\overrightarrow{OG}$ in terms of k , $\mathbf{a}$ and $\mathbf{b}$.

(ii) Show that $\overrightarrow{PQ} = \dfrac{1}{2}\mathbf{b} - \dfrac{2}{3}\mathbf{a}$.

(c) (i) Find $\mathbf{a} \cdot \mathbf{b}$ and hence find $\left|\overrightarrow{PQ}\right|$.

(ii) Find $\angle QGM$ correct to the nearest degree.

(2008-CE-A MATH #07) (5 marks)

7. It is given that $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OB} = 5\mathbf{i} + 6\mathbf{j}$. If P is a point on AB such that $\overrightarrow{PB} = 2\overrightarrow{AP}$, find the unit vector in the direction of $\overrightarrow{OP}$.

(2008-CE-A MATH #15) (12 marks)

15.

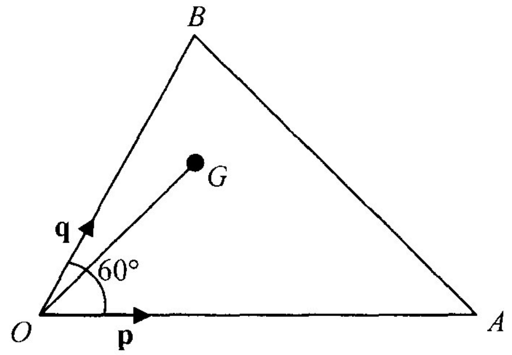

Figure 3

In Figure 3, $\mathbf{p}$ and $\mathbf{q}$ are unit vectors with angle between them $60°$. Let $\overrightarrow{OA} = 4\mathbf{p}$, $\overrightarrow{OB} = 3\mathbf{q}$ and $\overrightarrow{OG} = \frac{2}{3}\mathbf{p} + \frac{5}{3}\mathbf{q}$.

(a) Find $\mathbf{p} \cdot \mathbf{q}$.

(b) Show that $OG \perp AB$.
Hence show that G is the orthocentre of ΔOAB .

(c)

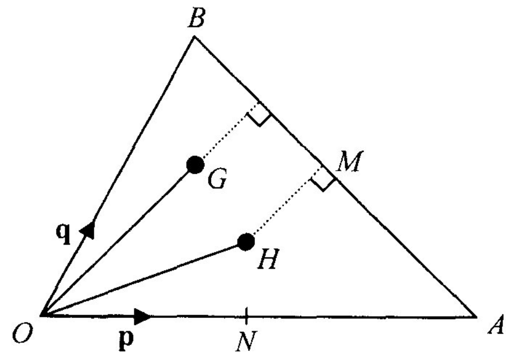

Figure 4

In Figure 4, H is the circumcentre of ΔOAB , M and N are the mid-points of AB and OA respectively. Let $HM : OG = t : 1$.

By expressing $\overrightarrow{HM}$ and $\overrightarrow{HN}$ in terms of t , $\mathbf{p}$ and $\mathbf{q}$, find $\overrightarrow{OH}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

(2009-CE-A MATH #07) (4 marks)

7.

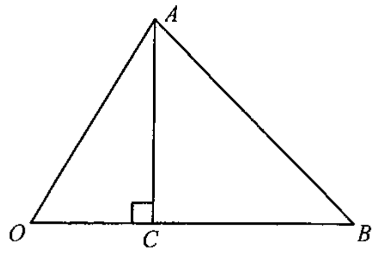

Figure 1

In Figure 1, AC is an altitude of ΔOAB . Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively. It is given that $|\mathbf{a}| = 6$, $|\mathbf{b}| = 8$ and $\mathbf{a} \cdot \mathbf{b} = 24$. Find

(a) $\angle AOB$,

(b) $|\mathbf{c}|$.

(2009-CE-A MATH #14) (12 marks)

14.

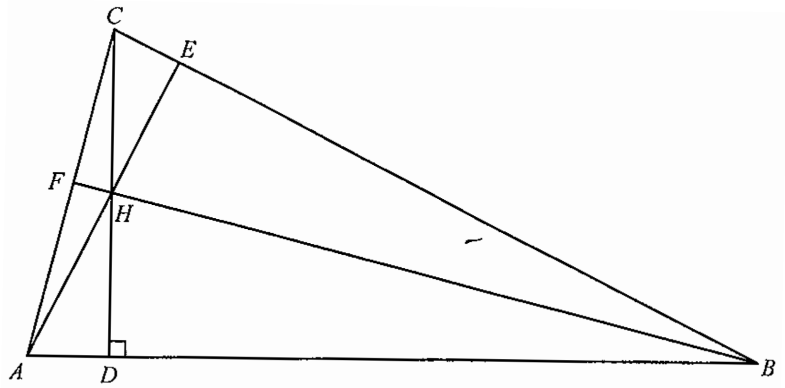

Figure 4

In Figure 4, CD is an altitude of ΔABC and H is the mid-point of CD . AH and BH are produced to meet BC and AC at E and F respectively.

Let $\mathbf{p}$, $\lambda\mathbf{p}$ ($\lambda > 1$) and $\mathbf{q}$ be $\overrightarrow{AD}$, $\overrightarrow{AB}$ and $\overrightarrow{DH}$ respectively. Let $\dfrac{BE}{EC} = r$.

(a) Find $\overrightarrow{AH}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

(b) Express $\overrightarrow{AE}$ in terms of λ , r , $\mathbf{p}$ and $\mathbf{q}$. Hence show that $r = \lambda$.

(c) It is given that $|\mathbf{p}| = 1$, $|\mathbf{q}| = 2$ and H is the orthocentre of ΔABC .

(i) Find $\overrightarrow{AE}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

(ii) Find $\dfrac{AF}{FC}$.

(2010-CE-A MATH #12) (7 marks)

12. It is given that $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j}$, $\left|\overrightarrow{OB}\right| = 5\sqrt{2}$ and $\cos\angle AOB = \dfrac{3}{\sqrt{10}}$.

(a) Evaluate $\overrightarrow{OA} \cdot \overrightarrow{OB}$.

