2007-CE
A MATH

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2007

ADDITIONAL MATHEMATICS

Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

1. Write your Candidate Number in the space provided on Page 1.
2. Stick barcode labels in the spaces provided on Pages 1, 3, 5, 7 and 9.
3. This paper consists of **TWO** sections, Section A and Section B. Section A carries 62 marks and Section B carries 48 marks.
4. Answer **ALL** questions in Section A. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number and stick a barcode label on each sheet, and fasten them with string **INSIDE** this book.
5. Answer any **FOUR** questions in Section B. Write your answers in the CE(B) answer book.
6. The Question-Answer Book and the CE(B) answer book must be handed in separately at the end of the examination.
7. All working must be clearly shown.
8. Unless otherwise specified, numerical answers must be **exact**.
9. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
10. The diagrams in the paper are not necessarily drawn to scale.

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Candidate Number							

	Marker's Use Only	Examiner's Use Only
	Marker No.	Examiner No.
Section A Question No.	Marks	Marks
1		
2		
3		
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7		
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9		
10		
11		
12		
13		
Section A Total		

Checker's Use Only	Section A Total		

Checker No.	

Page total

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (62 marks)
Answer **ALL** questions in this section and write your answers in the spaces provided in this Question-Answer Book.

1. Find $\int \frac{x^4+1}{x^2}\,dx$.

(3 marks)

2. It is given that the four points $A(0,-2)$, $B(1,-3)$, $C(2,0)$ and $D(k,k)$ form a quadrilateral $ABCD$. Find the area of this quadrilateral.

(3 marks)

3. Find the general solution of the equation $\cos x - \sqrt{2}\cos 2x + \cos 3x = 0$.

(4 marks)

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4. Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2+1\right)$ from first principles.

(4 marks)

5. Let $a \neq 0$ and $a \neq 1$. Prove by mathematical induction that

$$\frac{1}{a-1}-\frac{1}{a}-\frac{1}{a^2}-\cdots-\frac{1}{a^n}=\frac{1}{a^n(a-1)}$$

for all positive integers n.

(5 marks)

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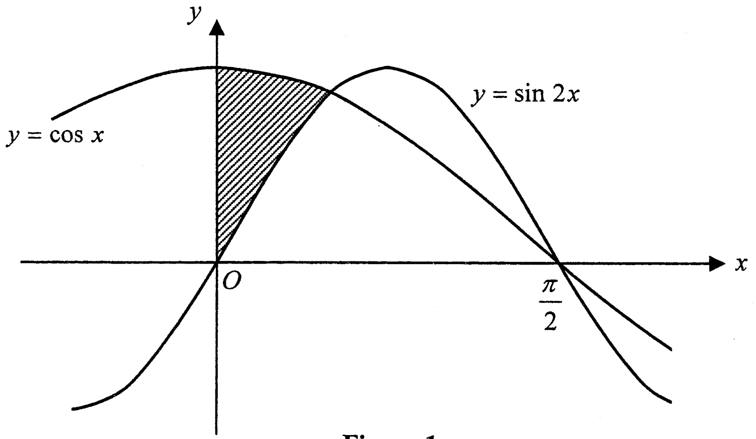

Figure 1

Figure 1 shows the graphs of $y = \sin 2x$ and $y = \cos x$. Find the area of the shaded region.

(5 marks)

7. It is given the points $A(2,1)$ and $B(-2,4)$. C is a point on AB such that $AC:CB = 1:2$.

(a) Find the coordinates of C .

(b) Show that OC bisects $\angle AOB$, where O is the origin.

(5 marks)

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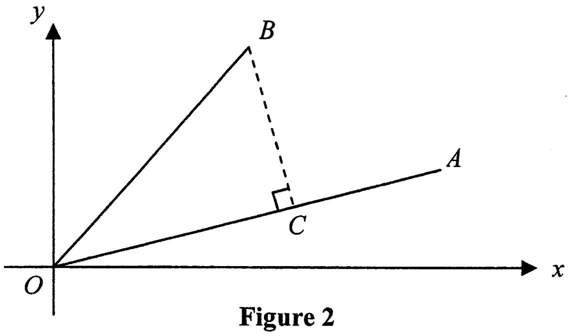

Figure 2

In Figure 2, OCA is a straight line and $BC \perp OA$. It is given that $\overrightarrow{OA} = 6\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} + 6\mathbf{j}$. Let $\overrightarrow{OC} = k\overrightarrow{OA}$.

