3. Introduction to Vectors

(1991-CE-A MATH 1 #05) (7 marks)

5.

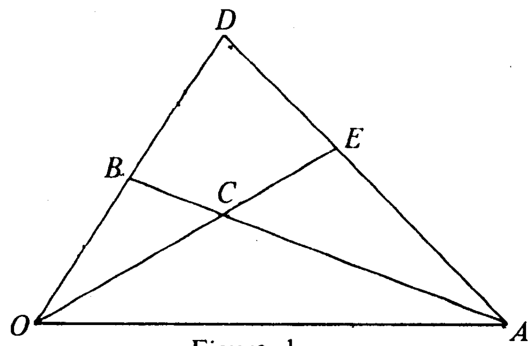

Figure 1

In Figure 1, OAD is a triangle and B is the mid-point of OD. The line OE cuts the line AB at C such that $AC : CB = 3 : 1$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) (i) Let $OC : CE = k : 1$. Express $\overrightarrow{OE}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.

(ii) Let $AE : ED = m : 1$. Express $\overrightarrow{OE}$ in terms of m, $\mathbf{a}$ and $\mathbf{b}$.

Hence find k and m.

(1992-CE-A MATH 1 #01) (5 marks)

1. Given $\overrightarrow{OA} = 5\mathbf{i} - \mathbf{j}$, $\overrightarrow{OB} = -3\mathbf{i} + 5\mathbf{j}$ and APB is a straight line.

(a) Find $\overrightarrow{AB}$ and $\left|\overrightarrow{AB}\right|$.

(b) If $\left|\overrightarrow{AP}\right| = 4$, find $\overrightarrow{AP}$.

(1999-CE-A MATH 1 #10) (16 marks)

10.

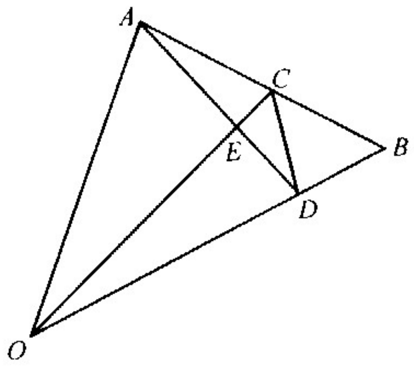

Figure 3

In Figure 3, OAB is a triangle. C and D are points on AB and OB respectively such that $AC : CB = 8 : 7$ and $OD : DB = 16 : 5$. OC and AD intersect at a point E . Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OC}$ and $\overrightarrow{AD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Let $\overrightarrow{OE} = r\overrightarrow{OC}$ and $\overrightarrow{AE} = k\overrightarrow{AD}$.

(i) Express $\overrightarrow{OE}$ in terms of r , $\mathbf{a}$ and $\mathbf{b}$.

(ii) Express $\overrightarrow{OE}$ in terms of k , $\mathbf{a}$ and $\mathbf{b}$.

Hence show that $r = \dfrac{6}{7}$ and $k = \dfrac{3}{5}$.

(c) It is given that $EC : ED = 1 : 2$.

(i) Using (b), or otherwise, find $EA : EO$.

(ii) Explain why $OACD$ is a cyclic quadrilateral.

(2001-CE-A MATH #14) (12 marks)

14. (a)

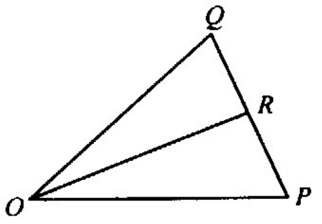

Figure 1(a)

In Figure 1 (a), OPQ is a triangle. R is a point on PQ such that $PR : RQ = r : s$. Express $\overrightarrow{OR}$ in terms of r , s , $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.

Hence show that if $\overrightarrow{OR} = m\overrightarrow{OP} + n\overrightarrow{OQ}$, then $m + n = 1$.