(b) Find $\overrightarrow{OB}$.

(2010-CE-A MATH #14) (12 marks)

14.

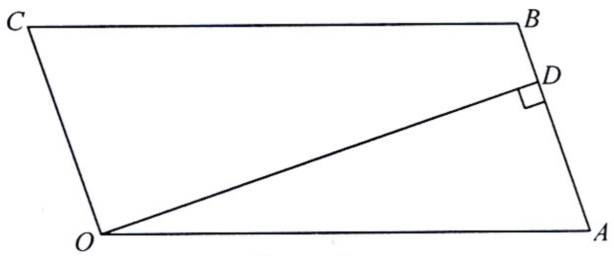

Figure 5

In Figure 5, $OABC$ is a parallelogram with $OA = 7$, $OC = 3$ and $\angle AOC = \theta$ where $\cos\theta = -\dfrac{1}{3}$. D is a point on AB such that $OD \perp AB$ and $AD : DB = 1 : r$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

(a) By expressing $\overrightarrow{OD}$ in terms of $\mathbf{a}$, $\mathbf{c}$ and r , find the value of r .

(b) E is a point on OD produced such that C , B and E are collinear.

(i) Express $\overrightarrow{OE}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

(ii) Are A , O , C and E concyclic? Explain your answer.

(2011-CE-A MATH #09) (6 marks)

9. It is given that $\overrightarrow{OA} = \mathbf{i} + \mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} + \mathbf{j}$.

(a) Find the value of $\cos\angle AOB$.

(b) Let $\overrightarrow{OC} = k\mathbf{i} + \mathbf{j}$. If OB is the angle bisector of ΔAOC , find the value of k .

(2011-CE-A MATH #12) (12 marks)

12.

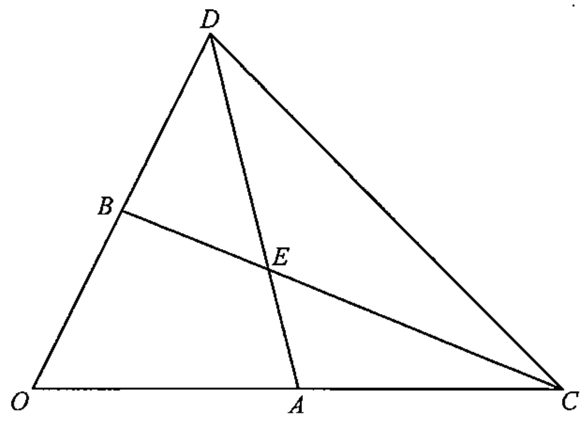

Figure 2

Figure 2 shows a triangle OCD . A and B are points on OC and OD respectively such that $OA : AC = OB : BD = 1 : h$, where $h > 0$. AD and BC intersect at E such that $AE : ED = \mu : (1 - \mu)$ and $BE : EC = \lambda : (1 - \lambda)$, where $0 < \mu < 1$ and $0 < \lambda < 1$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) By considering $\overrightarrow{OE}$, show that $\mu = \lambda$.

(b) F is a point on CD such that O , E and F are collinear. Show that OF is a median of ΔOCD .

(c) Using the above results, show that in a triangle, the centroid divides every median in $2 : 1$.

(SP-DSE-MATH-EP(M2) #09) (6 marks)

9.

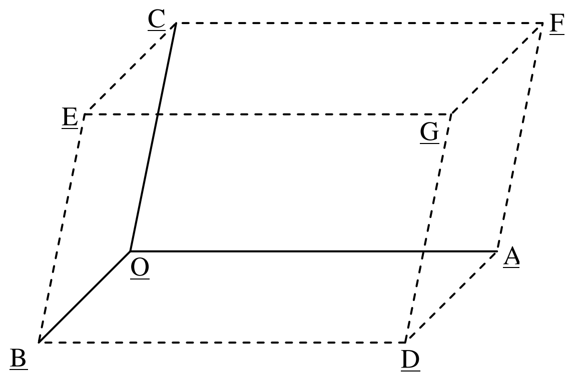

Figure 2

Let $\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j}$, $\overrightarrow{OB} = 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 3\mathbf{i} + \mathbf{j} + 5\mathbf{k}$. Figure 2 shows the parallelepiped $OADBECFG$ formed by $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$.

(a) Find the area of the parallelogram $OADB$.

(b) Find the volume of the parallelepiped $OADBECFG$.

(c) If C' is a point different from C such that the volume of the parallelepiped formed by $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC'}$ is the same as that of $OADBECFG$, find a possible vector of $\overrightarrow{OC'}$.

(SP-DSE-MATH-EP(M2) #14) (10 marks)

14.

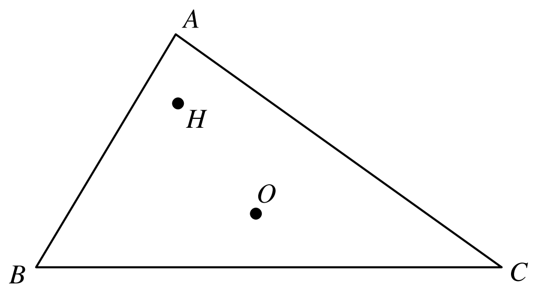

Figure 3

In Figure 3, ΔABC is an acute-angled triangle, where O and H are the circumcentre and orthocentre respectively. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OH} = \mathbf{h}$.

(a) Show that $(\mathbf{h} - \mathbf{a}) \mathbin{//} (\mathbf{b} + \mathbf{c})$.

(b) Let $\mathbf{h} - \mathbf{a} = t(\mathbf{b} + \mathbf{c})$, where t is a non-zero constant.
Show that
(i) $t(\mathbf{b} + \mathbf{c}) + \mathbf{a} - \mathbf{b} = s(\mathbf{c} + \mathbf{a})$ for some scalars,
(ii) $(t - 1)(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{c} - \mathbf{a}) = 0$.

(c) Express $\mathbf{h}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

(PP-DSE-MATH-EP(M2) #12) (13 marks)

12.

