香 港 考 試 及 評 核 局

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2 0 1 1 年 香 港 中 學 會 考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2011

附加數學

ADDITIONAL MATHEMATICS

評 卷 參 考

MARKING SCHEME

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員 / 教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

6. In the marking scheme, marks are classified into the following three categories:

'M' marks	–	awarded for applying correct methods
'A' marks	–	awarded for the accuracy of the answers
Marks without 'M' or 'A'	–	awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.

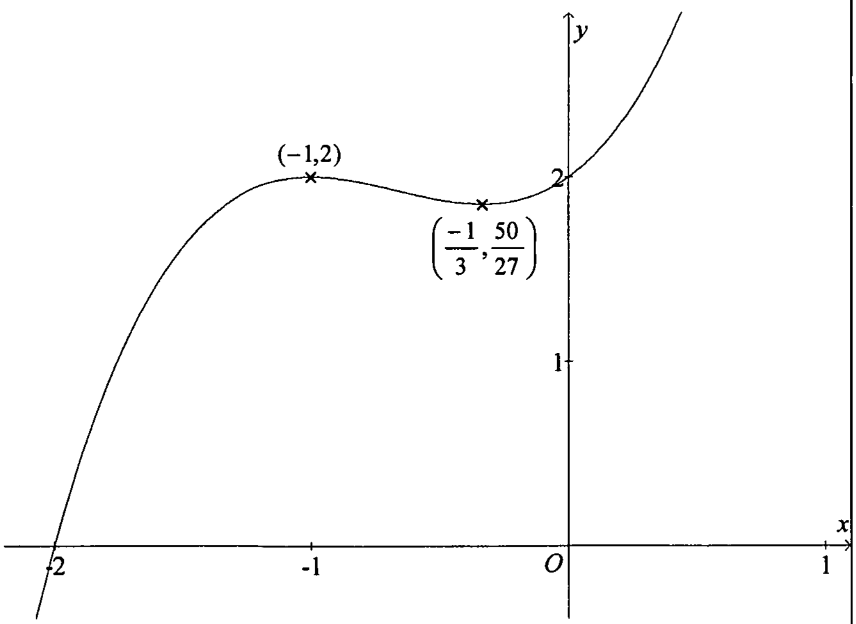
7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.

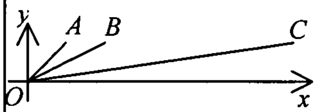
8. In section B, marks may be deducted for poor presentation (*pp*), including wrong / no unit. The symbol (*pp*-1) should be used to denote 1 mark deducted for *pp*.
   (a) At most deduct 1 mark for *pp* in EACH question in section B.
   (b) In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

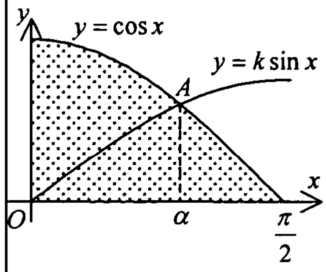
9. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted. For answers in Section B with an excess degree of accuracy, deduct 1 mark (*pp*). In any case, do not deduct any marks for excess degree of accuracy in those steps where candidates could not score any marks.