(a) Express $\overrightarrow{BC}$ in terms of k, $\mathbf{i}$ and $\mathbf{j}$.

(b) Find the value of k.

(5 marks)

9.

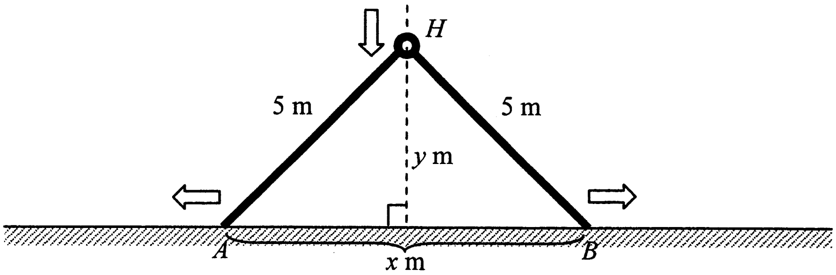

Figure 3

Two rods HA and HB, each of length 5 m, are hinged at H. The rods slide such that A, B, H are on the same vertical plane and A, B move in opposite directions on the horizontal floor, as shown in Figure 3. Let AB be x m and the distance of H from the floor be y m.

(a) Write down an equation connecting x and y.

(b) When H is 3 m from the ground, its falling speed is 2 m s^{-1} . Find the rate of change of the distance between A and B with respect to time at that moment.

(5 marks)

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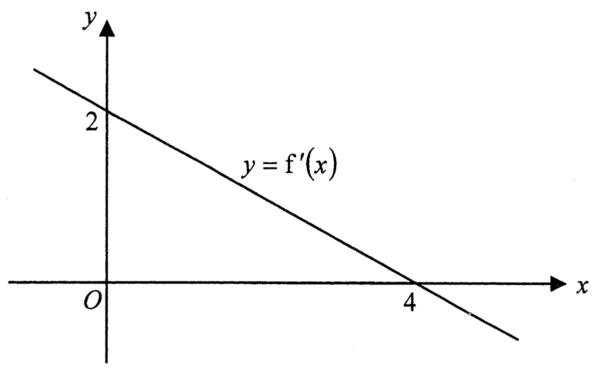

Figure 4

Let $f(x)$ be a function of x. Figure 4 shows the graph of $y = f'(x)$ which is a straight line with x- and y-intercepts 4 and 2 respectively.

(a) Find the slope of the tangent to the curve $y = f(x)$ at $x = 1$.

(b) Find the x-coordinate(s) of all the turning point(s) of the curve $y = f(x)$. For each turning point, determine whether it is a minimum point or a maximum point.

(5 marks)

11. (a) Solve $|x-1| = |x| - 1$, where $0 \le x \le 1$.

(b) Solve $|x-1| = |x| - 1$.

(5 marks)

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12. If the coefficient of x^2 in the expansion of $(1-2x+x^2)^n$ is 66, find the value of n and the coefficient of x^3.

(6 marks)

13. The curve $C: y = x^2$ and the straight line $L: y = mx - 2m$ intersect at two distinct points A and B.

(a) Find the range of values of m.

(b) (i) Show that the coordinates of the mid-point of AB are $\left(\frac{m}{2}, \frac{m^2-4m}{2}\right)$.

(ii) It is given that the straight line $x + y = 5$ bisects the line segment AB. Using (b)(i), or otherwise, find the value(s) of m.

(7 marks)

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SECTION B (48 marks)
Answer any **FOUR** questions in this section. Each question carries 12 marks.
Write your answers in the CE(B) answer book.