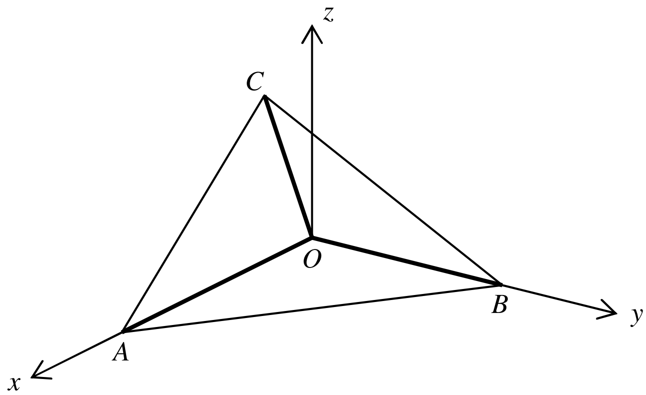

Figure 2

Let $\overrightarrow{OA} = \mathbf{i}$, $\overrightarrow{OB} = \mathbf{j}$ and $\overrightarrow{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ (see Figure 2). Let M and N be points on the straight lines AB and OC respectively such that $AM : MB = a : (1-a)$ and $ON : NC = b : (1-b)$, where $0 < a < 1$ and $0 < b < 1$. Suppose that MN is perpendicular to both AB and OC .

(a) (i) Show that $\overrightarrow{MN} = (a+b-1)\mathbf{i} + (b-a)\mathbf{j} + b\mathbf{k}$.

(ii) Find the values of a and b .

(iii) Find the shortest distance between the straight lines AB and OC .

(b) (i) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

(ii) Let G be the projection of O on the plane ABC , find the coordinates of the intersecting point of the two straight lines OG and MN .

(2012-DSE-MATH-EP(M2) #07) (5 marks)

7.

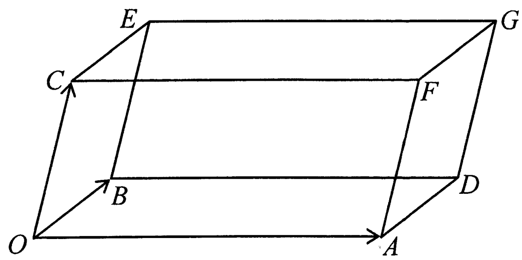

Figure 3

Figure 3 shows a parallelepiped $OADBECFG$. Let $\overrightarrow{OA} = 6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\overrightarrow{OB} = 2\mathbf{i} + \mathbf{j}$ and $\overrightarrow{OC} = 5\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

(a) Find the area of the parallelogram $OADB$.

(b) Find the distance between point C and the plane $OADB$.

(2012-DSE-MATH-EP(M2) #12) (12 marks)

12.

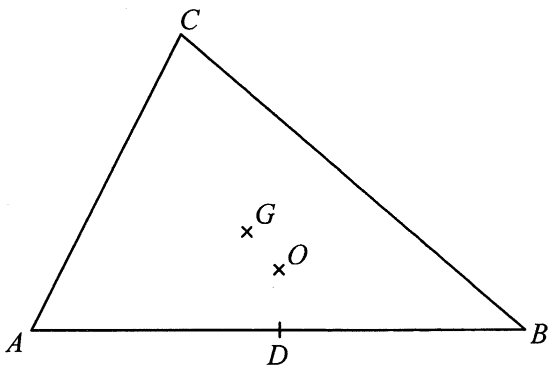

Figure 6

Figure 6 shows an acute angled scalene triangle ABC , where D is the mid-point of AB , G is the centroid and O is the circumcentre. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$.

(a) Express $\overrightarrow{AG}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

(b) It is given that E is a point on AB such that CE is an altitude. Extend OG to meet CE at F .

(i) Prove that $\Delta DOG \sim \Delta CFG$.
Hence find $FG : GO$.

(ii) Show that $\overrightarrow{AF} = \mathbf{b} + \mathbf{c}$.
Hence prove that F is the orthocentre of ΔABC .

(2013-DSE-MATH-EP(M2) #10) (5 marks)

10.

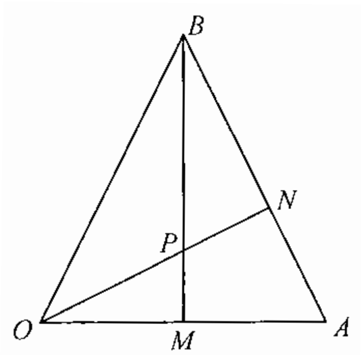

Figure 2

Let $\overrightarrow{OA} = 2\mathbf{i}$ and $\overrightarrow{OB} = \mathbf{i} + 2\mathbf{j}$. M is the mid-point of OA and N lies on AB such that $BN : NA = k : 1$. BM intersects ON at P .

(a) Express $\overrightarrow{ON}$ in terms of k .

(b) If A , N , P and M are concyclic, find the value of k .

(2013-DSE-MATH-EP(M2) #14) (12 marks)

14.

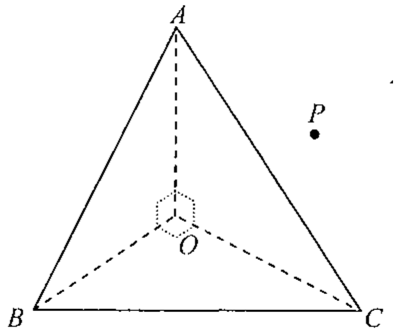

Figure 5

Figure 5 shows a fixed tetrahedron $OABC$ with $\angle AOB = \angle BOC = \angle COA = \frac{\pi}{2}$. P is a variable point such that $\overrightarrow{AP} \cdot \overrightarrow{BP} + \overrightarrow{BP} \cdot \overrightarrow{CP} + \overrightarrow{CP} \cdot \overrightarrow{AP} = 0$. Let D be a fixed point such that $\overrightarrow{OD} = \frac{\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}}{3}$.

Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = \mathbf{c}$, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OD} = \mathbf{d}$.

(a) (i) Show that $\overrightarrow{AP} \cdot \overrightarrow{BP} = \mathbf{p} \cdot \mathbf{p} - (\mathbf{a} + \mathbf{b}) \cdot \mathbf{p}$.