(b)

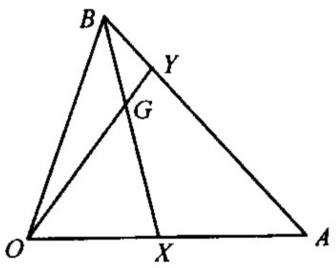

Figure 1(b)

In Figure 1 (b), OAB is a triangle. X is the mid-point of OA and Y is a point on AB . BX and OY intersect at point G where $BG : GX = 1 : 3$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(i) Express $\overrightarrow{OG}$ in terms of **a** and **b**.

(ii) Using (a), express $\overrightarrow{OY}$ in terms of **a** and **b**.

(Hint: Put $\overrightarrow{OY} = k\overrightarrow{OG}$.)

(iii) Moreover, AG is produced to a point Z on OB . Let $\overrightarrow{OZ} = h\overrightarrow{OB}$.

(1) Find the value of h .

(2) Explain whether ZY is parallel to OA or not.

(2003-CE-A MATH #06) (5 marks)

6.

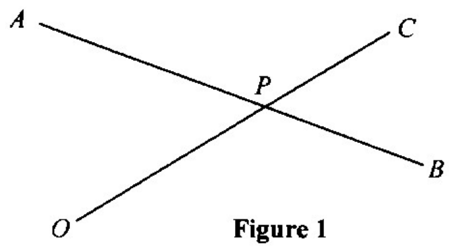

Figure 1

In Figure 1, point P divides both line segments AB and OC in the same ratio $3 : 1$. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Express $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Hence show that OA is parallel to BC .

(2005-CE-A MATH #14) (12 marks)

14.

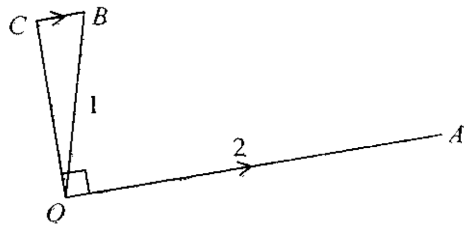

Figure 4

In Figure 4, $OA = 2$, $OB = 1$ and $\cos\angle AOB = \frac{1}{4}$. C is a point such that $CB \parallel OA$ and $OC \perp OA$.

Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$.

(a) Find $\overrightarrow{CB}$ in terms of $\mathbf{a}$.

Hence, or otherwise, show that $\mathbf{c} = \mathbf{b} - \frac{1}{8}\mathbf{a}$.

(b)

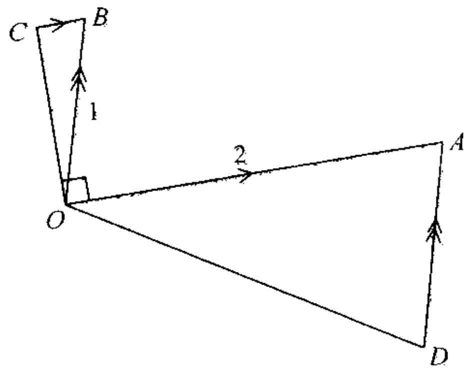

Figure 5

D is a point such that $DA \parallel OB$ and $OD = OA$ (see Figure 5). Let $\overrightarrow{OD} = \mathbf{d}$.

(i) By finding DA , or otherwise, express $\mathbf{d}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) P is a point on the line segment CD such that $CP : PD = r : 1$. Express $\overrightarrow{OP}$ in terms of r , $\mathbf{a}$ and $\mathbf{b}$.

(iii) If M is the mid-point of AB , find the ratio in which OM divides CD .

(2008-CE-A MATH #07) (5 marks)

7. It is given that $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OB} = 5\mathbf{i} + 6\mathbf{j}$. If P is a point on AB such that $\overrightarrow{PB} = 2\overrightarrow{AP}$, find the unit vector in the direction of $\overrightarrow{OP}$.

(2011-CE-A MATH #12) (12 marks)

12.

