2. System of Linear Equations

(1991-AL-P MATH 1 #03) (4 marks)

3. Consider the following system of linear equations:

$$\begin{cases} x + 2y + z = 1 \\ x + y + 2z = 2 \\ \quad - y + q^2 z = q \end{cases}$$

Determine all values of q for each of the following cases:

(a) The system has no solution.

(b) The system has infinitely many solutions.

(1992-AL-P MATH 1 #01) (6 marks)

1. Consider the following system of linear equations:

$$(*) \begin{cases} x + (t+3)y + 5z = 3 \\ -3x + 9y - 15z = s \\ 2x + ty + 10z = 6 \end{cases}$$

(a) If $(*)$ is consistent, find s and t .

(b) Solve $(*)$ when it is consistent.

(1993-AL-P MATH 1 #03) (6 marks)

1 Suppose the following system of linear equation is consistent:

$$(*) \begin{cases} ax + by + cz = 1 \\ bx + cy + az = 1 \\ cx + ay + bz = 1 \\ x + y + z = 3 \end{cases}, \text{ where } a, b, c \in \mathbf{R}.$$

(a) Show that $a+b+c=1$.

(b) Show that (*) has a unique solution if and only if a , b and c are not all equal.

(c) If $a=b=c$, solve $(*)$.

(1994-AL-P MATH 1 #02) (6 marks)

1. Consider the following system of linear equations:

$$(*)\ \begin{cases} 4x + 3y + z = \lambda x \\ 3x - 4y + 7z = \lambda y \\ x + 7y - 6z = \lambda z \end{cases}$$

Suppose λ is an integer and $(*)$ has nontrivial solutions.

Find λ and solve $(*)$.

(1994-AL-P MATH 1 #09) (15 marks)

9. (a) Consider

$$(I)\ :\ \begin{cases} a_{11}x + a_{12}y + a_{13}z = 0 \\ a_{21}x + a_{22}y + a_{23}z = 0 \\ a_{31}x + a_{32}y + a_{33}z = 0 \end{cases} \text{ and } (II)\ :\ \begin{cases} a_{11}x + a_{12}y + a_{13} = 0 \\ a_{21}x + a_{22}y + a_{23} = 0 \\ a_{31}x + a_{32}y + a_{33} = 0 \end{cases}.$$

(i) Show that if (I) has a unique solution, then (II) has no solution.

(ii) Show that (u, v) is a solution of (II) if and only if (ut, vt, t) are solutions of (I) for all $t \in \mathbf{R}$.

(iii) If (II) has no solution and (I) has nontrivial solutions, what can you say about the solutions of (I) ?

(b) Consider

$$(III)\ :\ \begin{cases} -(3+k)x + y - z = 0 \\ -7x + (5-k)y - z = 0 \\ -6x + 6y + (k-2)z = 0 \end{cases}$$

and

$$(IV)\ :\ \begin{cases} -(3+k)x + y - 1 = 0 \\ -7x + (5-k)y - 1 = 0 \\ -6x + 6y + (k-2) = 0 \end{cases}$$

(i) Find the values of k for which (III) has non-trivial solutions.

(ii) Find the values of k for which (IV) is consistent. Solve (IV) for each of these values of k .

(iii) Solve (III) for each k such that (III) has non-trivial solutions.

(1995-AL-P MATH 1 #09) (15 marks)

9. Consider the following system of linear equations

$$(S)\ :\ \begin{cases} 2x + 2y - z = k \\ hx - 3y - z = 0 \\ -3x + hy + z = 0 \end{cases}$$

and

$$(T)\ :\ \begin{cases} 6x + 6y - 3z = 2 \\ hx - 3y - z = 0 \\ -3x + hy + z = 0 \\ -5x - 2y + 6z = h \end{cases}$$

(a) Show that (S) has a unique solution if and only if $h^2 \neq 9$. Solve (S) in this case.

(b) For each of the following cases, find the value(s) of k for which (S) is consistent, and solve (S) :

(i) $h = 3$,

(ii) $h = -3$.

(c) Find the values of h for which (T) is consistent. Solve (T) for each of these values of h .

(1996-AL-P MATH 1 #05) (6 marks)

1. (a) Solve $\begin{cases} Z + Y = a \\ Z + X = b \\ Y + X = c \end{cases}$ for X , Y and Z .

(b) If $a + b - c > 0$, $b + c - a > 0$ and $c + a - b > 0$,

solve $\begin{cases} xy + xz = a \\ xy + yz = b \\ xz + yz = c \end{cases}$ for x , y and z .

(1996-AL-P MATH 1 #09) (15 marks)

9. Consider the system of linear equations

$$(*) : \begin{cases} x + 2y - z = 3 \\ x + y + 2z = 4 \end{cases}$$

(a) Solve (*) .

(b) Find the solutions of (*) that satisfy $xy + yz + zx = 2$.

(c) Find all possible values of a and λ (a , $\lambda \in \mathbf{R}$) so that

$$\begin{cases} x + 2y - z = 3 \\ x + y + 2z = 4 \\ ax + y + z = \lambda \end{cases}$$

is solvable.

(d) Using (b), or otherwise, find all possible values of a and λ (a , $\lambda \in \mathbf{R}$) so that

$$\begin{cases} x + 2y - z = 3 \\ x + y + 2z = 4 \\ xy + yz + zx = 2 \\ ax + y + z = \lambda \end{cases}$$

has at least one solution.

(1997-AL-P MATH 1 #03) (6 marks)

3. Suppose the system of linear equations

$$(*) \begin{cases} \lambda x + ky = 0 \\ -\lambda y + z = 0 \\ x + ky + z = 0 \end{cases}$$

has nontrivial solutions.

(a) Show that λ satisfies the equation $\lambda^2 + k\lambda - k = 0$.

(b) If the quadratic equations in λ in (a) has equal roots, find k .
Solve (*) for each of these values of k .

(1997-AL-P MATH 1 #08) (15 marks)

8. Consider the following two systems of linear equations:

$$(S):\begin{cases}(a+1)x + 2y - 2z = 0\\ x + ay + 2z = 0\\ 3x - y + (a-7)z = 0\end{cases}$$

$$(T):\begin{cases}(a+1)x + 2y - 2z = 6\\ x + ay + 2z = 5b-1\\ 3x - y + (a-7)z = 1-b\end{cases}$$

(a) If (S) has infinitely many solutions, find all the values of a . Solve (S) for each of these values of a .