(ii) Using (a)(i), show that $\mathbf{p} \cdot \mathbf{p} = 2\mathbf{p} \cdot \mathbf{d}$.

(iii) Show that $\left|\mathbf{p} - \mathbf{d}\right| = \left|\mathbf{d}\right|$.

hence show that P lies on the sphere centred at D with fixed radius.

(b) (i) Alice claims that O lies on the sphere mentioned in (a)(iii). Do you agree? Explain your answer.

(ii) Suppose P_1 , P_2 and P_3 are three distinct points on the sphere in (a)(iii) such that $\overrightarrow{DP_1} \times \overrightarrow{DP_2} = \overrightarrow{DP_2} \times \overrightarrow{DP_3}$. Alice claims that the radius of the circle passing through P_1 , P_2 and P_3 is OD . Do you agree? Explain your answer.

(2014-DSE-MATH-EP(M2) #08) (8 marks)

8. Let $\overrightarrow{OP} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OQ} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OR} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$.

(a) Find $\overrightarrow{OP} \times \overrightarrow{OQ}$.

Hence find the volume of tetrahedron *OPQR*.

(b) Find the acute angle between the plane *OPQ* and the line *OR*, correct to the nearest 0.1°.

(2014-DSE-MATH-EP(M2) #11) (13 marks)

11.

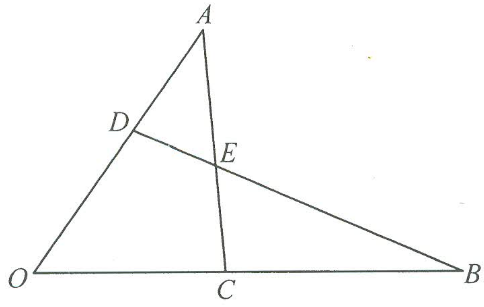

Figure 4

In Figure 4, C and D are points on OB and OA respectively such that $AD : DO = OC : CB = t : (1-t)$, where $0 < t < 1$, BD and AC intersect at E such that $AE : EC = m : 1$ and $BE : ED = n : 1$, where m and n are positive. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) (i) By considering ΔOAC , express $\overrightarrow{OE}$ in terms of m , t , $\mathbf{a}$ and $\mathbf{b}$.

(ii) By considering ΔOBD , express $\overrightarrow{OE}$ in terms of n , t , $\mathbf{a}$ and $\mathbf{b}$.

(iii) Show that $m = \dfrac{t}{(1-t)^2}$ and $n = \dfrac{1-t}{t^2}$.

(iv) Chris claims that

"if $m = n$, then E is the centroid of ΔOAB ".

Do you agree? Explain your answer.

(b) It is given that $OA = 1$ and $OB = 2$. Francis claims that

"if AC is perpendicular to OB , then BD is always perpendicular to OA ".

Do you agree? Explain your answer.

(2015-DSE-MATH-EP(M2) #10) (12 marks)

10. OAB is a triangle. P is the mid-point of OA . Q is a point lying on AB such that $AQ : QB = 1 : 2$ while R is a point lying on OB such that $OR : RB = 3 : 1$. PR and OQ intersect at C .

(a) (i) Let t be a constant such that $PC : CR = t : (1-t)$.

By expressing $\overrightarrow{OQ}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$, find the value of t .

(ii) Find $CQ : OQ$.

(b) Suppose that $\overrightarrow{OA} = 20\mathbf{i} - 6\mathbf{j} - 12\mathbf{k}$, $\overrightarrow{OB} = 16\mathbf{i} - 16\mathbf{j}$ and $\overrightarrow{OD} = \mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$, where O is the origin.

Find

(i) the area of ΔOAB ,

(ii) the volume of the tetrahedron $ABCD$.

(2016-DSE-MATH-EP(M2) #12) (13 marks)

12. Let $\overrightarrow{OA} = 2\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\overrightarrow{OP} = \mathbf{i} + t\mathbf{j}$, where t is a constant and O is the origin. It is given that P is equidistant from A and B .

(a) Find t .

(b) Let $\overrightarrow{OC} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{OD} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$. Denote the plane which contains A , B and C by Π .

(i) Find a unit vector which is perpendicular to Π .

(ii) Find the angle between CD and Π .

(iii) It is given that E is a point lying on Π such that $\overrightarrow{DE}$ is perpendicular to Π . Let F be a point such that $\overrightarrow{PF} = \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$. Describe the geometric relationship between D , E and F . Explain your answer.

(2017-DSE-MATH-EP(M2) #03) (5 marks)

3. P is a point lying on AB such that $AP : PB = 3 : 2$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, where O is the origin.

(a) Express $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) It is given that $|\mathbf{a}| = 45$, $|\mathbf{b}| = 20$ and $\cos\angle AOB = \frac{1}{4}$. Find

(i) $\mathbf{a} \cdot \mathbf{b}$,

(ii) $\left|\overrightarrow{OP}\right|$.

(2017-DSE-MATH-EP(M2) #10) (12 marks)

10. ABC is a triangle. D is the mid-point of AC . E is a point lying on BC such that $BE : EC = 1 : r$. AB produced and DE produced meet at the point F . It is given that $DE : EF = 1 : 10$. Let $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $\overrightarrow{OC} = 8\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$, where O is a the origin.

(a) By expressing $\overrightarrow{AE}$ and $\overrightarrow{AF}$ in terms of r , find r .

(b) (i) Find $\overrightarrow{AD} \cdot \overrightarrow{DE}$.

(ii) Are B , D , C and F concyclic? Explain your answer.

(c) Let $\overrightarrow{OP} = 3\mathbf{i} + 10\mathbf{j} - 4\mathbf{k}$. Denote the circumcenter of ΔBCF by Q . Find the volume of the tetrahedron $ABPQ$.