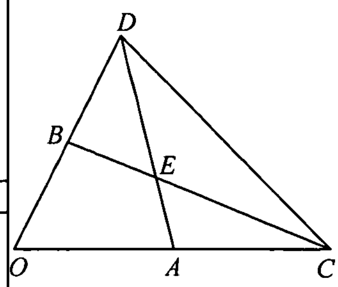
Solution	Marks	Remarks
1. (a) $(1+x+kx^2)^3 = 1+3(x+kx^2)+3(x+kx^2)^2+\cdots$	1M	
$= 1+3x+3(k+1)x^2+\cdots$	1A	
$\therefore\ b=3(k+1)$	1A	
(b) If 1, 3, $3(k+1)$ form a geometric sequence, then $\frac{3}{1}=\frac{3(k+1)}{3}$	1M	
i.e $k=2$	1A	
	(5)	
2. For $n=1$,		
$5^1-2^1=3$ which is divisible by 3.		
$\therefore$ the statement is true for $n=1$.	1	
Assume 5^k-2^k is divisible by 3, where k is a positive integer.	1	
i.e. let $5^k-2^k=3N$, where N is an integer.		Withdraw the last mark if "N is an integer" was omitted
$5^{k+1}-2^{k+1}=5(5^k)-2(2^k)$	1	
$=3(5^k)+2(5^k)-2(2^k)$		OR $=5(5^k)-5(2^k)+3(2^k)$
$=3(5^k)+2(3N)$ (by the assumption)		$=5(3N)+3(2^k)$
$=3(5^k+2N)$ which is divisible by 3	1	$=3(5N+2^k)$
Hence the statement is true for $n=k+1$.		
By the principle of mathematical induction, the statement is true for all positive integers n.	1	
	(5)	
3. (a) $5x-3>2x+9$		
$x>4$	1A	
(b) $x(x-8)\le 20$		
$x^2-8x-20\le 0$		
$(x+2)(x-10)\le 0$	1M	
$-2\le x\le 10$	1A	
(c) $5x-3>2x+9$ or $x(x-8)\le 20$		
$x>4$ or $-2\le x\le 10$		
$x\ge -2$	1M+1A	1M for operation of "or"
	(5)	
4. (a) Let P be (x,y).		
$\left\|\frac{1}{2}\begin{vmatrix}3&0\\x&y\\0&4\\3&0\end{vmatrix}\right\|=6$	1A	For $\frac{1}{2}\begin{vmatrix}3&0\\x&y\\0&4\\3&0\end{vmatrix}$
$4x+3y-12=\pm 12$	1M	For $4x+3y-12$
i.e. the equation of locus of P is $4x+3y=0$ or $4x+3y-24=0$.	1A	For either one
(b) The distance between the two lines $4x+3y=0$ and $4x+3y-24=0$		
$=\left\|\frac{0-(-24)}{\sqrt{4^2+3^2}}\right\|$	1M	

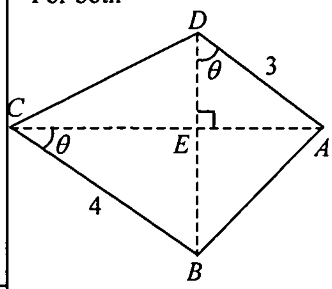
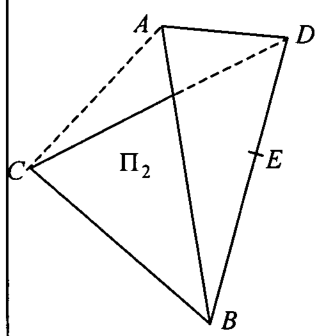
Solution	Marks	Remarks
Alternative Soultion (1) Since $(0,0)$ lies on $4x+3y=0$, the required distance $=\left\|\dfrac{4(0)+3(0)-24}{\sqrt{4^2+3^2}}\right\|$	1M	OR other suitable points
Alternative Soultion (2) Since $(3,4)$ lies on $4x+3y-24=0$, the required distance $=\left\|\dfrac{4(3)+3(4)}{\sqrt{4^2+3^2}}\right\|$	1M	OR other suitable points
$=\dfrac{24}{5}$	1A	
	(5)	
5. (a) $\int(2x+1)^2\,dx=\dfrac{(2x+1)^3}{3(2)}+C$	1M	For $\int x^n\,dx=\dfrac{x^{n+1}}{n+1}$
$=\dfrac{(2x+1)^3}{6}+C$	1A	
Alternative Solution $\int(2x+1)^2\,dx=\int(4x^2+4x+1)\,dx$	1M	
$=\dfrac{4}{3}x^3+2x^2+x+C$	1A	
(b) $\dfrac{dy}{dx}=(2x+1)^2$		
$y=\dfrac{(2x+1)^3}{6}+C$	1M	OR $y=\dfrac{4}{3}x^3+2x^2+x+C'$
Since the curve passes through $(-1,0)$, $0=\dfrac{[2(-1)+1]^3}{6}+C$.	1M	OR $0=\dfrac{-4}{3}+2-1+C'$
$C=\dfrac{1}{6}$		OR $C'=\dfrac{1}{3}$
Hence the equation of the curve is $y=\dfrac{(2x+1)^3+1}{6}$.	1A	OR $y=\dfrac{4}{3}x^3+2x^2+x+\dfrac{1}{3}$
	(5)	
6. $y=\dfrac{x^2+1}{x+1}$		
$\dfrac{dy}{dx}=\dfrac{(x+1)(2x)-(x^2+1)(1)}{(x+1)^2}$	1M	
$=\dfrac{x^2+2x-1}{(x+1)^2}$	1A	
$\therefore$ the slope of the tangent when $x=1$ is $\dfrac{1}{2}$		
$\therefore$ the slope of the normal when $x=1$ is -2	1M	
When $x=1$, $y=1$.		
Hence the equation of the normal when $x=1$ is $y-1=-2(x-1)$.	1M	
i.e. $2x+y-3=0$	1A	
	(5)	