14. (a)

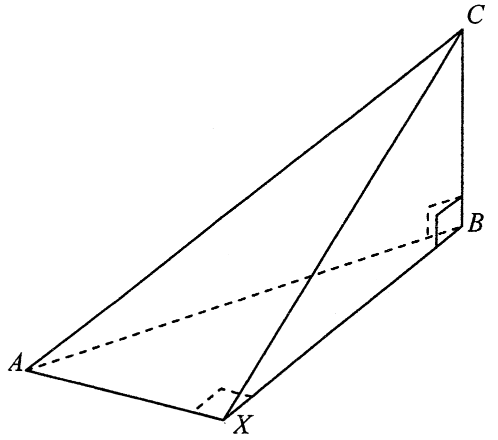

Figure 5

Figure 5 shows a tetrahedron with CB perpendicular to the plane ABX.
Suppose $AX \perp XB$, prove that $AX \perp XC$.

(3 marks)

(b)

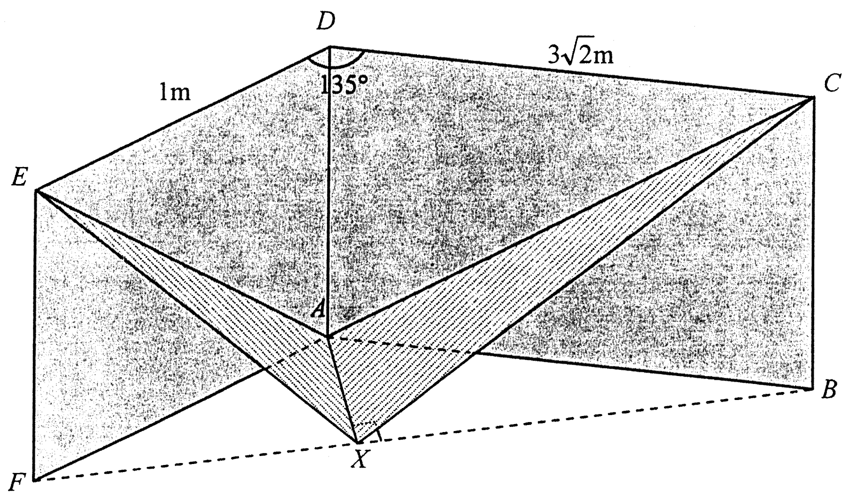

Figure 6

Figure 6 shows two rectangular display boards $ABCD$ and $ADEF$, both perpendicular to the ground. FXB is a straight line and $AX \perp FB$. ACX and AEX are two wooden boards supporting the display boards. It is given that $CD = 3\sqrt{2}$ m, $DE = 1$ m and $\angle CDE = 135°$.

(i) Show that $XB = \dfrac{21}{5}$ m.

(ii) Let θ be the angle between the boards ACX and AEX. If $EF = \dfrac{7}{5}$ m, find $\tan\theta$.

(9 marks)

15.

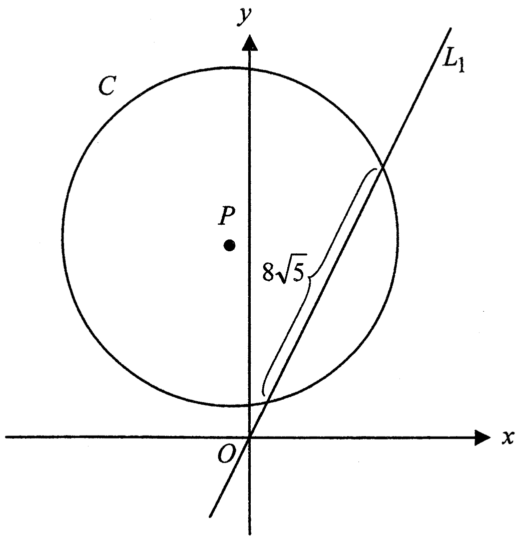

Figure 7

A straight line $L_1 : y = 2x$ intersects a circle C at two points to form a chord of length $8\sqrt{5}$. Let $P(a,b)$ and r be the centre and radius of C respectively (see Figure 7).

