香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2007年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2007

附加數學

ADDITIONAL MATHEMATICS

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative answers. In general, a correct alternative answer merits all the marks allocated to that part, unless a particular method was specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it

   'A' marks – awarded for the accuracy of the answer

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol $pp-1$ should be used to denote marks deducted for poor presentation (*pp*). Note the following points:

   (a) In Section A, at most deduct 1 mark for *pp* in each question, up to a maximum of 2 marks.
   In Section B, at most deduct 1 mark for *pp* in the whole section.

   (b) In Section A, deduct only 1 mark for similar *pp*s for the first time that it occurs, i.e. do not penalise candidates twice in Section A for the same *pp*.

   (c) In any case, do not deduct any marks for *pp* in those steps where candidates failed to score any marks.

   (d) Some cases in which marks should be deducted for pp are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgment to give *pp*s whenever applicable.

5. In Section A, The symbol $u-1$ should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points:

   (a) In Section A, at most deduct 1 mark for wrong/no units.

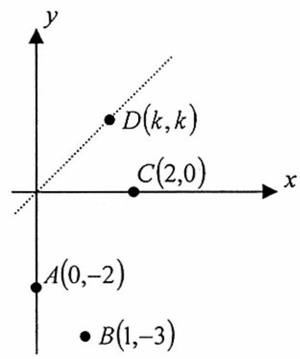
   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted. For answers with an excess degree of accuracy, deduct 1 mark for the first time if happened. In any case, do not deduct any marks for excess degree of accuracy in those steps where candidates failed to score any marks.

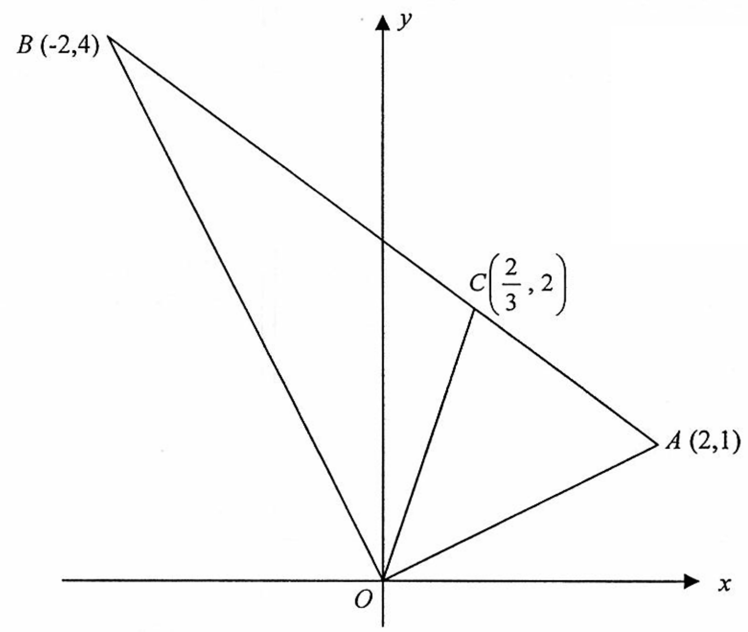
7. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

8. Unless the form of answer was specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

Solution	Marks	Remarks
1. $\int \frac{x^4+1}{x^2}\,dx = \int \left(x^2 + \frac{1}{x^2} \right) dx$	1A	
$= \frac{x^3}{3} - \frac{1}{x} + c$	1M+1A	1M for $\int x^n dx = \frac{x^{n+1}}{n+1}$ (pp-1) if c was omitted
	(3)	
2. The area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & -2 \\ 1 & -3 \\ 2 & 0 \\ 0 & -2 \end{vmatrix}$	1M	Either one
$= \frac{1}{2}(-4+2+6)$ $= 2$	1A	
The area of $\Delta ACD = \frac{1}{2}\begin{vmatrix} 0 & -2 \\ 2 & 0 \\ k & k \\ 0 & -2 \end{vmatrix}$		
$= \frac{1}{2}(2k-2k+4)$ $= 2$		
Hence the area of the quadrilateral $ABCD = 2 + 2 = 4$	1A	
Alternative Solution The area of the quadrilateral $ABCD = \frac{1}{2}\begin{vmatrix} 0 & -2 \\ 1 & -3 \\ 2 & 0 \\ k & k \\ 0 & -2 \end{vmatrix}$	1M	
$= \frac{1}{2}(2k-2k+2+6)$	1A	
$= 4$	1A	
	(3)	
3. $\cos x - \sqrt{2}\cos 2x + \cos 3x = 0$		
$2\cos 2x \cos x - \sqrt{2}\cos 2x = 0$	1M	For sum to product formula
Alternative solution $\cos x - \sqrt{2}\cos 2x + \cos 3x = 0$ $\cos x - \sqrt{2}\cos 2x + \cos 2x \cos x - \sin 2x \sin x = 0$ $\cos x - \cos 2x(\sqrt{2} - \cos x) - 2\sin^2 x \cos x = 0$ $\cos x(1 - 2\sin^2 x) + \cos 2x(\cos x - \sqrt{2}) = 0$	1M	For compound angle formula
$\cos 2x(2\cos x - \sqrt{2}) = 0$		
$\cos 2x = 0$ or $\cos x = \frac{\sqrt{2}}{2}$	1A	
$2x = 360^\circ n \pm 90^\circ$ (or $180^\circ n + 90^\circ$) or $x = 360^\circ n \pm 45^\circ$	1M	For any general solution
$x = 180^\circ n \pm 45^\circ$ (or $90^\circ n + 45^\circ$) or $360^\circ n \pm 45^\circ$	1A	Accept radian measure:
i.e. $x = 180^\circ n \pm 45^\circ$ (or $90^\circ n + 45^\circ$)		$n\pi \pm \frac{\pi}{4}$ or $2n\pi \pm \frac{\pi}{4}$
	(4)	