(b) For the smallest value of a found in (a), find the values of b so that (T) is consistent. Solve (T) for these values of a and b .

(c) Solve the system of equations

$$\begin{cases}-x + 2y - 2\sqrt{z} = 6\\ x - 2y + 2\sqrt{z} = -6\\ 3x - y - 9\sqrt{z} = 2\\ 3x - 4y - z = -11\end{cases}$$

(1998-AL-P MATH 1 #01) (6 marks)

1. Consider the system of linear equations

$$(*)\begin{cases}2x + y + 2z = 0\\ x + (k+1)z = 0\\ kx - y + 4z = 0\end{cases}$$

Suppose $(*)$ has infinitely many solutions.

(a) Find k .

(b) Solve $(*)$.

(1998-AL-P MATH 1 #08) (15 marks)

8. Consider the system of linear equations in

$$(E):\begin{cases} ax + y + bz = 1 \\ x + ay + bz = 1 \\ x + y + abz = b \end{cases}$$

(a) Show that (E) has a unique solution if and only if $a \neq -2$, $a \neq 1$ and $b \neq 0$. Solve (E) in this case.

(b) For each of the following cases, determine the value(s) of b for which (E) is consistent. Solve (E) in each case.

(i) $a = -2$,

(ii) $a = 1$.

(c) Determine whether (E) is consistent or not for $b = 0$.

(1999-AL-P MATH 1 #01) (6 marks)

1. Suppose the system of linear equations

$$(*)\begin{cases} x + y - \lambda z = 0 \\ x + \lambda y - z = 0 \\ \lambda x + y - z = 0 \end{cases}$$

has non-trivial solutions.

(a) Find all values of λ .

(b) Solve $(*)$ for each of the values of λ obtained in (a).

(1999-AL-P MATH 1 #08) (15 marks)

8. Consider the system of linear equations

$$(E) : \begin{cases} x + \lambda y + z = \lambda \\ 3x - y + (\lambda + 2)z = 7 \\ x - y + z = 3 \end{cases} \text{ where } \lambda \in \mathbf{R} .$$

(a) Show that (E) has a unique solution if and only if $\lambda \neq \pm 1$.

(b) Solve (E) for

(i) $\lambda \neq \pm 1$,

(ii) $\lambda = -1$,

(iii) $\lambda = 1$.

(c) Find the conditions on a , b , c and d so that the system of linear equations

$$\begin{cases} x + y + z = 1 \\ 3x - y + 3z = 7 \\ x - y + z = 3 \\ ax + by + cz = d \end{cases}$$

is consistent.

(2000-AL-P MATH 1 #08) (15 marks)

8. Consider the system of linear equations

$$(S) : \begin{cases} x - y - z = a \\ 2x + \lambda y - 2z = b \\ x + (2\lambda + 3)y + \lambda^2 z = c \end{cases} \text{ where } \lambda \in \mathbf{R} .$$

(a) Show that (S) has a unique solution if and only if $\lambda \neq -2$. Solve (S) for $\lambda = -1$.

(b) Let $\lambda = -2$.

(i) Find the conditions on a , b and c so that (S) has infinitely many solutions.

(ii) Solve (S) when $a = -1$, $b = -2$ and $c = -3$.

(c) Consider the system of linear equations

$$(T) : \begin{cases} x - y - z + 3\mu - 5 = 0 \\ 2x - 2y - 2z + 2\mu - 2 = 0 \\ x - y + 4z - \mu - 1 = 0 \end{cases} \text{ where } \mu \in \mathbf{R} .$$

Using the results in (b), or otherwise, solve (T) .

(2001-AL-P MATH 1 #09) (15 marks)

9. Consider the system of linear equations

$$(S) : \begin{cases} x + \lambda y + z = k \\ \lambda x - y + z = 1 \\ 3x + y + 2z = -1 \end{cases} \text{ where } \lambda , k \in \mathbf{R} .$$

(a) Show that (*S*) has a unique solution if and only if $\lambda \neq 0$ and $\lambda \neq 2$.

(b) For each of the following cases, determine the value(s) of k for which (S) is consistent. Solve (S) in each case.

(i) $\lambda \neq 0$ and $\lambda \neq 2$,

(ii) $\lambda = 0$,

(iii) $\lambda = 2$.

(c) If some solution of (x, y, z) of

$$\begin{cases} x + z = 0 \\ - y + z = 1 \\ 3x + y + 2z = -1 \end{cases}$$

satisfies $(x - p)^2 + y^2 + z^2 = 1$, find the range of values of p .

(2002-AL-P MATH 1 #08) (15 marks)

8. (a) Consider the system of linear equations in x , y , z .

$$(S) : \begin{cases} ax - 2y + z = 0 \\ x - y + 2z = b \\ y + az = b \end{cases} \text{, where a , } b \in \mathbf{R} .$$

(i) Show that (S) has a unique solution if and only if $a^2 \neq 1$. Solve (S) in this case.

(ii) For each of the following cases, determine the value(s) of b for which (S) is consistent, and solve (S) for such value(s) of b .

(1) $a = 1$,

(2) $a = -1$.

(b) Consider the system of linear equations in x , y , z

$$(T) : \begin{cases} ax - 2y + z = 0 \\ x - y + 2z = -1 \\ y + az = -1 \\ 5x - 2y + z = a \end{cases} \text{, where } a \in \mathbf{R} .$$

Find all the values of a for which (T) is consistent. Solve (T) for each of these values of a .

(2003-AL-P MATH 1 #07) (15 marks)

7. (a) Consider the system of linear equations in x , y , z .

$$(E)\ :\ \begin{cases} x + ay - z = 0 \\ 2x - y + az = -2a \\ -x + 2a^2y + (a-3)z = 2a \end{cases}, \text{ where } a \in \mathbf{R} .$$

(i) Find the range of values of a for which (E) has a unique solution. Solve (E) when (E) has a unique solution.

(ii) Solve (E) for

(1) $a = 1$,

(2) $a = -4$.

(b) Suppose (x, y, z) satisfy

$$\begin{cases} x + y - z = 0 \\ 2x - y + z = -2 \\ -x + 2y - 2z = 2 \end{cases} .$$

Find the least value of $24x^2 + 3y^2 + 2z$ and the corresponding values of x , y , z .

