2011-CE
A MATH

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Candidate Number							

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2011

ADDITIONAL MATHEMATICS

Question-Answer Book

8.30 am – 11.00 am (2½ hours)

This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7 and 9.

2. This paper consists of Section A and Section B.

3. Answer **ALL** questions in Section A. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Answer any **FOUR** questions in Section B. Write your answers in the CE(A) answer book.

5. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** the book.

6. The Question-Answer book and the CE(A) answer book must be handed in separately at the end of the examination.

7. All working must be clearly shown.

8. Unless otherwise specified, numerical answers must be **exact**.

9. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.

10. The diagrams in this paper are not necessarily drawn to scale.

11. No extra time will be given to candidates for sticking the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2 \sin \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2 \cos \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2 \cos \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$
$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2 \sin \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$
$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Answers written in the margins will not be marked.

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Section A (62 marks)
Answer **ALL** questions in this section and write your answers in the spaces provided in this Question-Answer Book.

1. It is given that $(1+x+kx^2)^3 = 1+ax+bx^2 +$ terms involving higher powers of x.

   (a) Express b in terms of k.

   (b) If 1, a, b form a geometric sequence, find the value of k.

   (5 marks)

2. Prove that $5^n - 2^n$ is divisible by 3 for all positive integers n.

   (5 marks)

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3. Solve the following inequalities:

(a) $5x-3>2x+9$;

(b) $x(x-8)\leq 20$;

(c) $5x-3>2x+9$ or $x(x-8)\leq 20$.

(5 marks)

4. Let $A(3,0)$, $B(0,4)$ and P be three points on the rectangular coordinate plane such that the area of ΔABP is 6 . It is known that the locus of P is a pair of parallel straight lines.

(a) Find the equation of any one of these two lines.

(b) Find the distance between these two lines.

(5 marks)

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5. (a) Find $\int (2x+1)^2 \, dx$.

(b) The slope at any point (x, y) of a curve is given by $\frac{dy}{dx} = (2x+1)^2$. If the curve passes through the point $(-1, 0)$, find its equation.

(5 marks)

6. Find the equation of the normal to the curve $y = \frac{x^2+1}{x+1}$ at $x = 1$.

(5 marks)

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7. Solve $\sin 5x + \sin x = \cos 2x$ for $0° \le x \le 90°$.

(6 marks)

8. Let $f(x) = (x+2)(x^2+1)$.

(a) Find the maximum and minimum points of the graph of $y = f(x)$.

(b) Sketch the graph of $y = f(x)$.

(6 marks)

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9. It is given that $\overrightarrow{OA} = \mathbf{i} + \mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} + \mathbf{j}$.

(a) Find the value of $\cos \angle AOB$.

(b) Let $\overrightarrow{OC} = k\mathbf{i} + \mathbf{j}$. If OB is the angle bisector of $\angle AOC$, find the value of k .

(6 marks)

10. Let α and β be the roots of the quadratic equation $x^2 + (k+2)x + k = 0$, where k is real.

(a) Prove that α and β are real and distinct.

(b) If $\alpha = |\beta|$, find the value of β .

(7 marks)

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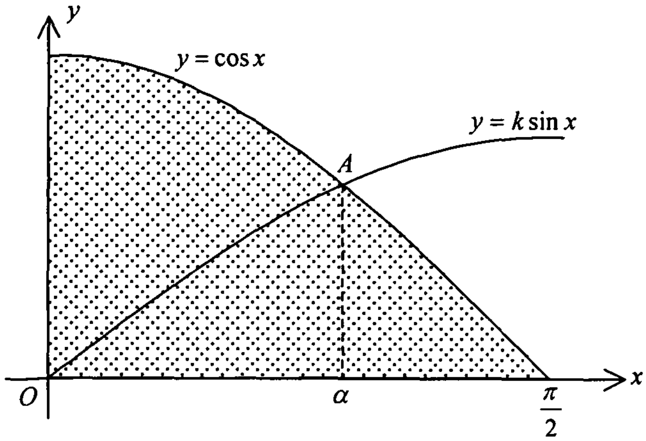

Figure 1

In Figure 1, the curves $y = \cos x$ and $y = k\sin x$, where $k > 0$, intersect at the point A . It is given that the x-coordinate of A is α , where $0 < \alpha < \frac{\pi}{2}$.

(a) Show that $\tan\alpha = \frac{1}{k}$.

(b) If the curve $y = k\sin x$ bisects the shaded region bounded by the curves $y = \cos x$, x-axis and y-axis, find the value of k .

(7 marks)

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Section B (48 marks)
Answer any **FOUR** questions in this section and write your answers in the CE(A) answer book.
Each question carries 12 marks.