(a) By considering the distance from P to L_1, or otherwise, show that

$$r^2 = \frac{4a^2 - 4ab + b^2 + 400}{5}.$$

(3 marks)

(b)

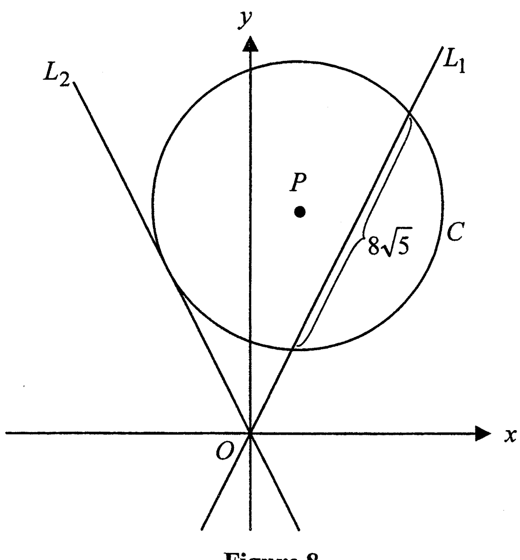

Figure 8

$L_2 : y = -2x$ is another straight line. Suppose P and r vary such that L_2 is always a tangent to C (see Figure 8).

(i) Find the equation of the locus of P.

(ii) If the area of C attains its least value, find the equation(s) of C.

(9 marks)

16.

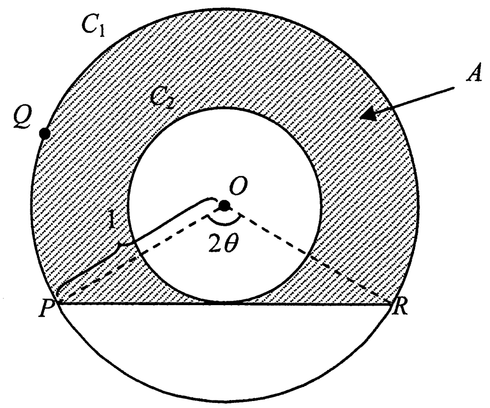

Figure 9

C_1 is a circle with centre O and radius 1. PR is a variable chord which subtends an angle 2θ at O, where $0 < \theta < \frac{\pi}{2}$. C_2 is a circle with centre O and touches PR. Let the area of the shaded region bounded by C_1, C_2 and PR be A (see Figure 9).

(a) Show that

(i) $A = \pi \sin^2 \theta - \theta + \frac{1}{2} \sin 2\theta$,

(ii) $\frac{dA}{d\theta} = (\pi - \tan\theta) \sin 2\theta$.

(5 marks)

(b) When A attains its greatest value, find the value of $\tan\theta$.

(3 marks)

(c) A student guesses that when A attains its greatest value, the perimeter of the shaded region will also attain its greatest value. Explain whether the student's guess is correct or not.

[Note: the perimeter of the shaded region = $\overset{\frown}{PQR}$ + PR + circumference of C_2 .]

(4 marks)

17.

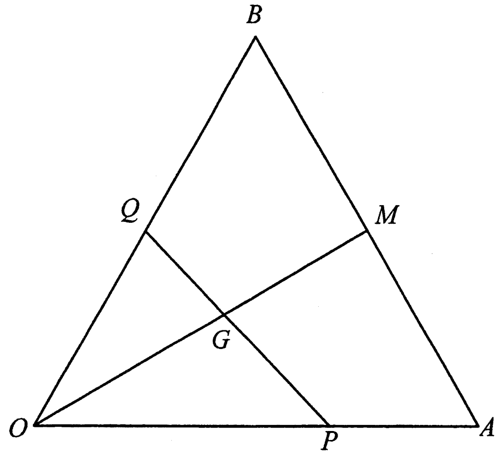

Figure 10

In Figure 10, OAB is an equilateral triangle with $OA = 1$. M is the mid-point of AB and P divides the line segment OA in the ratio 2:1 . Q is a point on OB such that PQ intersects OM at G and $PG : GQ = 4 : 3$. Let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be $\mathbf{a}$ and $\mathbf{b}$ respectively.