(2004-AL-P MATH 1 #07) (15 marks)

7. (a) Consider the system of linear equations in x , y , z .

$$(E)\ :\ \begin{cases} x + (a-2)y + az = 1 \\ x + 2y + 4z = 1 \\ ax - y + 3z = b \end{cases}, \text{ where } a\ , b \in \mathbf{R} .$$

(i) Prove that (E) has a unique solution if and only if $a \neq 2$ and $a \neq 4$. Solve (E) in this case.

(ii) For each of the following cases, determine the value(s) of b for which (E) is consistent, and solve (E) for such value(s) of b .

(1) $a = 2$,

(2) $a = 4$.

(b) If all solutions (x, y, z) of

$$\begin{cases} x + + 2z = 1 \\ x + 2y + 4z = 1 \\ 2x - y + 3z = 2 \end{cases}$$

satisfy $k\left(x^2 - 3\right) > yz$, find the range of values of k .

(2005-AL-P MATH 1 #07) (15 marks)

7. (a) Consider the system of linear equations in x , y , z .

$$(E)\ :\ \begin{cases} x + ay + z = b \\ 2x + (a+3)y + (a-1)z = 0 \\ 3x + a^2y + (4a+1)z = -b \end{cases}\text{, where } a\ ,\ b \in \mathbf{R}\ .$$

(i) Find the range of values of a for which (E) has a unique solution. Solve (E) when (E) has a unique solution.

(ii) For each of the following cases, find the value(s) of b for which (E) is consistent, and solve (E) for such value(s) of b .

(1) $a = 1$,

(2) $a = -2$.

(b) Suppose that a real solution of

$$\begin{cases} x - 2y + z = b \\ 2x + y - 3z = 0 \\ 3x + 4y - 7z = -b \end{cases}$$

satisfies $x^2 + y^2 + z^2 = b + 3$, where $b \in \mathbf{R}$. Find the range of values of b .

(2006-AL-P MATH 1 #07) (15 marks)

7. Consider the system of linear equations in x , y , z .

$$(E)\ :\ \begin{cases} x + ay + z = 4 \\ x + (2-a)y + (3b-1)z = 3 \\ 2x + (a+1)y + (b+1)z = 7 \end{cases}\text{, where } a\ ,\ b \in \mathbf{R}\ .$$

(a) Prove that (E) has a unique solution if and only if $a \neq 1$ and $b \neq 0$. Solve (E) in this case.

(b) (i) For $a = 1$, find the value(s) of b for which (E) is consistent, and solve (E) for such value(s) of b .

(ii) Is there a real solution (x, y, z) of

$$\begin{cases} x + y + z = 4 \\ 2x + 2y + z = 6 \\ 4x + 4y + 3z = 14 \end{cases}$$

satisfying $x^2 - 2y^2 - z = 14$? Explain your answer.

(c) Is (E) consistent for $b = 0$? Explain your answer.

(2007-AL-P MATH 1 #07) (15 marks)

7. (a) Consider the system of linear equations in x, y, z.

$$(E): \begin{cases} x - 3y = 1 \\ x + 5y + az = b \\ 2x + ay - z = 2 \end{cases}, \text{ where } a, b \in \mathbf{R}.$$

(i) Find the range of values of a for which (E) has a unique solution. Solve (E) when (E) has a unique solution.

(ii) Suppose that $a = -2$. Find the value(s) of b for which (E) is consistent, and solve (E) for such value(s) of b.

(b) Is the system of linear equations

$$\begin{cases} x - 3y = 1 \\ x + 5y + z = 16 \\ 2x + y - z = 2 \\ x - y - z = 3 \end{cases}$$

consistent? Explain your answer.

(c) Solve the system of linear equations

$$\begin{cases} x - 3y = 1 \\ x + 5y - 2z = 16 \\ 2x - 2y - z = 2 \\ x - y - z = 3 \end{cases}.$$

(2008-AL-P MATH 1 #07) (15 marks)

7. (a) Consider the system of linear equations in x, y, z.

$$(E): \begin{cases} x + (a+2)y + (a+1)z = 1 \\ x - 3y - z = b \\ 3x - 2y + (a-1)z = 1 \end{cases}, \text{ where } a, b \in \mathbf{R}.$$

(i) Prove that (E) has a unique solution if and only if $a^2 \neq 4$. Solve (E) when (E) has a unique solution.

(ii) For each of the following cases, find the value(s) of b for which (E) is consistent, and solve (E) for such value(s) of b.

(1) $a = 2$,

(2) $a = -2$.

(b) Find the greatest value of $2x^2 + 15y^2 - 10z^2$, where x, y and z are real numbers satisfying

$$\begin{cases} x + 4y + 3z = 1 \\ x - 3y - z = 0 \\ 3x - 2y + z = 1 \end{cases}.$$

(2009-AL-P MATH 1 #07) (15 marks)

7. (a) Consider the system of linear equations in x , y , z .

$$(E) : \begin{cases} x + \lambda y + 2z = 1 \\ 5x - \lambda y + z = 5 \\ \lambda x - y + z = a \end{cases} \text{, where } \lambda \text{ , } a \in \mathbf{R} \text{ .}$$

(i) Find the range of values of λ for which (E) has a unique solution. Solve (E) when (E) has a unique solution.

(ii) Suppose that $\lambda = -1$. Find the value(s) of a for which (E) is consistent, and solve (E) for such value(s) of a .

(b) Is the system of linear equations

$$\begin{cases} x - 2y + 2z = 1 \\ 5x + 2y + z = 5 \\ 2x + y - z = -3 \\ 4x + 3y - 3z = 2 \end{cases}$$

consistent? Explain your answer.

(c) Find the solution(s) of the system of linear equations

$$\begin{cases} x - y + 2z = 1 \\ 5x + y + z = 5 \\ x + y - z = 1 \end{cases}$$

satisfying $4x^2 + 2y - z = 28$.

(2010-AL-P MATH 1 #07) (15 marks)

7. (a) Consider the system of linear equations in x , y , z

$$(E):\begin{cases} x + y + z = 2 \\ ax - 4z = 2 \\ 3x + 4y + (a+4)z = b \end{cases}, \text{ where } a\ ,\ b \in \mathbf{R}\ .$$

(i) Find the range of values of a for which (E) has a unique solution, and solve (E) when (E) has a unique solution.