(2018-DSE-MATH-EP(M2) #12) (13 marks)

12. The position vectors of the points A, B, C and D are $4\mathbf{i}-3\mathbf{j}+\mathbf{k}$, $-\mathbf{i}+3\mathbf{j}-3\mathbf{k}$, $7\mathbf{i}-\mathbf{j}+5\mathbf{k}$ and $3\mathbf{i}-2\mathbf{j}-5\mathbf{k}$ respectively. Denote the plane which contains A, B and C by Π. Let E be the projection of D on Π.

(a) Find

(i) $\overrightarrow{AB}\times\overrightarrow{AC}$,

(ii) the volume of the tetrahedron $ABCD$.

(iii) $\overrightarrow{DE}$.

(b) Let F be a point lying on BC such that DF is perpendicular to BC.

(i) Find $\overrightarrow{DF}$.

(ii) Is $\overrightarrow{BC}$ perpendicular to $\overrightarrow{EF}$? Explain your answer.

(c) Find the angle between ΔBCD and Π.

(2019-DSE-MATH-EP(M2) #12) (13 marks)

12. Let $\overrightarrow{OA}=\mathbf{i}-4\mathbf{j}+2\mathbf{k}$, $\overrightarrow{OB}=-5\mathbf{i}-4\mathbf{j}+8\mathbf{k}$ and $\overrightarrow{OC}=-5\mathbf{i}-12\mathbf{j}+t\mathbf{k}$, where O is the origin and t is a constant. It is given that $\left|\overrightarrow{AC}\right|=\left|\overrightarrow{BC}\right|$.

(a) Find t.

(b) Find $\overrightarrow{AB}\times\overrightarrow{AC}$.

(c) Find the volume of the pyramid $OABC$.

(d) Denote the plane which contains A, B and C by Π. It is given that P, Q and R are points lying on Π such that $\overrightarrow{OP}=p\mathbf{i}$, $\overrightarrow{OQ}=q\mathbf{j}$ and $\overrightarrow{OR}=r\mathbf{k}$. Let D be the projection of O on Π.

(i) Prove that $pqr\neq 0$.

(ii) Find $\overrightarrow{OD}$.

(iii) Let E be a point such that $\overrightarrow{OE}=\frac{1}{p}\mathbf{i}+\frac{1}{q}\mathbf{j}+\frac{1}{r}\mathbf{k}$. Describe the geometric relationship between D, E and O. Explain your answer.

(2020-DSE-MATH-EP(M2) #12) (12 marks)

12. Let $\overrightarrow{OP} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{OQ} = 5\mathbf{i} - 7\mathbf{j} - 4\mathbf{k}$, where O is the origin. R is a point lying on PQ such that $PR : RQ = 1 : 3$.

(a) Find $\overrightarrow{OP} \times \overrightarrow{OR}$.

(b) Define $\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{OR}$. Find the area of the quadrilateral $OPSR$.

(c) Let N be a point such that $\overrightarrow{ON} = \lambda\left(\overrightarrow{OP} \times \overrightarrow{OR}\right)$, where λ is a real number.

(i) Is $\overrightarrow{NR}$ perpendicular to $\overrightarrow{PQ}$? Explain your answer.

(ii) Let μ be a real number such that $\overrightarrow{NQ}$ is parallel to $11\mathbf{i} + \mu\mathbf{j} - 10\mathbf{k}$.

(1) Find λ and μ .

(2) Denote the angle between ΔOPQ and ΔNPQ by θ . Find $\tan\theta$.

(2021-DSE-MATH-EP(M2) #12) (13 marks)

12. The position vectors of the points A , B , C and D are $t\mathbf{i} + 14\mathbf{j} + s\mathbf{k}$, $12\mathbf{i} - s\mathbf{j} - 2\mathbf{k}$ and $(s+2)\mathbf{i} - 16\mathbf{j} + 10\mathbf{k}$ and $-t\mathbf{i} + (s+2)\mathbf{j} + 14\mathbf{k}$ respectively, where $s, t \in \mathbf{R}$. Suppose that $\overrightarrow{AB}$ is parallel to $5\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$. Denote the plane which contains A , B and C by Π .

(a) Find

(i) s and t .

(ii) the area of ΔABC ,

(iii) the volume of the tetrahedron $ABCD$,

(iv) the shortest distance from D to Π .

(b) Let E be the projection of D on Π . Is E the circumcentre of ΔABC ? Explain your answer.

ANSWERS

(1991-CE-A MATH 1 #08) (16 marks)

8. (a) $\overrightarrow{CA} = (3-x)\mathbf{i} - (y+1)\mathbf{j}$

$\overrightarrow{OB} = (x-7)\mathbf{i} + (y-1)\mathbf{j}$

$\overrightarrow{AB} = (x-1)\mathbf{i} + y\mathbf{j}$

(b) (ii) (1) $x = 4, y = 2$

(1992-CE-A MATH 1 #08) (16 marks)

8. (a) $\mathbf{a} \cdot \mathbf{a} = 4$

$\mathbf{a} \cdot \mathbf{b} = 3$

(b) $OD = 1$

$\overrightarrow{OD} = \frac{1}{3}\mathbf{b}$

(c) (i) $\overrightarrow{OH} = \frac{k}{k+1}\mathbf{a} + \frac{1}{3(k+1)}\mathbf{b}$

$k = 2$

(ii) (1) $\overrightarrow{OC} = \frac{m}{m+1}\mathbf{a} + \frac{1}{m+1}\mathbf{b}$

(2) $\overrightarrow{OC} = \frac{2(n+1)}{3}\mathbf{a} + \frac{(n+1)}{9}\mathbf{b}$

(3) $m = 6,\ n = \frac{2}{7}$

(1993-CE-A MATH 1 #06) (7 marks)

6. (a) $\overrightarrow{AB} = -2\mathbf{i} + 3\mathbf{j}$

(b) $\overrightarrow{AB} \cdot \overrightarrow{AB} = 13$

$\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 13$

(1993-CE-A MATH 1 #08) (16 marks)

8. (a) $\overrightarrow{OP} = \frac{1}{1+r}\mathbf{a} + \frac{r}{1+r}\mathbf{b}$

$\overrightarrow{OQ} = \frac{1}{(1+r)^2}\mathbf{a} + \frac{r(r+2)}{(1+r)^2}\mathbf{b}$