(a) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(1 mark)

(b) Let $OQ : QB = k : (1-k)$.

(i) Find $\overrightarrow{OG}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.

(ii) Show that $\overrightarrow{PQ} = \frac{1}{2}\mathbf{b} - \frac{2}{3}\mathbf{a}$.

(4 marks)

(c) (i) Find $\mathbf{a} \cdot \mathbf{b}$ and hence find $|\overrightarrow{PQ}|$.

(ii) Find $\angle QGM$ correct to the nearest degree.

(7 marks)

18. (a) It is given that the curve $y = 2\sqrt{x} - x$ has a horizontal tangent at $x = r$. Show that $r = 1$.

(2 marks)

(b)

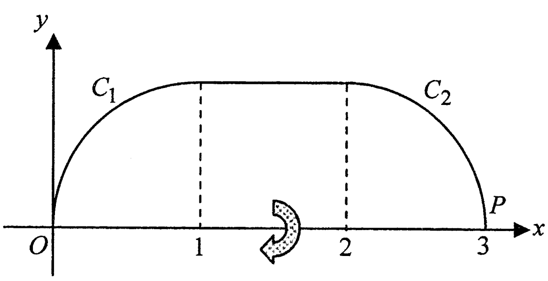

Figure 11

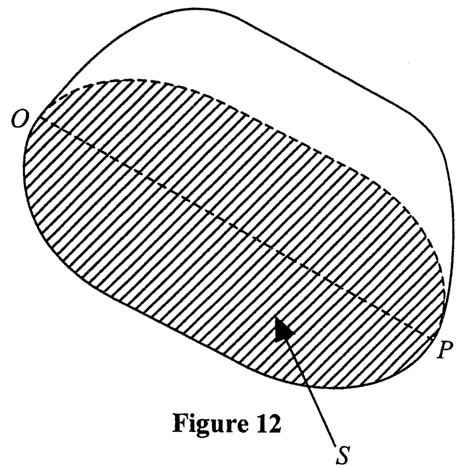

Figure 12

Let O be the origin and P be the point $(3, 0)$. Figure 11 shows a region bounded by:

[1] the curve $C_1 : y = 2\sqrt{x} - x$ (for $0 \le x \le 1$),

[2] the line segment $y = 1$ (for $1 \le x \le 2$),

[3] the curve $C_2 : y = 2\sqrt{3 - x} - (3 - x)$ (for $2 \le x \le 3$), and

[4] OP.

Figure 12 shows a solid formed by revolving the region about the x-axis by **180°**.

(i) The base of the solid is denoted by S in Figure 12. Find the area of S.

(ii) Show that the volume of the solid is $\frac{37}{30}\pi$.

(iii)

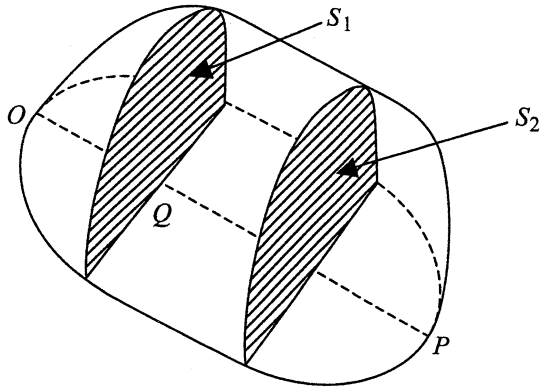

Figure 13

Mrs. Chan has baked a cake which is in the shape of the solid in Figure 12. She cuts the cake into three parts of equal volumes for her three children. The cross-sections formed, S_1 and S_2, are perpendicular to OP (see Figure 13).

Let the intersection of OP and S_1 be Q.

Find $OQ : OP$.

(10 marks)

END OF PAPER