1. FOUNDATION KNOWLEDGE AREA

1. Mathematical Induction

(1981-HL-GEN MATHS #02) (6 marks)

2. (a) Prove, by mathematical induction, that for any positive integer n ,

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{1}{4}n^2(n+1)^2.$$

(1988-HL-GEN MATHS #07) (8 marks) (Modified)

7. (b) Let $A_n = 1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{n-1}n^2$

and $B_n = 1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}$

where n is a positive integer.

Show, by mathematical induction, that $A_n = (-1)^{n-1}B_n$ for all positive integers n .

Hence, or otherwise, find $\displaystyle\sum_{n=1}^{2m} A_n$ and $\displaystyle\sum_{n=1}^{2m+1} A_n$.

(1990-HL-GEN MATHS #05) (8 marks)

5. (a) (i) Prove by mathematical induction that for any positive integer n ,

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2.$$

(ii) Find $1^3 - 2^3 + 3^3 - 4^3 + \ldots + (-1)^{r+1}r^3 + \ldots - (2n)^3$.

(1991-CE-A MATH 2 #07) (8 marks)

7. (a) Prove, by mathematical induction, that

$$1^2 + 2^2 + \ldots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

for all positive integers n .

(b) Using the formula in (a), find the sum of

$$1 \times 2 + 2 \times 3 + \ldots + n(n+1).$$

(1992-CE-A MATH 2 #01) (5 marks)

1. Prove, by mathematical induction, that

$$1 \times 2 + 2 \times 5 + 3 \times 8 + \ldots + n(3n-1) = n^2(n+1)$$

for all positive integers n .

(1993-CE-A MATH 2 #01) (5 marks)

1. Prove that

$$1^2 \times 2 + 2^2 \times 3 + \ldots + n^2(n+1) = \frac{n(n+1)(n+2)(3n+1)}{12}$$

for any positive integer n .

(1994-CE-A MATH 2 #05) (5 marks)

5. Prove that

$$\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\ldots+\frac{2n-1}{2^n}=3-\frac{2n+3}{2^n}$$

for any positive integer n .

(1997-CE-A MATH 2 #07) (6 marks)

7. Let $T_n=(n^2+1)(n!)$ for any positive integer n . Prove, by mathematical induction, that

$$T_1+T_2+\ldots+T_n=n\left[(n+1)!\right]$$

for any positive integer n .

(Note: $n!=n(n-1)(n-2)\ldots 3\times 2\times 1$)

(1998-CE-A MATH 2 #03) (5 marks)

3. Prove, by mathematical induction, that

$$1\times 2+2\times 3+2^2\times 4+\ldots+2^{n-1}(n+1)=2^n(n)$$

for all positive integers n .

(2000-CE-A MATH 2 #04) (6 marks)

4. Prove, by mathematical induction, that

$$1^2-2^2+3^2-4^2+\ldots+(-1)^{n-1}n^2=(-1)^{n-1}\frac{n(n+1)}{2}$$

for all positive integers n .

(2001-CE-A MATH #12) (8 marks)

12. Prove, by mathematical induction, that

$$1\times 2+2\times 3+3\times 4+\ldots+n(n+1)=\frac{1}{3}n(n+1)(n+2)$$

for all positive integers n .

Hence evaluate $1\times 3+2\times 4+3\times 5+\ldots+50\times 52$.

(2002-CE-A MATH #12) (8 marks)

12. (a) Prove, by mathematical induction, that

$$2(2)+3(2^2)+4(2^3)+\ldots+(n+1)(2^n)=n(2^{n+1})$$

for all positive integers n .

(b) Show that

$$1(2)+2(2^2)+3(2^3)+\ldots+98(2^{98})=97(2^{99})+2 .$$

(2003-CE-A MATH #07) (5 marks)

7. Prove, by mathematical induction, that

$$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\ldots+\frac{n}{2^n}=2-\frac{n+2}{2^n}$$

for all positive integers n .