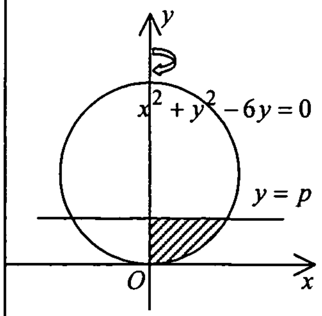
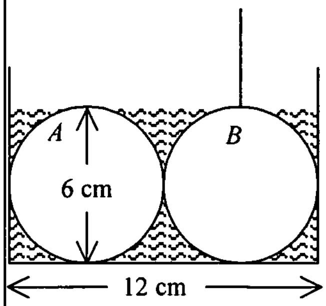
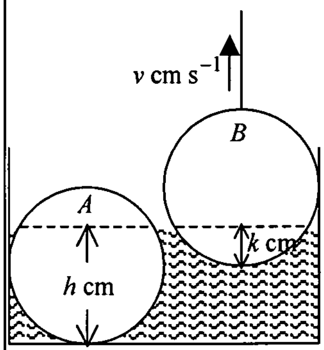
Solution	Marks	Remarks
7. $\sin 5x + \sin x = \cos 2x$		
$2\sin 3x \cos 2x - \cos 2x = 0$	1A	For $2\sin 3x \cos 2x$
$\cos 2x(2\sin 3x - 1) = 0$		
$\cos 2x = 0$ or $\sin 3x = \frac{1}{2}$	1M	1M for considering angle beyond 90°
$2x = 90^\circ$ or $(3x = 30^\circ$ or $150^\circ)$	1M+1A+1A	1A for 90°; 1A for 30°
$x = 10^\circ$ or 45° or 50°	1A	
	(6)	
8. (a) $f(x) = (x+2)(x^2+1)$		
$f'(x) = (x^2+1) + (x+2)(2x)$		
$= 3x^2 + 4x + 1$	1A	
$\therefore$ $f'(x) = 0$ when $x = -1$ or $\frac{-1}{3}$.	1M	For considering $f'(x) = 0$
$f''(x) = 6x + 4$		
$f''(-1) = -2 < 0$ and $f''\left(\frac{-1}{3}\right) = 2 > 0$	1M	OR by using sign test
Therefore $f(-1)$ is a maximum value and $f\left(\frac{-1}{3}\right)$ is a minimum value.		
Hence $(-1,2)$ is a maximum point and $\left(\frac{-1}{3}, \frac{50}{27}\right)$ is a minimum point.	1A	For both
(b) 	1M+1A	1M for shape 1A for all correct
	(6)	