(b) $\overrightarrow{OT} = \frac{1}{1+r}\mathbf{b}$

(c) $r = \frac{-1+\sqrt{5}}{2}$

(d) (i) $\mathbf{a} \cdot \mathbf{a} = 4$

$\mathbf{a} \cdot \mathbf{b} = 16$

(ii) $r = \frac{1}{2}$

(1994-CE-A MATH 1 #03) (6 marks)

3. (a) $\overrightarrow{PQ} = 2\mathbf{i} - \mathbf{j}$

$\left|\overrightarrow{PQ}\right| = \sqrt{5}$

(b) $\cos\angle QPR = \frac{-4}{\sqrt{63}}$

(1994-CE-A MATH 1 #10) (16 marks)

10. (a) $\overrightarrow{OC} = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

$\overrightarrow{DA} = \mathbf{a} - \frac{1}{2}\mathbf{b}$

(c) $k = \frac{1}{5}$

(d) (i) 1

(ii) $k = \frac{2}{7}$

Distance $= \frac{\sqrt{7}}{7}$

(1995-CE-A MATH 1 #07) (8 marks)

7. (a) $\overrightarrow{OR} = \frac{2-6k}{k+1}\mathbf{i} + \frac{3+4k}{k+1}\mathbf{j}$

(b) $\overrightarrow{OP} \cdot \overrightarrow{OR} = \frac{13}{k+1}$

$\overrightarrow{OQ} \cdot \overrightarrow{OR} = \frac{52k}{k+1}$

(c) $k = \frac{1}{2}$

(1995-CE-A MATH 1 #08) (16 marks)

8. (a) (i) $\overrightarrow{AE} = h\mathbf{p} + h\mathbf{q}$

(ii) $\overrightarrow{AE} = \frac{\lambda k}{1+\lambda}\mathbf{p} + \frac{1}{1+\lambda}\mathbf{q}$

(b) (i) 3

(ii) (1) $\overrightarrow{DF} = k\mathbf{p} - \mathbf{q}$

$k = \frac{7}{12}$

(2) $\frac{7}{\sqrt{19}}$

(1996-CE-A MATH 1 #07) (6 marks)

7. (a) Unit vector $=\frac{4}{5}\mathbf{i}+\frac{3}{5}\mathbf{j}$

$\overrightarrow{OC}=\frac{64}{25}\mathbf{i}+\frac{48}{25}\mathbf{j}$

(1996-CE-A MATH 1 #10) (16 marks)

10. (a) (i) $\overrightarrow{AE}=\frac{2}{1+t}\mathbf{a}+\frac{t}{1+t}\mathbf{b}$

(ii) $\overrightarrow{AE}=\frac{9-7t}{1+t}\mathbf{a}+\frac{8t}{1+t}\mathbf{b}$

(b) (i) $\frac{9}{7}$

(ii) (1) 2

(2) $\overrightarrow{AB}\cdot\overrightarrow{BC}=0$

$\overrightarrow{AD}\cdot\overrightarrow{DE}=0$

(1997-CE-A MATH 1 #07) (7 marks)

7. (a) $|\mathbf{a}|=2\sqrt{5}$

(b) $\mathbf{a}\cdot\mathbf{b}=8$

(c) $n=\frac{11}{5}, m=\frac{-2}{5}$

(1997-CE-A MATH 1 #09) (16 marks)

9. (a) (i) $\overrightarrow{AF}=\mathbf{a}+2\mathbf{b}$

(ii) $\overrightarrow{DP}=(m+1)\mathbf{a}-\mathbf{b}$

(b) (ii) (1) $\overrightarrow{AE}=\frac{1}{r+1}\mathbf{a}+\frac{2}{r+1}\mathbf{b}$

(2) $\overrightarrow{AE}=\frac{8}{k+1}\mathbf{a}+\frac{k}{k+1}\mathbf{b}$

$r=\frac{9}{8}, k=16$

(c) $\theta_1=76°$

(1998-CE-A MATH 1 #05) (6 marks)

5. (a) $\overrightarrow{AB}=3\mathbf{i}+5\mathbf{j}$

$\overrightarrow{AC}=-3\mathbf{i}+8\mathbf{j}$

(b) $\overrightarrow{AB}\cdot\overrightarrow{AC}=31$

$\angle BAC=52°$

(1998-CE-A MATH 1 #09) (16 marks)

9. (a) (i) $\mathbf{a}\cdot\mathbf{b}=3$

(ii) $\overrightarrow{OC}=(1-t)\mathbf{a}+t\mathbf{b}$

(iii) $\mathbf{a}\cdot\overrightarrow{OC}=4-t$

$\mathbf{b}\cdot\overrightarrow{OC}=3+6t$

(b) (ii) $k=\frac{4-t}{4}, s=\frac{1+2t}{3}$

(c) $t=\frac{8}{11}$

(1999-CE-A MATH 1 #07) (6 marks)

7. (a) 5

(b) 10

(c) −1.6

(1999-CE-A MATH 1 #10) (16 marks)

10. (a) $\overrightarrow{OC}=\frac{7}{15}\mathbf{a}+\frac{8}{15}\mathbf{b}$

$\overrightarrow{AD}=\frac{16}{21}\mathbf{b}-\mathbf{a}$

(b) (i) $\overrightarrow{OE}=\frac{7r}{15}\mathbf{a}+\frac{8r}{15}\mathbf{b}$

(ii) $\overrightarrow{OE}=(1-k)\mathbf{a}+\frac{16k}{21}\mathbf{b}$

(c) (i) 1 : 2

(2000-CE-A MATH 1 #08) (7 marks)

8. (b) (i) $\sqrt{5}-1$

(ii) $\angle BOD=48°$

(2000-CE-A MATH 1 #09) (16 marks)

9. (a) (i) $\overrightarrow{OD}=\frac{\mathbf{a}+\mathbf{b}}{2}$

(iii) $\overrightarrow{EF}=\mathbf{a}+(1-k)\mathbf{b}$

(b) (i) $\mathbf{a}\cdot\mathbf{b}=3$

$\mathbf{b}\cdot\mathbf{b}=4$

(ii) (1) $\frac{7}{4}$

(2001-AL-P MATH 1 #04) (5 marks)

4. (a) $\overrightarrow{AB}\times\overrightarrow{AC}=bc\,\mathbf{i}+ac\,\mathbf{j}+ab\,\mathbf{k}$.