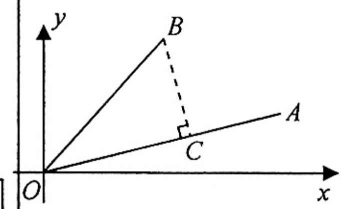
Solution	Marks	Remarks
4. $\frac{d}{dx}\left(x^2+1\right)=\lim_{\Delta x\to 0}\frac{[(x+\Delta x)^2+1]-[x^2+1]}{\Delta x}$	1M	For $\frac{f(x+\Delta x)-f(x)}{\Delta x}$
$=\lim_{\Delta x\to 0}\frac{x^2+2x\Delta x+(\Delta x)^2-x^2}{\Delta x}$	1M	For expanding $(x+\Delta x)^2$
OR		
$=\lim_{\Delta x\to 0}\frac{(x+\Delta x-x)(x+\Delta x+x)}{\Delta x}$	1M	For factorizing $(x+\Delta x)^2-x^2$
$=\lim_{\Delta x\to 0}(2x+\Delta x)$	1A	(pp-1) if $\lim_{\Delta x\to 0}$ was omitted
$=2x$	1A	or written improperly
	(4)	
5. For $n=1$,		
L.H.S. $=\frac{1}{a-1}-\frac{1}{a}=\frac{a-(a-1)}{a(a-1)}=\frac{1}{a(a-1)}$		
R.H.S. $=\frac{1}{a(a-1)}$		
$\therefore$ L.H.S. = R.H.S. and so the statement is true for $n=1$.	1	
Assume $\frac{1}{a-1}-\frac{1}{a}-\frac{1}{a^2}-\cdots-\frac{1}{a^k}=\frac{1}{a^k(a-1)}$, where k is a positive integer.	1	
$\therefore \frac{1}{a-1}-\frac{1}{a}-\frac{1}{a^2}-\cdots-\frac{1}{a^{k+1}}=\frac{1}{a^k(a-1)}-\frac{1}{a^{k+1}}$ by the assumption	1	
$=\frac{a-(a-1)}{a^{k+1}(a-1)}$		
$=\frac{1}{a^{k+1}(a-1)}$	1	Follow through
Hence the statement is also true for $n=k+1$.		
By the principle of mathematical induction, the statement is true for all positive integers n.	1	
	(5)	
6. $\sin 2x=\cos x$	1M	For finding intersection
$2\sin x\cos x=\cos x$		
$(2\sin x-1)\cos x=0$		
$\sin x=\frac{1}{2}$ or $\cos x=0$		
Alternative solution		
$\sin 2x=\cos x$		
$\cos\left(\frac{\pi}{2}-2x\right)=\cos x$ (or $\sin 2x=\cos\left(\frac{\pi}{2}-x\right)$)	1M	
$\frac{\pi}{2}-2x=x$ (or $2x=\frac{\pi}{2}-x$) (for $0<x<\frac{\pi}{2}$)		
$x=\frac{\pi}{6}$ (for $0<x<\frac{\pi}{2}$) or $x=\frac{\pi}{2}$	1A	Accept $x=30°$
Hence the shaded area $=\int_0^{\frac{\pi}{6}}(\cos x-\sin 2x)\,dx$	1M	For $A=\int_a^b(y_2-y_1)\,dx$
$=\left[\sin x+\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{6}}$	1A	For $\pm\left(\sin x+\frac{\cos x}{2}\right)$
$=\frac{1}{4}$	1A	
	(5)	

Solution	Marks	Remarks
7.		