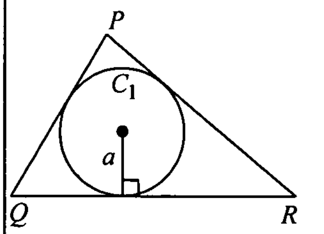
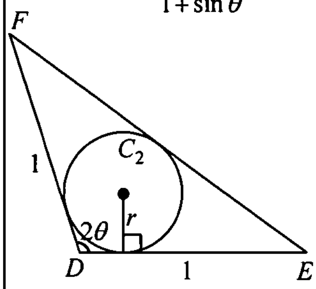
Solution	Marks	Remarks
9. (a) $\overrightarrow{OA}\cdot\overrightarrow{OB}=\left\|\overrightarrow{OA}\right\|\left\|\overrightarrow{OB}\right\|\cos\angle AOB$	1M	
$(\mathrm{i}+\mathrm{j})\cdot(2\mathrm{i}+\mathrm{j})=\sqrt{1^2+1^2}\cdot\sqrt{2^2+1^2}\cos\angle AOB$	1M	For magnitude of vector
$2+1=\sqrt{2}\cdot\sqrt{5}\cos\angle AOB$		
$\cos\angle AOB=\frac{3}{\sqrt{10}}$	1A	
(b) $\overrightarrow{OB}\cdot\overrightarrow{OC}=\left\|\overrightarrow{OB}\right\|\left\|\overrightarrow{OC}\right\|\cos\angle BOC$		
$(2\mathrm{i}+\mathrm{j})\cdot(k\mathrm{i}+\mathrm{j})=\sqrt{2^2+1^2}\cdot\sqrt{k^2+1^2}\cos\angle AOB$ (since $\angle BOC=\angle AOB$)	1M	For dot product
Alternative Solution $BC^2=OB^2+OC^2-2(OB)(OC)\cos\angle BOC$ $(k-2)^2=(2^2+1^2)+(k^2+1^2)-2\sqrt{2^2+1^2}\sqrt{k^2+1^2}\cos\angle BOC$ $k^2-4k+4=k^2+6-2\sqrt{5}\sqrt{k^2+1}\cos\angle AOB$ (since $\angle BOC=\angle AOB$)	1M	For cosine law
$2k+1=\sqrt{5}\cdot\sqrt{k^2+1}\cdot\frac{3}{\sqrt{10}}$	1M	For using (a)
$2(4k^2+4k+1)=9(k^2+1)$		
$k^2-8k+7=0$		
$k=7$ or 1 rejected	1A	
	(6)	
10. (a) $\Delta=(k+2)^2-4(1)(k)$	1M	
$=k^2+4$		
>0	1M	
Hence α and β are real and distinct.	1	
(b) $\alpha=\|\beta\|$		
$\alpha=\beta$ for $\beta\ge 0$ (rejected by (a)) or $\alpha=-\beta$ for $\beta<0$	1M	For $\alpha=\pm\beta$
$\therefore\ \alpha+\beta=0$		
$-(k+2)=0$	1M	
$k=-2$	1A	
Hence the equation becomes $x^2-2=0$.		
$x=\pm\sqrt{2}$		
Since $\alpha>0$, we have $\beta<0$ and hence $\beta=-\sqrt{2}$.	1A	
	(7)	