(ii) Suppose that $a = 2$. Find the value(s) of b for which (E) is consistent, and solve (E) for such value(s) of b .

(b) Consider the system of linear equation in x , y , z

$$(F):\begin{cases} x + y + z = 2 \\ x + 2z = -1 \\ 3x + 4y + 2z = \lambda \\ 7x + 17y - 3z = \mu \end{cases}, \text{ where } \lambda\ ,\ \mu \in \mathbf{R}\ .$$

Find the values of λ and μ for which (F) is consistent.

(c) Consider the system of linear equation in x , y , z

$$(G):\begin{cases} x + y + z = 2 \\ x - 6z = 3 \\ 9x + 12y + 14z = 15 \\ 5x - 2y - 18z = 16 \end{cases}$$

Is (G) consistent? Explain your answer.

(2011-AL-P MATH 1 #07) (15 marks)

7. (a) Consider the system of linear equations in x , y , z

$$(S):\begin{cases} y + (\lambda+1)z = 0 \\ \lambda x + 2y + 2z = \mu \\ x - \lambda y - 4z = \mu^2 \end{cases}, \text{ where } \lambda\ ,\ \mu \in \mathbf{R}\ .$$

(i) Suppose that $\mu = 0$.

(1) Prove that (S) has non-trivial solutions if and only if $\lambda^3 + \lambda^2 - 2\lambda = 0$.

(2) Solve (S) when $\lambda = 1$.

(ii) Suppose that $\mu \neq 0$.

(1) Find the range of values of λ for which (S) has a unique solution.

(2) Solve (S) when (S) has a unique solution.

(3) Find λ and μ for which (S) has infinitely many solutions.

(b) Is there a real solution (x, y, z) of the system of linear equations

$$\begin{cases} y + 2z = 0 \\ x + 2y + 2z = 1 \\ x - y - 4z = 1 \end{cases}$$

satisfying $3x^3 + 2y^2 - z^2 = 1$? Explain your answer.

(SP-DSE-MATH-EP(M2) #07) (5 marks)

7. Solve the system of linear equations

$$\begin{cases} x + 7y - 6z = -4 \\ 3x - 4y + 7z = 13 \\ 4x + 3y + z = 9 \end{cases} .$$

(PP-DSE-MATH-EP(M2) #02) (4 marks)

2. Consider the following system of linear equations in x , y , z

$$\begin{cases} x - 7y + 7z = 0 \\ x - ky + 3z = 0 \\ 2x + y + kz = 0 \end{cases} ,$$ where k is a real number.

If the system has non-trivial solutions, find the two possible values of k .

(2012-AL-P MATH 1 #07) (15 marks)

7. (a) Consider the following system of linear equations in x , y , z

$$(E) : \begin{cases} x + 2y - z = 3 \\ 2x + 5y + (a-1)z = 4 \\ (a+2)x + y + (2a+1)z = b \end{cases} ,$$ where a , $b \in \mathbf{R}$.

(i) Prove that (E) has a unique solution if and only if $a \neq -1$ and $a \neq -3$. Solve (E) when (E) has a unique solution.

(ii) Suppose that $a = -3$. Find b for which (E) is consistent, and solve (E) when (E) is consistent.

(b) Is the system of linear equations in real variables x , y , z

$$\begin{cases} x + 2y - z = 3 \\ 6x + 15y - 7z = 12 \\ 2x + 3y - 5z = -12 \\ 4x + 5y - 6z = 1 \end{cases}$$

consistent? Explain your answer.

(c) Find the least value of $3x^2 - 7y^2 + 8z^2$, where x , y and z are real numbers satisfying

$$\begin{cases} x + 2y - z = 3 \\ 2x + 5y - 4z = 4 \\ x - y + 5z = 9 \end{cases} .$$

(2012-DSE-MATH-EP(M2) #08) (5 marks)

8. (a) Solve the following system of linear equations:

$$\begin{cases} x + y + z = 0 \\ 2x - y + 5z = 6 \end{cases}.$$

(b) Using (a), or otherwise, solve the following system of linear equations:

$$\begin{cases} x + y + z = 0 \\ 2x - y + 5z = 6 \\ x - y + \lambda z = 4 \end{cases}, \text{ where } \lambda \text{ is a constant.}$$

(2013-AL-P MATH 1 #07) (15 marks)

7. (a) Consider the system of linear equations in real variables x , y , z

$$(E) : \begin{cases} 3x - 2y + z = 0 \\ (2a+7)x - 5y + az = b \\ -x + ay - z = 1 \end{cases}, \text{ where } a , b \in \mathbf{R} .$$

(i) Find the range of values of a for which (E) has a unique solution, and solve (E) when (E) has a unique solution.

(ii) Suppose that $a = 1$. Find b for which (E) is consistent, and solve (E) when (E) is consistent.

(b) Consider the system of linear equations in real variables x , y , z

$$(F) : \begin{cases} 3x - 2y + z = 0 \\ -9x + 5y - z = -3 \\ -x + y - z = 1 \\ \lambda x + \mu y - 7z = 4\lambda \end{cases}, \text{ where } \lambda , \mu \in \mathbf{R} .$$

Find λ and μ for which (F) has infinitely many solutions.

(c) If the real solution of the system of linear equations $\begin{cases} 3x - 2y + z = 0 \\ 11x - 5y + 2z = \beta \\ x - 2y + z = -1 \end{cases}$ satisfies

$4x^2 - y^2 + z^2 = 1$, find β .

(2013-DSE-MATH-EP(M2) #09) (5 marks)

9. Consider the following system of linear equations in x , y and z

$$(E) : \begin{cases} x - ay + z = 2 \\ 2x + (1-2a)y + (2-b)z = a+4 \\ 3x + (1-3a)y + (3-ab)z = 4 \end{cases}, \text{ where } a \text{ and } b \text{ are real numbers.}$$

It is given that (E) has infinitely many solutions.

(a) Find the values of a and b .

(b) Solve (E) .

(2014-DSE-MATH-EP(M2) #09) (6 marks)

9. (a) Solve the system of linear equations $\begin{cases} x + y + z = 100 \\ x + 6y + 10z = 200 \end{cases}$.