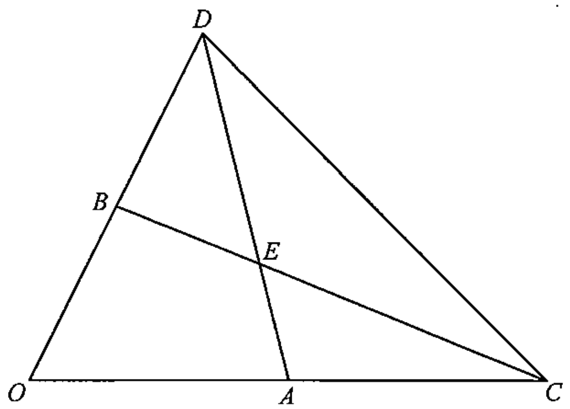

Figure 2

Figure 2 shows a triangle OCD . A and B are points on OC and OD respectively such that $OA : AC = OB : BD = 1 : h$, where $h > 0$. AD and BC intersect at E such that $AE : ED = \mu : (1-\mu)$ and $BE : EC = \lambda : (1-\lambda)$, where $0 < \mu < 1$ and $0 < \lambda < 1$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) By considering $\overrightarrow{OE}$, show that $\mu = \lambda$.

(b) F is a point on CD such that O , E and F are collinear. Show that OF is a median of ΔOCD .

(c) Using the above results, show that in a triangle, the centroid divides every median in $2 : 1$.

ANSWERS

(1991-CE-A MATH 1 #05) (7 marks)

5. (a) $\overrightarrow{OC} = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

(b) (i) $\overrightarrow{OE} = \frac{k+1}{4k}\mathbf{a} + \frac{3(k+1)}{4k}\mathbf{b}$

(ii) $\overrightarrow{OE} = \frac{1}{1+m}\mathbf{a} + \frac{2m}{1+m}\mathbf{b}$

$m = \frac{3}{2}$, $k = \frac{5}{3}$

(1992-CE-A MATH 1 #01) (5 marks)

1. (a) $\overrightarrow{AB} = -8\mathbf{i} + 6\mathbf{j}$

$\left|\overrightarrow{AB}\right| = 10$

(b) $\frac{-16}{5}\mathbf{i} + \frac{12}{5}\mathbf{j}$

(1999-CE-A MATH 1 #10) (16 marks)

10. (a) $\overrightarrow{OC} = \frac{7\mathbf{a} + 8\mathbf{b}}{15}$

$\overrightarrow{AD} = \frac{16}{21}\mathbf{b} - \mathbf{a}$

(b) (i) $\frac{r}{15}(7\mathbf{a} + 8\mathbf{b})$

(ii) $(1-k)\mathbf{a} + \frac{16k}{21}\mathbf{b}$

(c) (i) 1 : 2

(2001-CE-A MATH #14) (12 marks)

14. (a) $\overrightarrow{OR} = \frac{s\overrightarrow{OP} + r\overrightarrow{OQ}}{r+s}$

(b) (i) $\overrightarrow{OG} = \frac{1}{8}\mathbf{a} + \frac{3}{4}\mathbf{b}$

(ii) $\overrightarrow{OY} = \frac{1}{7}\mathbf{a} + \frac{6}{7}\mathbf{b}$

(iii) (1) $\frac{6}{7}$

(2003-CE-A MATH #06) (5 marks)

6. (a) $\overrightarrow{OP} = \frac{\mathbf{a} + 3\mathbf{b}}{4}$

(b) $\overrightarrow{OC} = \frac{1}{3}\mathbf{a} + \mathbf{b}$

(2005-CE-A MATH #14) (12 marks)

14. (a) $\overrightarrow{CB} = \frac{1}{8}\mathbf{a}$

(b) (i) $DA = 1$, $\mathbf{d} = \mathbf{a} - \mathbf{b}$

(ii) $\frac{8r-1}{8(r+1)}\mathbf{a} + \frac{1-r}{1+r}\mathbf{b}$

(iii) 9 : 16

(2008-CE-A MATH #07) (5 marks)

7. $\frac{3\mathbf{i} + 4\mathbf{j}}{5}$

(2011-CE-A MATH #12) (12 marks)

12. (a) $\overrightarrow{OE} = (1+\mu)\mathbf{a} + \mu(1+h)\mathbf{b}$

$\overrightarrow{OE} = (1-\lambda)\mathbf{b} + \lambda(1+h)\mathbf{a}$