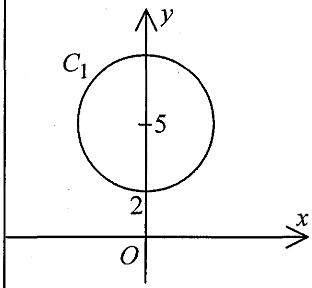
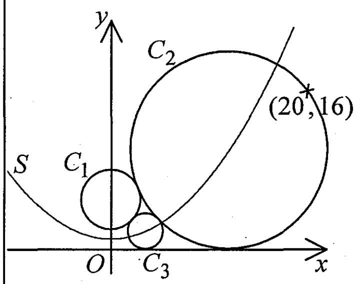
Solution	Marks	Remarks
11. (a) $\because$ $y=\cos x$ and $y=k\sin x$ intersect at $x=\alpha$		
$\therefore$ $\cos\alpha = k\sin\alpha$	1M	
i.e. $\tan\alpha=\dfrac{1}{k}$	1	
(b) Since $y=k\sin x$ bisects the shaded area,		
$\displaystyle\int_0^{\alpha}(\cos x-k\sin x)\,dx=\frac{1}{2}\int_0^{\frac{\pi}{2}}\cos x\,dx$	1M	
$[\sin x+k\cos x]_0^{\alpha}=\dfrac{1}{2}[\sin x]_0^{\frac{\pi}{2}}$	1A	
<u>Alternative Solution</u>		
$\displaystyle\int_0^{\alpha}(\cos x-k\sin x)\,dx=\int_0^{\alpha}k\sin x\,dx+\int_{\alpha}^{\frac{\pi}{2}}\cos x\,dx$	1M	
$[\sin x+k\cos x]_0^{\alpha}=[-k\cos x]_0^{\alpha}+[\sin x]_{\alpha}^{\frac{\pi}{2}}$ $\sin\alpha+k\cos\alpha-k=-k\cos\alpha+k+1-\sin\alpha$	1A	
$\sin\alpha+k\cos\alpha-k=\dfrac{1}{2}$	1A	
By (a), $\dfrac{1}{\sqrt{1+k^2}}+k\dfrac{k}{\sqrt{1+k^2}}=k+\dfrac{1}{2}$	1M	For either $\sin\alpha$ or $\cos\alpha$
$\sqrt{1+k^2}=k+\dfrac{1}{2}$		
$1+k^2=k^2+k+\dfrac{1}{4}$		
<u>Alternative Solution</u>		
By (a), $\sin\alpha+\left(\dfrac{1}{\tan\alpha}\right)\cos\alpha-\dfrac{1}{\tan\alpha}=\dfrac{1}{2}$		
$\dfrac{\sin^2\alpha}{\cos\alpha}+\dfrac{\cos^2\alpha}{\cos\alpha}-1=\dfrac{\tan\alpha}{2}$		
$2\sec\alpha=\tan\alpha+2$		
$4(1+\tan^2\alpha)=\tan^2\alpha+4\tan\alpha+4$	1M	For $\sec^2\alpha=1+\tan^2\alpha$
$3\tan^2\alpha-4\tan\alpha=0$		
$\tan\alpha=\dfrac{4}{3}$ or 0 (rejected as $0<\alpha<\dfrac{\pi}{2}$)		
By checking, $\tan\alpha=\dfrac{4}{3}$ is valid.		
i.e. $k=\dfrac{3}{4}$	1A	
	(7)	