(b) In a store, the prices of each of small, medium and large marbles are $\$0.5$, $\$3$ and $\$5$ respectively. Aubrey plans to spend all $\$100$ for exactly 100 marbles, which include m small marbles, n medium marbles and k large marbles.

Aubrey claims that there is only one set of combination of m , n and k . Do you agree? Explain your answer.

(2015-DSE-MATH-EP(M2) #05) (6 marks)

5. Solve the following systems of linear equations in real variables x , y and z :

(a) $\begin{cases} x + y + z = 2 \\ 2x + 3y - 3z = 4 \end{cases}$

(b) $\begin{cases} x + y + z = 2 \\ 2x + 3y - 3z = 4 \\ 3x + 2y + kz = 6 \end{cases}$

(2016-DSE-MATH-EP(M2) #11) (12 marks)

11. (a) Consider the system of linear equations in real variables x , y , z

$$(E) : \begin{cases} x + y - z = 3 \\ 4x + 6y + az = b \\ 5x + (1-a)y + (3a-1)z = b-1 \end{cases} \text{, where } a , b \in \mathbf{R} .$$

(i) Assume that (E) has a unique solution.

(1) Prove that $a \neq -2$ and $a \neq -12$.

(2) Solve (E) .

(ii) Assume that $a = -2$ and (E) is consistent.

(1) Find b .

(2) Solve (E) .

(b) Is there a real solution of the system of linear equations

$$\begin{cases} x + y - z = 3 \\ 2x + 3y - z = 7 \\ 5x + 3y - 7z = 13 \end{cases}$$

satisfying $x^2 + y^2 - 6z^2 > 14$? Explain your answer.

(2017-DSE-MATH-EP(M2) #05) (6 marks)

5. Consider the system of linear equations in real variables x , y , z

$$(E)\ :\ \begin{cases} x + 2y - z = 11 \\ 3x + 8y - 11z = 49 \\ 2x + 3y + hz = k \end{cases} \text{, where } h\ ,\ k \in \mathbf{R}\ .$$

(a) Assume that (E) has a unique solution.

(i) Find the range of values of h .

(ii) Express z in terms of h and k .

(b) Assume that (E) has infinitely many solutions. Solve (E) .

(2018-DSE-MATH-EP(M2) #11) (12 marks)

11. (a) Consider the system of linear equations in real variables x , y , z

$$(E)\ :\ \begin{cases} x + ay + 4(a+1)z = 18 \\ 2x + (a-1)y + 2(a-1)z = 20 \\ x - y - 12z = b \end{cases} \text{, where } a\ ,\ b \in \mathbf{R}\ .$$

(i) Assume that (E) has a unique solution.

(1) Find the range of values of a .

(2) Solve (E) .

(ii) Assume that $a = 3$ and (E) is consistent.

(1) Find b .

(2) Solve (E) .

(b) Consider the system of linear equations in real variables x , y , z

$$(F)\ :\ \begin{cases} x + 3y + 16z = 18 \\ x + y + 2z = 10 \\ x - y - 12z = s \\ 2x - 5y - 45z = t \end{cases} \text{, where } s\ ,\ t \in \mathbf{R}\ .$$

Assume that (F) is consistent. Find s and t .

(2019-DSE-MATH-EP(M2) #06) (7 marks)

6. Consider the system of linear equations in real variables x , y , z

$$(E)\ :\ \begin{cases} x - 2y - 2z = \beta \\ 5x + \alpha y + \alpha z = 5\beta \\ 7x + (\alpha-3)y + (2\alpha+1)z = 8\beta \end{cases} \text{, where } \alpha\ ,\ \beta \in \mathbf{R}\ .$$

(a) Assume that (E) has a unique solution.

(i) Find the range of values of α .

(ii) Express y in terms of α and β .

(b) Assume that $\alpha = -4$. If (E) is inconsistent, find the range of values of β .

(2020-DSE-MATH-EP(M2) #11) (13 marks)

11. (a) Consider the system of linear equations in real variables x , y , z

$$(E) : \begin{cases} x - y - 2z = 1 \\ x - 2y + hz = k \\ 4x + hy - 7z = 7 \end{cases} \text{, where } h , k \in \mathbf{R} .$$

(i) Assume that (E) has a unique solution.

(1) Prove that $h \neq -3$.

(2) Solve (E) .

(ii) Assume that $h = -3$ and (E) is consistent.

(1) Prove that $k = -2$.

(2) Solve (E) .

(b) Consider the system of linear equations in real variables x , y , z

$$(F) : \begin{cases} x - y - 2z = 1 \\ x - 2y + hz = -2 \\ 4x + hy - 7z = 7 \end{cases} \text{, where } h \in \mathbf{R} .$$

Someone claims that there are at least two values of h such that (F) has a real solution (x , y , z) satisfying $3x^2 + 4y^2 - 7z^2 = 1$. Do you agree? Explain your answer.

(2021-DSE-MATH-EP(M2) #08) (8 marks)

8. Consider the system of linear equations in real variables x , y , z

$$(E) : \begin{cases} x + (d-1)y + (d+3)z = 4-d \\ 2x + (d+2)y - z = 2d-5 \\ 3x + (d+4)y + 5z = 2 \end{cases} \text{, where } d \in \mathbf{R} .$$

It is given that (E) has infinitely many solutions

(a) Find d . Hence, solve (E) .

(b) Someone claims that (E) has a real solution (x, y, z) satisfying $xy + 2xz = 3$. Is the claim correct? Explain your answer.

ANSWERS

(1991-AL-P MATH 1 #03) (4 marks)

3. (a) $q = -1$

(b) $q = 1$

(1992-AL-P MATH 1 #01) (6 marks)

1. (a) $s = -9$, t can be any real number.

(b) When $s = -9$, $t = -6$, $\{(3+3m-5n, m, n) : m, n \in \mathbf{R}\}$;

When $s = -9$, $t \neq -6$, $\{(3-5n, 0, n) : n \in \mathbf{R}\}$.

(1993-AL-P MATH 1 #03) (6 marks)

1 (c) $\{(m, n, 3-m-n) : m, n \in \mathbf{R}\}$

(1994-AL-P MATH 1 #02) (6 marks)

1. $\lambda = 0$, $\{(-t, t, t) : t \in \mathbf{R}\}$

(1994-AL-P MATH 1 #09) (15 marks)

9. (b) (i) $k = -2$, 2 or 4

(ii) $k = -2$, (IV) is inconsistent; $k = 2$, $x = -\dfrac{1}{4}$, $y = \dfrac{1}{4}$; $k = 4$, $x = -\dfrac{2}{9}$, $y = \dfrac{-5}{9}$.