(2001-CE-A MATH #08) (6 marks)

8. (a) 6

(b) $\frac{1}{3}$

(2001-CE-A MATH #14) (12 marks)

14. (a) $\overrightarrow{OR} = \frac{s\overrightarrow{OP} + r\overrightarrow{OQ}}{r+s}$

(b) (i) $\overrightarrow{OG} = \frac{1}{8}\mathbf{a} + \frac{3}{4}\mathbf{b}$

(ii) $\overrightarrow{OY} = \frac{k}{8}\mathbf{a} + \frac{3k}{8}\mathbf{b}$

(iii) (1) $\frac{6}{7}$

(2) parallel

(2002-AL-P MATH 1 #04) (6 marks)

4. (b) $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{j}+\mathbf{k}), \mathbf{u} = -\frac{1}{\sqrt{2}}(\mathbf{j}+\mathbf{k})$

(2002-CE-A MATH #10) (6 marks)

10. (a) $\overrightarrow{OB} = 6\mathbf{i} + 6\mathbf{j}$

$\overrightarrow{AC} = 4\mathbf{i} - 2\mathbf{j}$

(b) $\theta = 72°$

(2002-CE-A MATH #13) (12 marks)

13. (a) 3

(b) $\overrightarrow{OE} = t\mathbf{a} + (1-t)\mathbf{b}$

(c) $\overrightarrow{BA} \cdot \overrightarrow{BF} = 1$

(2003-AL-P MATH 1 #05) (6 marks)

5. (b) (i) $|\mathbf{m} \times \mathbf{n}| = 6$

(ii) 4

(2003-CE-A MATH #06) (5 marks)

6. (a) $\overrightarrow{OP} = \frac{\mathbf{a}+3\mathbf{b}}{4}$

(b) $\overrightarrow{OC} = \frac{1}{3}\mathbf{a} + \mathbf{b}$

(2003-CE-A MATH #14) (12 marks)

14. (a) $\frac{-3}{2}$

(b) $k = \frac{35}{9}$

(2004-CE-A MATH #06) (5 marks)

6. (a) $\overrightarrow{OC} = \frac{2\mathbf{a}+\mathbf{b}}{3}$

(b) $\left|\overrightarrow{OC}\right| = \frac{2}{3}$

(2004-CE-A MATH #13) (12 marks)

13. (a) (i) $\overrightarrow{OC} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$

$\overrightarrow{OA} = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$

(c) $\theta = 24°$

(2005-AL-P MATH 1 #12) (15 marks)

12. (b) $\alpha = 5\sqrt{3}, \beta = -2\sqrt{2}, \gamma = 3\sqrt{6}$

(2005-CE-A MATH #11) (6 marks)

11. (a) $\frac{-15}{2}$

(b) $\sqrt{19}$

(2005-CE-A MATH #14) (12 marks)

14. (a) $\overrightarrow{CB} = \frac{1}{8}\mathbf{a}$

(b) (i) $DA = 1$

$\mathbf{d} = \mathbf{a} - \mathbf{b}$

(ii) $\overrightarrow{OP} = \frac{8r-1}{8(r+1)}\mathbf{a} + \frac{1-r}{1+r}\mathbf{b}$

(iii) 9 : 16

(2006-CE-A MATH #07) (5 marks)

7. (a) -3

(b) $\sqrt{7}$

(2006-CE-A MATH #18) (12 marks)

18. (a) $\overrightarrow{OG} = \dfrac{\mathbf{a}+\mathbf{b}}{3}$

(b) $\overrightarrow{OT} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{b} = 0$

(c) $\overrightarrow{OC} \cdot \mathbf{b} = \dfrac{|\mathbf{b}|^2}{2}$

(d) (i) $(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{a} = 0$

$(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{b} = 0$

(2007-CE-A MATH #08) (5 marks)

8. (a) $\overrightarrow{BC} = (6k-2)\mathbf{i} + (3k-6)\mathbf{j}$

(b) $\dfrac{2}{3}$

(2007-CE-A MATH #17) (12 marks)

17. (a) $\overrightarrow{OM} = \dfrac{\mathbf{a}+\mathbf{b}}{2}$

(b) (i) $\overrightarrow{OG} = \dfrac{2\mathbf{a}+4k\mathbf{b}}{7}$

(c) (i) $\mathbf{a} \cdot \mathbf{b} = \dfrac{1}{2}$

$\left|\overrightarrow{PQ}\right| = \dfrac{\sqrt{13}}{6}$

(ii) $\angle QGM = 104°$

(2008-CE-A MATH #07) (5 marks)

7. $\dfrac{3\mathbf{i}+4\mathbf{j}}{5}$

(2008-CE-A MATH #15) (12 marks)

15. (a) $\dfrac{1}{2}$

(c) $\overrightarrow{HM} = \dfrac{2t}{3}\mathbf{p} + \dfrac{5t}{3}\mathbf{q}$

$\overrightarrow{HN} = \dfrac{2t}{3}\mathbf{p} + \left(\dfrac{5t}{3} - \dfrac{3}{2}\right)\mathbf{q}$

$\overrightarrow{OH} = \dfrac{5}{3}\mathbf{p} + \dfrac{2}{3}\mathbf{q}$

(2009-CE-A MATH #07) (4 marks)

7. (a) $\angle AOB = 60°$

(b) $|\mathbf{c}| = 3$

(2009-CE-A MATH #14) (12 marks)

14. (a) $\overrightarrow{AH} = \mathbf{p} + \mathbf{q}$

(b) $\overrightarrow{AE} = \dfrac{(r+\lambda)\mathbf{p} + 2r\mathbf{q}}{r+1}$

(c) (i) $\dfrac{9}{5}(\mathbf{p}+\mathbf{q})$

(ii) $\dfrac{AF}{FC} = \dfrac{9}{8}$

(2010-CE-A MATH #12) (7 marks)