Solution	Marks	Remarks
12. (a) $\overrightarrow{OC}=(1+h)\mathrm{a}$ and $\overrightarrow{OD}=(1+h)\mathrm{b}$	1A	
Consider ΔOAD , $\overrightarrow{OE}=\dfrac{(1-\mu)\overrightarrow{OA}+\mu\overrightarrow{OD}}{1-\mu+\mu}$ $=(1+\mu)\mathrm{a}+\mu(1+h)\mathrm{b}$	1M	For both
Consider ΔOBC , $\overrightarrow{OE}=\dfrac{(1+\lambda)\overrightarrow{OB}+\lambda\overrightarrow{OC}}{1-\lambda+\lambda}$ $=(1-\lambda)\mathrm{b}+\lambda(1+h)\mathrm{a}$	1A	
Hence $\begin{cases}1-\mu=\lambda(1+h)\\ \mu(1+h)=1-\lambda\end{cases}$	1M	
Adding the equations, we have $1-\mu+\mu+\mu h=\lambda+\lambda h+1-\lambda$.		
$\therefore\ \mu=\lambda$ (since $h\neq 0$)	1	
	(5)	
(b) By (a), $\overrightarrow{OE}=(1-\mu)(\mathrm{a}+\mathrm{b})$	1A	
Let $CF:FD=\omega:1-\omega$		
$\therefore\overrightarrow{OF}=\dfrac{(1-\omega)(1+h)\mathrm{a}+\omega(1+h)\mathrm{b}}{1-\omega+\omega}$		
$=(1+h)[(1-\omega)\mathrm{a}+\omega\mathrm{b}]$	1A	
Since O , E and F are collinear, $\dfrac{(1+h)(1+\omega)}{1-\mu}=\dfrac{(1+h)\omega}{1-\mu}$.	1M	
i.e. $\omega=\dfrac{1}{2}$		
Hence $CF=FD$ and therefore OF is a median of ΔOCD .	1	
	(4)	
(c) If BC and AD are also medians of ΔOCD , then $h=1$.	1M	
By the equations in (a), we have $\mu=\lambda=\dfrac{1}{3}$.		
Hence $BE:EC=AE:ED=1:2$.		
By the equations in (b), we have $\overrightarrow{OE}=\dfrac{2}{3}(\mathrm{a}+\mathrm{b})$ and $\overrightarrow{OF}=\mathrm{a}+\mathrm{b}$.	1M	
Hence $OE:EF=2:1$.		
i.e. the centroid divides all the medians in $2:1$.	1	
	(3)	

Solution	Marks	Remarks
13. (a) (i) $AE = 3\sin\theta$ and $BE = 4\sin\theta$	1A	For both 
$\therefore\ AB = \sqrt{(3\sin\theta)^2 + (4\sin\theta)^2}$		
$= 5\sin\theta$	1	
(ii) $DE = 3\cos\theta$ and $CE = 4\cos\theta$		
$\therefore\ CD = \sqrt{(3\cos\theta)^2 + (4\cos\theta)^2}$		
$= 5\cos\theta$	1A	
	(3)	
(b) (i) In ΔABC , $AC = \sqrt{AB^2 + BC^2 - 2(AB)(BC)\cos\alpha}$		
$= \sqrt{(5\sin\theta)^2 + 4^2 - 2(5\sin\theta)(4)\cos\alpha}$	1M	
In ΔAEC , $AC = \sqrt{AE^2 + EC^2 - 2(AE)(EC)\cos\alpha}$		
$= \sqrt{(3\sin\theta)^2 + (4\cos\theta)^2 - 2(3\sin\theta)(4\cos\theta)\cos\alpha}$	1M	
$\therefore\ 25\sin^2\theta + 16 - 40\sin\theta\cos\alpha = 9\sin^2\theta + 16\cos^2\theta - 24\sin\theta\cos\theta\cos\alpha$		
$32\sin^2\theta = 40\sin\theta\cos\alpha - 24\sin\theta\cos\theta\cos\alpha$		
$\cos\alpha = \dfrac{4\sin\theta}{5 - 3\cos\theta}$	1	
(ii) For $0° < \theta < 90°$, $4\sin\theta > 0$ and $5 - 3\cos\theta > 0$.	1M	
Hence $\cos\alpha > 0$ and therefore α must be acute.	1	
(iii) Let the angle between AB and Π_2 = angle between AD and $\Pi_2 = \beta$.		
Hence the distance between A and $\Pi_2 = AB\sin\beta = AD\sin\beta$.		
$\therefore\ 5\sin\theta\sin\beta = 3\sin\beta$	1M	
$\sin\theta = \dfrac{3}{5}$		
$\therefore\ \cos\theta = \dfrac{4}{5}$		
By (i), $\cos\alpha = \dfrac{4\left(\frac{3}{5}\right)}{5 - 3\left(\frac{4}{5}\right)} = \dfrac{12}{13}$.	1A	
$AC = \sqrt{25\sin^2\theta + 16 - 40\sin\theta\cos\alpha}$		
$= \sqrt{25\left(\frac{3}{5}\right)^2 + 16 - 40\left(\frac{3}{5}\right)\left(\frac{12}{13}\right)}$		
$= \sqrt{\dfrac{37}{13}}$	1A	OR ≈ 1.687
Since $AB = 3$, $AC < AB$.		
Hence the angle between AC and Π_2 > the angle between AB and Π_2 .	1	
	(9)	