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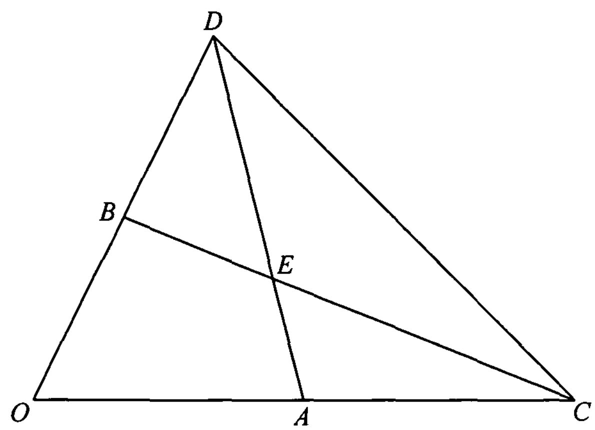

Figure 2

Figure 2 shows a triangle OCD . A and B are points on OC and OD respectively such that $OA:AC = OB:BD = 1:h$, where $h > 0$. AD and BC intersect at E such that $AE:ED = \mu:(1-\mu)$ and $BE:EC = \lambda:(1-\lambda)$, where $0 < \mu < 1$ and $0 < \lambda < 1$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) By considering $\overrightarrow{OE}$, show that $\mu = \lambda$.

(5 marks)

(b) F is a point on CD such that O, E and F are collinear. Show that OF is a median of ΔOCD .

(4 marks)

(c) Using the above results, show that in a triangle, the centroid divides every median in $2:1$.

(3 marks)

13.

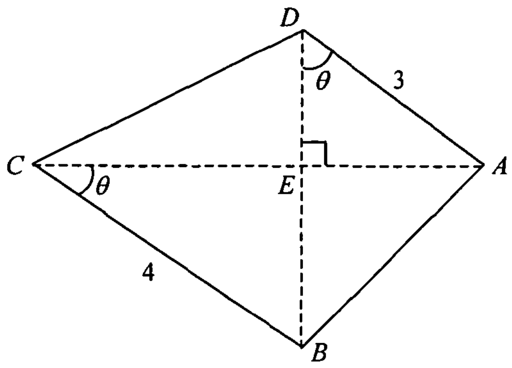

Figure 3

In Figure 3, $ABCD$ is a quadrilateral with diagonals AC and BD perpendicular to each other and intersecting at E. It is given that $AD = 3$, $BC = 4$ and $\angle ADE = \angle BCE = \theta$, where $0° < \theta < 90°$.

(a) (i) Show that $AB = 5\sin\theta$.

(ii) Express CD in terms of θ.

(3 marks)

(b)

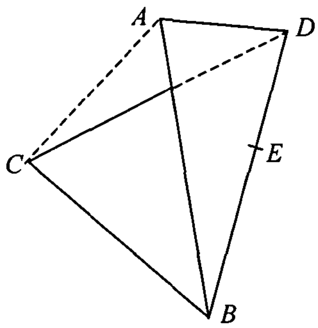

Figure 4

The quadrilateral is folded along BD as shown in Figure 4. Let the planes ABD and BCD be Π_1 and Π_2 respectively. Let $\angle ABC = \alpha$. It is given that

the angle between the lines AB and BC = the angle between the planes Π_1 and Π_2.

(i) By considering the length of AC, show that $\cos\alpha = \dfrac{4\sin\theta}{5 - 3\cos\theta}$.

(ii) Prove that α is acute.

(iii) Furthermore, it is given that

the angle between the line AB and Π_2 = the angle between the line AD and Π_2.

State with reasons whether the angle between the line AC and Π_2 is greater than, less than or equal to the angle between the line AB and Π_2.

(9 marks)

14. (a)

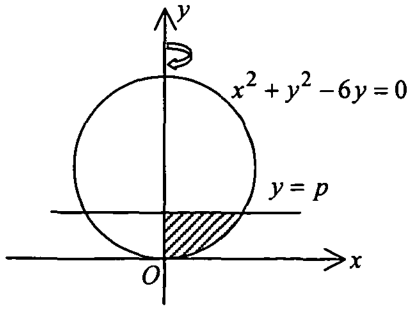

Figure 5

In Figure 5, a shaded region is bounded by the circle $x^2+y^2-6y=0$, the y-axis and the straight line $y=p$, where $0 \le p \le 6$. Show that the volume of the solid generated by revolving the shaded region about the y-axis is $\dfrac{\pi p^2(9-p)}{3}$.

(3 marks)

(b)

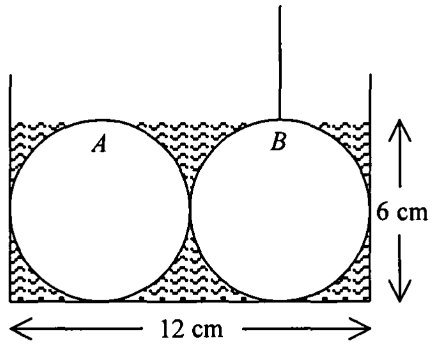

Figure 6

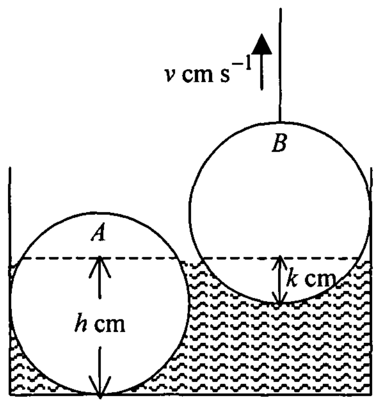

Figure 7

Two metal spheres, A and B, both of diameters 6 cm are placed inside a circular cylindrical container of base diameter 12 cm. The sphere B is attached by a wire. Water is poured into the container until the depth is 6 cm (see Figure 6).
Sphere B is then being pulled vertically out of the water. Let h cm and k cm respectively be the height of the parts of spheres A and B those are immersed in the water (see Figure 7).