12. (a) 15

(b) $\overrightarrow{OB} = 5\mathbf{i} + 5\mathbf{j}$ or $\mathbf{i} + 7\mathbf{j}$

(2010-CE-A MATH #14) (12 marks)

14. (a) $\overrightarrow{OD} = \mathbf{a} + \dfrac{1}{1+r}\mathbf{c}$

$r = \dfrac{2}{7}$

(b) (i) $\overrightarrow{OE} = \dfrac{9}{7}\mathbf{a} + \mathbf{c}$

(2011-CE-A MATH #09) (6 marks)

9. (a) $\cos\angle AOB = \dfrac{3}{\sqrt{10}}$

(b) 7

(2011-CE-A MATH #12) (12 marks)

12. (a) $\overrightarrow{OE} = (1+\mu)\mathbf{a} + \mu(1+h)\mathbf{b}$

$\overrightarrow{OE} = (1-\lambda)\mathbf{b} + \lambda(1+h)\mathbf{a}$

(SP-DSE-MATH-EP(M2) #09) (6 marks)

9. (a) 13

(b) 65

(c) $\overrightarrow{OC'} = (3+4s)\mathbf{i} + (1+3s+3t)\mathbf{j} + (5+t)\mathbf{k}$

where s and t are not both zero.

(SP-DSE-MATH-EP(M2) #14) (10 marks)

14. (c) $\mathbf{h} = \mathbf{a} + \mathbf{b} + \mathbf{c}$

(PP-DSE-MATH-EP(M2) #12) (13 marks)

12. (a) (ii) $a = \frac{1}{2}, b = \frac{1}{3}$

(iii) $\frac{\sqrt{6}}{6}$

(b) (i) $\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i} + \mathbf{j} - \mathbf{k}$

(ii) $P = (1, 1, -1)$

(2012-DSE-MATH-EP(M2) #07) (5 marks)

7. (a) 3

(b) $\frac{11}{3}$

(2012-DSE-MATH-EP(M2) #12) (12 marks)

12. (a) $\overrightarrow{AG} = \frac{-2\mathbf{a} + \mathbf{b} + \mathbf{c}}{3}$

(b) (i) $FG : GO = 2 : 1$

(2013-DSE-MATH-EP(M2) #10) (5 marks)

10. (a) $\overrightarrow{ON} = \frac{2(k+1)\mathbf{i} + 2\mathbf{j}}{k+1}$

(b) $k = \frac{3}{2}$

(2013-DSE-MATH-EP(M2) #14) (12 marks)

(2014-DSE-MATH-EP(M2) #08) (8 marks)

8. (a) $\overrightarrow{OP} \times \overrightarrow{OQ} = 6\mathbf{i} + 4\mathbf{j} - \mathbf{k}$.

Volume = 1

(b) 6.8°

(2014-DSE-MATH-EP(M2) #11) (13 marks)

11. (a) (i) $\overrightarrow{OE} = \frac{\mathbf{a} + mt\mathbf{b}}{1+n}$

(ii) $\overrightarrow{OE} = \frac{n(1-t)\mathbf{a} + \mathbf{b}}{1+n}$

(iv) $t = \frac{1}{2}$

(b) $\overrightarrow{BD} \cdot \overrightarrow{OA} \neq 0$

(2015-DSE-MATH-EP(M2) #10) (12 marks)

10. (a) (i) $\overrightarrow{OQ} = \frac{1-t}{2}\overrightarrow{OA} + \frac{3t}{4}\overrightarrow{OB}$

(ii) 7 : 16

(b) (i) 176

(ii) 42

(2016-DSE-MATH-EP(M2) #12) (13 marks)

12. (a) -1

(b) (i) $\frac{-1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$

(ii) $\sin^{-1}\left(\frac{3\sqrt{11}}{11}\right)$

(iii) D is the mid-point of the line segment joining E and F.

(2017-DSE-MATH-EP(M2) #03) (5 marks)

3. (a) $\overrightarrow{OP} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

(b) (i) 225

(ii) 24

(2017-DSE-MATH-EP(M2) #10) (12 marks)

10. (a) $\overrightarrow{AE} = \frac{2r+6}{r+1}\mathbf{i} + \frac{r-6}{r+1}\mathbf{j} + \frac{r}{r+1}\mathbf{k}$

$\overrightarrow{AF} = \frac{-8r+36}{r+1}\mathbf{i} + \frac{41r-36}{r+1}\mathbf{j} + \frac{11r}{r+1}\mathbf{k}$

$r = \frac{6}{5}$

(b) (i) 0

(c) 7

(2018-DSE-MATH-EP(M2) #12) (13 marks)

12. (a) (i) $32\mathbf{i} + 8\mathbf{j} - 28\mathbf{k}$

(ii) 24

(iii) $-\frac{32}{13}\mathbf{i} - \frac{8}{13}\mathbf{j} + \frac{28}{13}\mathbf{k}$

(b) (i) $-2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$

(ii) $\overrightarrow{BC} \cdot \overrightarrow{EF} = 0$

(c) $\cos^{-1}\left(\frac{3\sqrt{13}}{13}\right)$

(2019-DSE-MATH-EP(M2) #12) (13 marks)

12. (a) 2

(b) $48\mathbf{i} - 36\mathbf{j} + 48\mathbf{k}$

(c) 48 cubic units

(d) (i) $p = 6$, $q = -8$, $r = 6$

(ii) $\frac{24}{41}(4\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$

(iii) D , E and O are collinear

(2020-DSE-MATH-EP(M2) #12) (12 marks)

12. (a) $6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$

(b) 9

(c) (i) ... Yes

(ii) (1) $\lambda = \frac{2}{9}$, $\mu = -25$

(2) $\frac{2}{3}$

(2021-DSE-MATH-EP(M2) #12) (13 marks)

12. (a) (i) $s = 10$, $t = 18$

(ii) 270

(iii) 2 160,

(iv) 24

(b) E is not the circumcentre of ΔABC