Solution	Marks	Remarks
14. (a) Volume of shaded region $= \pi \int_0^p x^2 \mathrm{d}y$		
$= \pi \int_0^p (6y - y^2)\mathrm{d}y$	1M	
$= \pi \left[3y^2 - \frac{y^3}{3} \right]_0^p$	1A	
$= \frac{\pi p^2 (9-p)}{3}$	1	
	(3)	
(b) (i) Volume of water $= \pi(6)^2(6) - 2 \cdot \frac{4}{3}\pi(3)^3$		
$= 144\pi \quad \text{cm}^3$	1A	
(ii) $\therefore \quad \pi(6)^2 h - \frac{\pi h^2 (9-h)}{3} - \frac{\pi k^2 (9-k)}{3} = 144\pi$	1M	
$108h - 9h^2 + h^3 - 9k^2 + k^3 = 432$		
$k^3 - 9k^2 + h^3 - 9h^2 + 108h - 432 = 0$ ----------------------------------(*)	1	
(iii) When $h = 5$, $k^3 - 9k^2 + 8 = 0$.		
$(k-1)(k^2 - 8k - 8) = 0$		
$k = 1$ or $4 \pm 2\sqrt{6}$ (rejected as $0 \le k \le 6$)	1A	
Differentiate (*) with respect to t :		
$3k^2 \frac{\mathrm{d}k}{\mathrm{d}t} - 18k \frac{\mathrm{d}k}{\mathrm{d}t} + 3h^2 \frac{\mathrm{d}h}{\mathrm{d}t} - 18h \frac{\mathrm{d}h}{\mathrm{d}t} + 108 \frac{\mathrm{d}h}{\mathrm{d}t} = 0$	1M	
$\because \quad \frac{\mathrm{d}h}{\mathrm{d}t} = -5$	1A	
$\therefore \quad [3(1)^2 - 18(1)] \frac{\mathrm{d}k}{\mathrm{d}t} + [3(5)^2 - 18(5) + 108](-5) = 0$		
i.e. $\frac{\mathrm{d}k}{\mathrm{d}t} = -31$	1A	
Since v represents the rate of change of the distance between the bottom of the sphere B and the base of the container,		
$v = \frac{\mathrm{d}}{\mathrm{d}t}(h-k)$	1M	
$= \frac{\mathrm{d}h}{\mathrm{d}t} - \frac{\mathrm{d}k}{\mathrm{d}t}$		
$= (-5) - (-31)$		
$= 26$	1A	
	(9)	