(i) Find the volume of the water.

(ii) By considering the volume of water in Figure 7, or otherwise, prove that

$$k^3-9k^2+h^3-9h^2+108h-432=0 \ .$$

(iii) Suppose sphere B is being pulled at a uniform speed v cm s^{-1} and the depth of water is decreasing at a rate of 5 cm s^{-1} at the instant when $h=5$. Find the value of v .

(9 marks)

15. (a)

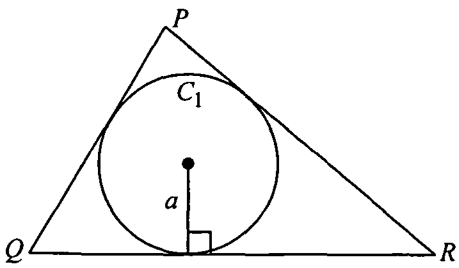

Figure 8

Figure 8 shows a triangle PQR with perimeter $2s$ and area A. A circle C_1 of radius a is inscribed in the triangle. Show that $a = \frac{A}{s}$.

(2 marks)

(b)

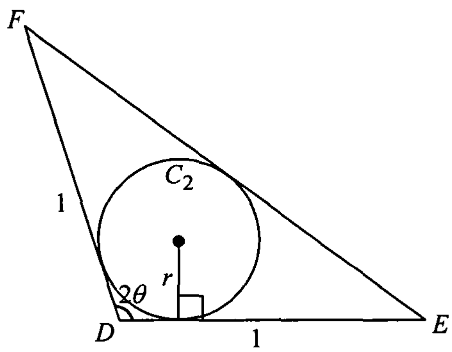

Figure 9

Figure 9 shows an isosceles triangle DEF with $DE = DF = 1$ and $\angle EDF = 2\theta$, where $0 < \theta < \frac{\pi}{2}$. A circle C_2 of radius r is inscribed in the triangle.

(i) Using (a), show that $r = \cos\theta - \frac{\cos\theta}{1+\sin\theta}$.

(ii) Find θ, correct to 3 decimal places, which maximizes the area of C_2.

(iii) Frankie studies the relationship between the area of C_2 and the perimeter of ΔDEF when $\frac{\pi}{12} \le \theta \le \frac{5\pi}{12}$. Frankie claims that:

"When the perimeter of ΔDEF is the least, the area of the inscribed circle is also the least."

Do you agree with Frankie? Explain your answer.

(10 marks)

16.

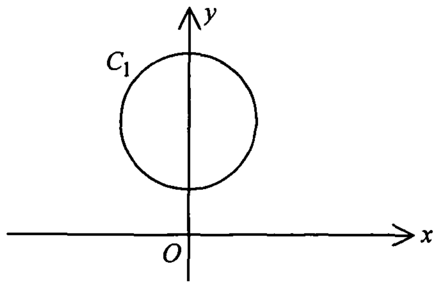

Figure 10

Figure 10 shows a circle $C_1 : x^2 + y^2 - 10y + 16 = 0$. Let Γ be the family of circles which touch the x-axis and C_1 externally, and S be the locus of the centres of the circles in Γ .

(a) Show that the equation of S is $y = \frac{1}{16}x^2 + 1$.

(4 marks)

(b)

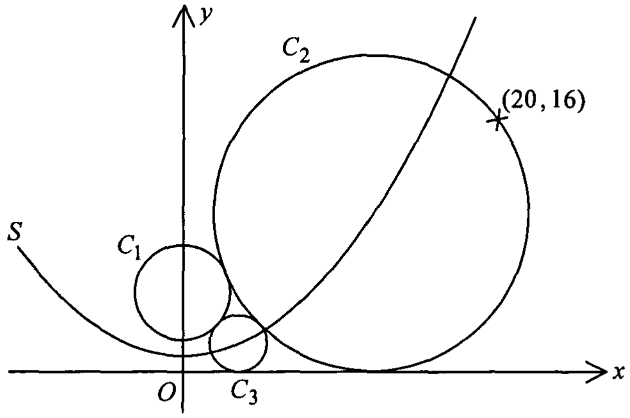

Figure 11

Let C_2 and C_3 be circles in Γ . It is given that C_2 passes through the point $(20, 16)$ and it touches C_3 externally. Suppose that both the centres of C_2 and C_3 lie in the first quadrant (see Figure 11).

(i) Find the equation of C_2 .

(ii) Without any algebraic manipulation, determine whether the following sentence is correct:

"The point of contact of C_2 and C_3 lies on S ."

(6 marks)

(c) Can we draw a circle satisfying all the following conditions?

- Its centre lies on S .
- It touches the x-axis.
- It touches C_1 internally.

Explain your answer.

(2 marks)

END OF PAPER