(iii) $k = -2$, $\{(t, t, 0) : t \in \mathbf{R}\}$; $k = 2$, $\{(t, t, -4t) : t \in \mathbf{R}\}$; $k = 4$, $\{(2t, 5t, -9t) : t \in \mathbf{R}\}$.

(1995-AL-P MATH 1 #09) (15 marks)

9. (a) $\left\{\left(\dfrac{-k}{h+3}, \dfrac{k}{h+3}, -k\right)\right\}$

(b) (i) k can be all values, $\left\{\left(\dfrac{3k+5t}{12}, \dfrac{3k+t}{12}, t\right) : t \in \mathbf{R}\right\}$

(ii) $k = 0$, $\{(-t, t, 0) : t \in \mathbf{R}\}$

(c) For $h^2 \neq 9$, $k = \dfrac{2}{3}$, $h = -2$, $x = -\dfrac{2}{3}$, $y = \dfrac{2}{3}$, $z = -\dfrac{2}{3}$;

$k = \dfrac{2}{3}$, $h = -5$, $x = \dfrac{1}{3}$, $y = -\dfrac{1}{3}$, $z = -\dfrac{2}{3}$.

For $h = 3$, $k = \dfrac{2}{3}$, $t = \dfrac{17}{27}$, $x = \dfrac{17}{27}$, $y = \dfrac{7}{27}$, $z = \dfrac{10}{9}$.

(1996-AL-P MATH 1 #05) (6 marks)

1. (a) $X = \dfrac{1}{2}(b+c-a)$, $Y = \dfrac{1}{2}(a+c-b)$, $Z = \dfrac{1}{2}(a+b-c)$

(b) $x = \pm\sqrt{\dfrac{(a+b-c)(a+c-b)}{2(b+c-a)}}$, $y = \pm\sqrt{\dfrac{(a+b-c)(b+c-a)}{2(a+c-b)}}$, $z = \pm\sqrt{\dfrac{(a+c-b)(b+c-a)}{2(a+b-c)}}$

(1996-AL-P MATH 1 #09) (15 marks)

9. (a) $\{(5-5t, 3t-1, t) : t \in \mathbf{R}\}$

(b) $\left\{(0,2,1), \left(\frac{50}{17}, \frac{4}{17}, \frac{7}{17}\right)\right\}$

(c) If $a \neq \frac{4}{5}$, then $\lambda \in \mathbf{R}$; or If $a = \frac{4}{5}$, then $\lambda = 3$

(d) If $\lambda = 3$, then $a \in \mathbf{R}$; or $a = \frac{17\lambda - 11}{50}$

(1997-AL-P MATH 1 #03) (6 marks)

3. (b) When $k = -4$, $\lambda = 2$, $\{(2t, t, 2t) : t \in \mathbf{R}\}$; When $k = 0$, $\lambda = 0$, $\{(0, t, 0) : t \in \mathbf{R}\}$.

(1997-AL-P MATH 1 #08) (15 marks)

8. (a) $a = -2$, $\{(4t, 3t, t) : t \in \mathbf{R}\}$; $a = 3$, $\{(t, -t, t) : t \in \mathbf{R}\}$; $a = 5$, $\{(t, -t, 2t) : t \in \mathbf{R}\}$.

(b) $a = -2$, $b = -1$, $\{(2+4t, 4+3t, t) : t \in \mathbf{R}\}$

(c) $x = 6$, $y = 7$, $z = 1$

(1998-AL-P MATH 1 #01) (6 marks)

1. (a) $k = -4$ or 1

(b) $k = -4$, $\{(3t, -8t, t) : t \in \mathbf{R}\}$; $k = 1$, $\{(-2t, 2t, t) : t \in \mathbf{R}\}$.

(1998-AL-P MATH 1 #08) (15 marks)

8. (a) $x = \frac{-(a-b)}{(1-a)(2+a)}$, $y = \frac{-(a-b)}{(1-a)(2+a)}$, $z = \frac{2-b-ab}{(1-a)(2+a)b}$

(b) (i) $b = -2$, $\{(-1-2t, -1-2t, t) : t \in \mathbf{R}\}$

(ii) $b = 1$, $\{(1-s-t, s, t) : s, t \in \mathbf{R}\}$

(c) Inconsistent

(1999-AL-P MATH 1 #01) (6 marks)

1. (a) $\lambda = -2$ or 1

(b) $\lambda = -2$, $\{(-t, -t, t) : t \in \mathbf{R}\}$; $\lambda = 1$, $\{(t-s, s, t) : s, t \in \mathbf{R}\}$.

(1999-AL-P MATH 1 #08) (15 marks)

8. (b) (i) $x = 4$, $y = \frac{\lambda - 3}{\lambda + 1}$, $z = \frac{-4}{\lambda + 1}$

(ii) Inconsistent

(iii) $\{(2-t, -1, t) : t \in \mathbf{R}\}$

(c) $a \neq c$, $b - 2c + d = 0$

(2000-AL-P MATH 1 #08) (15 marks)

8. (a) $x=\dfrac{c+a}{2}$, $y=b-2a$, $z=\dfrac{c-2b+3a}{2}$

(b) $\lambda=-2$, $b-2a=0$; $a=-1$, $b=-2$ and $c=3$, $\left\{\left(t-\dfrac{1}{5},t,\dfrac{4}{5}\right):t\in\mathbf{R}\right\}$

(c) (i) $\mu=2$, $a=-1$, $b=-2$ and $c=3$, $\left\{\left(t-\dfrac{1}{5},t,\dfrac{4}{5}\right):t\in\mathbf{R}\right\}$

(ii) $\mu\neq 2$, inconsistent

(2001-AL-P MATH 1 #09) (15 marks)

9. (b) (i) $x=\dfrac{3(\lambda+k)}{2\lambda(\lambda-2)}$, $y=\dfrac{2\lambda k+\lambda-3k}{2\lambda(\lambda-2)}$, $z=\dfrac{(\lambda+3)(\lambda+k)}{-2\lambda(\lambda-2)}$