Solution	Marks	Remarks
15. (a) $2s = PQ + QR + RP$		
$\therefore \quad A = \frac{1}{2} \times PQ \times a + \frac{1}{2} \times QR \times a + \frac{1}{2} \times RP \times a$	1M	
$= \frac{1}{2} a(PQ + QR + RP)$		
$= as$		
i.e. $a = \frac{A}{s}$	1	
	(2)	
(b) (i) $EF = 2\sin\theta$		
By (a), $r = \frac{\frac{1}{2}(1)(1)\sin 2\theta}{\frac{1}{2}(1+1+2\sin\theta)}$	1M	
$= \frac{\sin\theta\cos\theta}{1+\sin\theta}$		
On the other hand, $\cos\theta - \frac{\cos\theta}{1+\sin\theta} = \frac{\cos\theta + \cos\theta\sin\theta - \cos\theta}{1+\sin\theta}$		OR $r = \frac{\sin\theta\cos\theta + \cos\theta - \cos\theta}{1+\sin\theta}$
$= \frac{\sin\theta\cos\theta}{1+\sin\theta}$		$= \frac{\cos\theta(1+\sin\theta) - \cos\theta}{1+\sin\theta}$
$\therefore \quad r = \cos\theta - \frac{\cos\theta}{1+\sin\theta}$	1	$= \cos\theta - \frac{\cos\theta}{1+\sin\theta}$
(ii) $\frac{dr}{d\theta} = -\sin\theta - \frac{(1+\sin\theta)(-\sin\theta) - \cos\theta(\cos\theta)}{(1+\sin\theta)^2}$	1M	
$= -\sin\theta + \frac{\sin\theta + \sin^2\theta + \cos^2\theta}{(1+\sin\theta)^2}$		
$= \frac{1}{1+\sin\theta} - \sin\theta$	1A	
$\frac{dr}{d\theta} = 0$ when $\frac{1}{1+\sin\theta} = \sin\theta$		
$\sin^2\theta + \sin\theta - 1 = 0$	1A	
$\sin\theta = \frac{-1+\sqrt{5}}{2}$ or $\frac{-1-\sqrt{5}}{2}$ (rejected as $0 < \theta < \frac{\pi}{2}$)		
$\theta \approx 0.666$ rad		
$\frac{d^2r}{d\theta^2} = \frac{-\cos\theta}{(1+\sin\theta)^2} - \cos\theta < 0$ for $0 < \theta < \frac{\pi}{2}$	1M	OR using sign test
Hence the area of C_2 is maximum when $\theta \approx 0.666$ rad.	1A	
(iii) The perimeter is the least when $\theta = \frac{\pi}{12}$.	1A	
By (b)(ii), r has only one maximum value when $\frac{\pi}{12} \le \theta \le \frac{5\pi}{12}$	} 1M	
Hence r is the least when $\theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$.		
When $\theta = \frac{\pi}{12}$, $r \approx 0.199$.		
When $\theta = \frac{5\pi}{12}$, $r \approx 0.127$.		
Therefore r , and hence the area of the circle, is the least when $\theta = \frac{5\pi}{12}$ and Frankie is wrong.	} 1	
	(10)	

Solution	Marks	Remarks
16. (a) Centre of C_1 is (0,5) and radius of C_1 is 3.	1A	
Let (x, y) be the centre of a circle in Γ .		
Since the circle touches the x-axis, its radius is y .	1A	
Since the circle touches C_1 externally, $\sqrt{(x-0)^2+(y-5)^2}=y+3$	1M	
$x^2+y^2-10y+25=y^2+6y+9$		
i.e. $y=\frac{1}{16}x^2+1$ which is the equation of S .	1	
	(4)	
(b) (i) Let (h,k) be the centre of C_2 .		
Since C_2 touches the x-axis, its radius is k .		
Since C_2 passes through (20, 16), $\sqrt{(h-20)^2+(k-16)^2}=k$.	1M	
$h^2-40h+400+k^2-32k+256=k^2$		
$h^2-40h-32k+656=0$ -- (1)	1A	
Since C_2 is a circle in Γ , by (a), $k=\frac{1}{16}h^2+1$ ---------- (2)		
Substitute (2) into (1): $h^2-40h-2h^2-32+656=0$		
$h^2+40h-624=0$		
$h=12$ or –52 (rejected as (h,k) lies in the first quadrant)	1A	
By (2), $k=10$		
Hence the equation of C_2 is $(x-12)^2+(y-10)^2=10^2$		
i.e. $x^2+y^2-24x-20y+144=0$	1A	
(ii) Let the centres of C_2 and C_3 and the point of contact of C_2 and C_3 be A , B and P respectively.		
Since P lies on AB and the quadratic curve S intersects any straight	1	1 for " P lies on AB "
line at no more than two points, it is impossible for S to pass through P .	1	1 for " S intersects any straight line at no more than two points
i.e. the sentence is incorrect.		
	(6)	
(c) For a circle satisfying the first two conditions, it touches C_1 externally and it is impossible for it to touch C_1 internally.	} 1	
Hence there is no circle satisfying all three conditions.	1	
	(2)	