(2005-CE-A MATH #08) (5 marks)

8. Prove, by mathematical induction, that

$$\frac{1\times 2}{2\times 3}+\frac{2\times 2^2}{3\times 4}+\frac{3\times 2^3}{4\times 5}+\frac{n\times 2^n}{(n+1)(n+2)}=\frac{2^{n+1}}{n+2}-1$$

for all positive integers n .

(2007-CE-A MATH #05) (5 marks)

5. Let $a\neq 0$ and $a\neq 1$. Prove by mathematical induction that

$$\frac{1}{a-1}-\frac{1}{a}-\frac{1}{a^2}-\ldots-\frac{1}{a^n}=\frac{1}{a^n(a-1)}$$

for all positive integers n .

(2008-CE-A MATH #05) (5 marks)

5. Prove, by mathematical induction, that

$$1^3+2^3+3^3+\ldots+n^3=\frac{1}{4}n^2(n+1)^2$$

for all positive integers n .

(2009-CE-A MATH #05) (5 marks)

5. Prove, by mathematical induction, that

$$1\times 4+2\times 5+3\times 6+\ldots+n(n+3)=\frac{1}{3}n(n+1)(n+5)$$

for all positive integers n .

(2012-DSE-MATH-EP(M2) #03) (5 marks)

3. Prove, by mathematical induction, that for all positive integer n ,

$$1\times 2+2\times 5+3\times 8+\ldots+n(3n-1)=n^2(n+1).$$

(2013-DSE-MATH-EP(M2) #03) (5 marks)

3. Prove, by mathematical induction, that for all positive integers n ,

$$1+\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\ldots+\frac{1}{(3n-2)\times(3n+1)}=\frac{4n+1}{3n+1}.$$

(2016-DSE-MATH-EP(M2) #05) (6 marks)

5. (a) Using mathematical induction, prove that $\sum_{k=1}^{n}(-1)^k k^2=\frac{(-1)^n n(n+1)}{2}$ for all positive integers n .

(b) Using (a), evaluate $\sum_{k=3}^{333}(-1)^{k+1}k^2$.

(2018-DSE-MATH-EP(M2) #06) (7 marks)

6. (a) Using mathematical induction, prove that $\sum_{k=1}^{n} k(k+4) = \frac{n(n+1)(2n+13)}{6}$ for all positive integers n .

   (b) Using (a), evaluate $\sum_{k=333}^{555} \left(\frac{k}{112}\right)\left(\frac{k+4}{223}\right)$.

(2019-DSE-MATH-EP(M2) #05) (7 marks)

5. (a) Using mathematical induction, prove that $\sum_{k=n}^{2n} \frac{1}{k(k+1)} = \frac{n+1}{n(2n+1)}$ for all positive integers n .

   (b) Using (a), evaluate $\sum_{k=50}^{200} \frac{1}{k(k+1)}$.

(2020-DSE-MATH-EP(M2) #05) (7 marks)

5. (a) Using mathematical induction, prove that $\sum_{k=1}^{n} \frac{1}{k(k+1)(k+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$ for all positive integers n .

   (b) Using (a), evaluate $\sum_{k=4}^{123} \frac{50}{k(k+1)(k+2)}$.

(2021-DSE-MATH-EP(M2) #02) (5 marks)

2. Using mathematical induction, prove that $\sum_{k=1}^{n} \left(3k^5 + k^3\right) = \frac{n^3(n+1)^3}{2}$ for all positive integers n .

ANSWERS

(1991-CE-A MATH 2 #07)

7. (b) $\frac{1}{3}n(n+1)(n+2)$

(2001-CE-A MATH #12)

12. 45 475

(2016-DSE-MATH-EP(M2) #05)

5. (b) 55 614

(2018-DSE-MATH-EP(M2) #06)

6. (b) 1 813

(2019-DSE-MATH-EP(M2) #05)

5. (b) $\frac{151}{10050}$

(2020-DSE-MATH-EP(M2) #05)

5. (b) $\frac{387}{310}$