(ii) $k=0$, $\{(-t,t-1,t):t\in\mathbf{R}\}$

(iii) $k=-2$, $\left\{\left(-\dfrac{3t}{5},-\dfrac{t+5}{5},t\right):t\in\mathbf{R}\right\}$

(c) $\dfrac{-1-\sqrt{3}}{2}\le p\le\dfrac{-1+\sqrt{3}}{2}$

(2002-AL-P MATH 1 #08) (15 marks)

8. (a) (i) $x=\dfrac{-2b}{a+1}$, $y=\dfrac{-b(a-1)}{a+1}$, $z=\dfrac{2b}{a+1}$

(ii) (1) b can be any number, $\{(2b-3t,b-t,t):t\in\mathbf{R}\}$

(2) $b=0$, $\{(-t,t,t):t\in\mathbf{R}\}$

(b) (i) $b=-1$

When $a=2$, $x=\dfrac{2}{3}$, $y=\dfrac{1}{3}$, $z=\dfrac{-2}{3}$; $a=-5$, $x=-\dfrac{1}{2}$, $y=\dfrac{3}{2}$, $z=\dfrac{1}{2}$.

(ii) $b=-1$

When $a=1$, $x=\dfrac{1}{4}$, $y=\dfrac{-1}{4}$, $z=\dfrac{-3}{4}$

(2003-AL-P MATH 1 #07) (15 marks)

7. (a) (i) $a\neq 1$, $a\neq\dfrac{-1}{2}$ and $a\neq -4$

$x=\dfrac{-2a(4a+1)}{(2a+1)(a+4)}$, $y=\dfrac{4a}{(2a+1)(a+4)}$, $z=\dfrac{-2a}{a+4}$

(ii) (1) $\left\{\left(\dfrac{-2}{3},\dfrac{2+3t}{3},t\right):t\in\mathbf{R}\right\}$

(2) Inconsistent

(b) least value = 9 , $x=\dfrac{-2}{3}$, $y=\dfrac{-1}{3}$, $z=-1$

(2004-AL-P MATH 1 #07) (15 marks)

7. (a) (i) $x=\dfrac{b-2}{a-2}$, $y=\dfrac{b-a}{2(a-2)}$, $z=\dfrac{a-b}{2(a-2)}$

(ii) (1) $b = 2$, $\{(1-2t,\ -t, t) : t \in \mathbf{R}\}$

(2) b can be any value, $\left\{\left(\frac{2b+1-10t}{9}, \frac{4-b-13t}{9}, t\right) : t \in \mathbf{R}\right\}$

(b) $\frac{-1}{6} < k < 0$

(2005-AL-P MATH 1 #07) (15 marks)

7. (a) (i) $a \neq -2$, $a \neq 1$ and $a \neq 3$

$x = \frac{b\left(a^2-6a-3\right)}{(a-1)(a-3)}$, $y = \frac{4b}{(a-1)(a-3)}$, $z = \frac{-2b}{a-1}$

(ii) (1) $b = 0$, $\{(-2t, t, t) : t \in \mathbf{R}\}$

(2) b can be any value, $\left\{\left(\frac{b+5t}{5}, \frac{-2b+5t}{5}, t\right) : t \in \mathbf{R}\right\}$

(b) $\frac{-15}{7} \leq b \leq \frac{15}{2}$

(2006-AL-P MATH 1 #07) (15 marks)

7. (a) $a \neq 1$ and $b \neq 0$

$x = \frac{2ab-4b+1}{(a-1)b}$, $y = \frac{2b-1}{(a-1)b}$, $z = \frac{1}{b}$

(b) (i) $b = \frac{1}{2}$, $\{(2-t, t, 2) : t \in \mathbf{R}\}$

(2007-AL-P MATH 1 #07) (15 marks)

7. (a) (i) $a \neq -2$ and $a \neq -4$

$x = \frac{a^2+6a+3b+5}{(a+2)(a+4)}$, $y = \frac{b-1}{(a+2)(a+4)}$, $z = \frac{(a+6)(b-1)}{(a+2)(a+4)}$

(ii) $b = 1$, $\left\{\left(\frac{4+3t}{4}, \frac{t}{4}, t\right) : t \in \mathbf{R}\right\}$

(c) $t = -1$, $x = -2$, $y = -1$, $z = -4$

(2008-AL-P MATH 1 #07) (15 marks)

7. (a) (i) $x = \dfrac{-a^2b - 3ab - a + 2}{4 - a^2}$, $y = \dfrac{2b}{a - 2}$, $z = \dfrac{3ab - a + 8b + 2}{4 - a^2}$

(ii) (1) $b = 0$, $\left\{\left(\dfrac{3 - 5t}{7}, \dfrac{1 - 4t}{7}, t\right) : t \in \mathbf{R}\right\}$

(2) $b = -2$, $\{(1 + t, 1, t) : t \in \mathbf{R}\}$

(b) $\dfrac{3}{2}$

(2009-AL-P MATH 1 #07) (15 marks)

7. (a) (i) $\lambda \neq -1$ and $\lambda \neq 3$

$x = \dfrac{a\lambda - 2\lambda - 3}{(\lambda + 1)(\lambda - 3)}$, $y = \dfrac{3(a - \lambda)}{(\lambda + 1)(\lambda - 3)}$, $z = \dfrac{2\lambda(\lambda - a)}{(\lambda + 1)(\lambda - 3)}$

(ii) $a = -1$, $\left\{\left(\dfrac{1 - t}{2}, \dfrac{3t}{2}, t\right) : t \in \mathbf{R}\right\}$

(b) $\lambda = -2$, $a = 3$, inconsistent

(c) $\lambda = -1$, $a = -1$,

when $t = -2$, $x = 3$, $y = -6$, $z = -4$; when $t = 3$, $x = -2$, $y = 9$, $z = 6$.

(2010-AL-P MATH 1 #07) (15 marks)

7. (a) (i) $a \neq -2$ and $a \neq 2$

$x = \dfrac{2(16 - 2b - a)}{4 - a^2}$, $y = \dfrac{-22 - 6a + 4b + ab - 2a^2}{4 - a^2}$, $z = \dfrac{-2 + 8a - ab}{4 - a^2}$

(ii) $b = 7$, $\{(1 + 2t, 1 - 3t, t) : t \in \mathbf{R}\}$

(b) $\lambda = 9$, $\{(-1 - 2t, t + 3, t) : t \in \mathbf{R}\}$, $\mu = 44$

(c) $a = \dfrac{2}{3}$, $b = 5$, inconsistent

(2011-AL-P MATH 1 #07) (15 marks)

7. (a) (i) (2) $\{(2t, -2t, t) : t \in \mathbf{R}\}$

(ii) (1) $\lambda \neq -2$, $\lambda \neq 0$ and $\lambda \neq 1$

(2) $x = \dfrac{\left(\lambda^2 + 2\lambda\mu + \lambda - 4\right)\mu}{\lambda(\lambda - 1)(\lambda + 2)}$, $y = \dfrac{(\lambda + 1)(1 - \lambda\mu)\mu}{\lambda(\lambda - 1)(\lambda + 2)}$, $z = \dfrac{(\lambda\mu - 1)\mu}{\lambda(\lambda - 1)(\lambda + 2)}$

(3) When $\lambda = 1$, $\mu = 1$; when $\lambda = -2$, $\mu = \dfrac{-1}{2}$.

(b) $\lambda = 1$, $\mu = 1$, there is no real solution.

(SP-DSE-MATH-EP(M2) #07) (5 marks)

7. $\{(3 - t, t - 1, t) : t \in \mathbf{R}\}$

(PP-DSE-MATH-EP(M2) #02) (4 marks)

2. $k = 19$ or 2

(2012-AL-P MATH 1 #07) (15 marks)

7. (a) (i) $x = \dfrac{2ab + 11a + 3b + 6}{2(a+1)(a+3)}$, $y = \dfrac{3a - b}{2(a+3)}$, $z = \dfrac{b - 7a - 12}{2(a+1)(a+3)}$

(ii) $b = -9$, $\{(7 - 3t, 2t - 2, t) : t \in \mathbf{R}\}$

(b) $a = -\dfrac{4}{3}$, $b = -4$, inconsistent

(c) $a = -3$, $b = -9$, least value $= -56$

(2012-DSE-MATH-EP(M2) #08) (5 marks)

8. (a) $\{(2 - 2t, t - 2, t) : t \in \mathbf{R}\}$

(b) When $\lambda \neq 3$, $x = 2$, $y = -2$, $z = 0$; when $\lambda = 3$, $\{(2 - 2t, t - 2, t) : t \in \mathbf{R}\}$.

(2013-AL-P MATH 1 #07) (15 marks)

7. (a) (i) $a \neq 1$ and $a \neq 4$

$x = \dfrac{2a + 2b - ab - 5}{(a-1)(a-4)}$, $y = \dfrac{a + 2b - 7}{(a-1)(a-4)}$, $z = \dfrac{1 + 3ab - 4a - 2b}{(a-1)(a-4)}$

(ii) $b = 3$, $\{(t, 2t - 1, t - 2) : t \in \mathbf{R}\}$

(b) $a = 1$, $b = 3$, $\lambda = 3$, $\mu = 2$

(c) $\beta = 1$ or 2

(2013-DSE-MATH-EP(M2) #09) (5 marks)

9. (a) $a = 2$ or $b = 0$

(b) $\{(6 - t, -2, t) : t \in \mathbf{R}\}$

(2014-DSE-MATH-EP(M2) #09) (6 marks)

9. (a) $\left\{\left(80 + \dfrac{4t}{5}, 20 - \dfrac{9t}{5}, t\right) : t \in \mathbf{R}\right\}$

(b) $t = 0$, 5 or 10 . Disagreed.

(2015-DSE-MATH-EP(M2) #05) (6 marks)

5. (a) $\{(2 - 6t, 5t, t) : t \in \mathbf{R}\}$

(b) $k = 8$, $\{(2 - 6t, 5t, t) : t \in \mathbf{R}\}$

$k \neq 8$, $t = 0$, $x = 2$, $y = 0$, $z = 0$

(2016-DSE-MATH-EP(M2) #11) (12 marks)

11. (a) (i) (2) $x = \dfrac{3a^2 - ab + 50a + 6b - 24}{(a+2)(a+12)}$, $y = \dfrac{2(ab - 10a + 8)}{(a+2)(a+12)}$, $z = \dfrac{ab - 12a + 6b - 80}{(a+2)(a+12)}$

(ii) (1) $b = 14$

(2) $\{(2t+2, 1-t, t) : t \in \mathbf{R}\}$

(b) $a = -2$, $b = 14$, Greatest value is 14 , no real solution.

(2017-DSE-MATH-EP(M2) #05) (6 marks)

5. (a) (i) $h \neq 2$

(ii) $z = \dfrac{k - 14}{h - 2}$

(b) $h = 2$, $k = 14$

$\{(-7t - 5, 4t + 8, t) : t \in \mathbf{R}\}$

(2018-DSE-MATH-EP(M2) #11) (12 marks)

11. (a) (i) (1) $a \neq 3$ and $a \neq -1$

(2) $x = \dfrac{a^2b + ab + 10a - 2b - 50}{(a+1)(a-3)}$, $y = \dfrac{-3ab + 22a - 5b - 38}{(a+1)(a-3)}$, $z = \dfrac{b-2}{2(a-3)}$

(ii) (1) $b = 2$

(2) $\{(5m + 6, -7m + 4, m) : m \in \mathbf{R}\}$

(b) $s = 2$, $t = -8$

(2019-DSE-MATH-EP(M2) #06) (7 marks)

6. (a) (i) $\alpha \neq -4$ and $\alpha \neq -10$

(ii) $y = -\dfrac{\beta}{\alpha + 4}$

(b) $\beta \neq 0$

(2020-DSE-MATH-EP(M2) #11) (13 marks)

11. (a) (i) (2) $x = \dfrac{h^2 + 2hk + 7h + 7k + 14}{(h+3)^2}$, $y = \dfrac{2h - k + 7}{(h+3)^2}$, $z = \dfrac{hk - h + 4k - 1}{(h+3)^2}$

(ii) (2) $\{(t+4, 3-t, t) : t \in \mathbf{R}\}$

(b) The claim is agreed.

(2021-DSE-MATH-EP(M2) #08) (8 marks)

8. (a) $\{(3 - 32t, 13t - 1, t) : t \in \mathbf{R}\}$

(b) The claim is not correct.