Solution	Marks	Remarks
(a) C is $\left(\frac{1(-2)+2(2)}{1+2},\frac{1(4)+2(1)}{1+2}\right)=\left(\frac{2}{3},2\right)$	1A	
(b) The slopes of OA, OC and OB are $\frac{1}{2}$, 3 and -2 respectively.	1M	For finding 2 of the slopes
$\tan\angle COA=\left\lvert\frac{(3)-\left(\frac{1}{2}\right)}{1+(3)\left(\frac{1}{2}\right)}\right\rvert=1$ $\tan\angle BOC=\left\lvert\frac{(-2)-(3)}{1+(-2)(3)}\right\rvert=1$	1M+ 1A	1M for $\tan\theta=\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$ (accept without absolute sign) 1A for either value correct
$\therefore \tan\angle COA=\tan\angle BOC$ Hence $\angle COA=\angle BOC$ and so OC is the angle bisector of $\angle AOB$.	1	Follow through
Alternative solution (1) $OA=\sqrt{5}$, $OC=\frac{2\sqrt{10}}{3}$, $OB=2\sqrt{5}$, $AC=\frac{5}{3}$, $CB=\frac{10}{3}$	1M	For finding 2 of the lengths
$\cos\angle COA=\frac{(\sqrt{5})^2+\left(\frac{2\sqrt{10}}{3}\right)^2-\left(\frac{5}{3}\right)^2}{2(\sqrt{5})\left(\frac{2\sqrt{10}}{3}\right)}=\frac{1}{\sqrt{2}}$ $\cos\angle BOC=\frac{(2\sqrt{5})^2+\left(\frac{2\sqrt{10}}{3}\right)^2-\left(\frac{10}{3}\right)^2}{2(2\sqrt{5})\left(\frac{2\sqrt{10}}{3}\right)}=\frac{1}{\sqrt{2}}$	1M+ 1A	1M for cosine formula 1A for either value correct
$\therefore \cos\angle COA=\cos\angle BOC$ Hence $\angle COA=\angle BOC$ and so OC is the angle bisector of $\angle AOB$.	1	Follow through

Solution	Marks	Remarks
Alternative solution (2)		
The equations of OA and OB are $y=\frac{1}{2}x$ and $y=-2x$ respectively.	1M	For finding both equations
The distance from C to OA is $\left\lvert\dfrac{\left(\frac{2}{3}\right)-2(2)}{\sqrt{1^2+2^2}}\right\rvert=\dfrac{2\sqrt{5}}{3}$	1M+ 1A	1M for distance formula 1A for either value correct
The distance from C to OB is $\left\lvert\dfrac{2\left(\frac{2}{3}\right)+(2)}{\sqrt{2^2+1^2}}\right\rvert=\dfrac{2\sqrt{5}}{3}$		
Since C is equidistant from OA and OB, so $\angle COA=\angle BOC$.		
Hence OC is the angle bisector of $\angle AOB$.	1	Follow through
Alternative solution (3)		
The equations of OA, OB and OC are $y=\frac{1}{2}x$, $y=-2x$ and $y=3x$ respectively.	1M	For finding 2 of the equations
The equation(s) of the angle bisector(s) of lines OA and OB is / are $\dfrac{2x+y}{\sqrt{2^2+1^2}}=-\dfrac{x-2y}{\sqrt{2^2+1^2}}$ or $\dfrac{2x+y}{\sqrt{2^2+1^2}}=\dfrac{x-2y}{\sqrt{2^2+1^2}}$	1M	
i.e. $y=3x$ or $x=-3y$	1A	
Hence OC is the angle bisector of $\angle AOB$.	1	Follow through
Alternative solution (4)		
$\overrightarrow{OA}=2\mathbf{i}+\mathbf{j}$, $\overrightarrow{OC}=\frac{2}{3}\mathbf{i}+2\mathbf{j}$, $\overrightarrow{OB}=-2\mathbf{i}+4\mathbf{j}$	1M	For finding 2 of the position vectors
$\overrightarrow{OA}\cdot\overrightarrow{OC}=\lvert\overrightarrow{OA}\rvert\cdot\lvert\overrightarrow{OC}\rvert\cos\angle COA$		
$\frac{4}{3}+2=\sqrt{5}\sqrt{\frac{40}{9}}\cos\angle COA$	1M	
$\cos\angle COA=\frac{1}{\sqrt{2}}$	1A	For either one
$\overrightarrow{OB}\cdot\overrightarrow{OC}=\lvert\overrightarrow{OB}\rvert\cdot\lvert\overrightarrow{OC}\rvert\cos\angle BOC$		
$\frac{-4}{3}+8=\sqrt{20}\sqrt{\frac{40}{9}}\cos\angle BOC$		
$\cos\angle BOC=\frac{1}{\sqrt{2}}$		
$\therefore\cos\angle COA=\cos\angle BOC$		
Hence $\angle COA=\angle BOC$ and so OC is the angle bisector of $\angle AOB$.	1	Follow through
Alternative solution (5)		
The areas of $\Delta COA:\Delta BOC=AC:CB$	1M	
$\therefore\frac{1}{2}(OC)(OA)\sin\angle COA:\frac{1}{2}(OC)(OB)\sin\angle BOC=AC:CB$	1M	For area formula
$\frac{1}{2}(OC)\sqrt{2^2+1^2}\sin\angle COA:\frac{1}{2}(OC)\sqrt{(-2)^2+4^2}\sin\angle BOC=AC:CB$		
$\sqrt{5}\sin\angle COA:2\sqrt{5}\sin\angle BOC=1:2$		
$\sin\angle COA=\sin\angle BOC$	1A	
Hence $\angle COA=\angle BOC$ and so OC is the angle bisector of $\angle AOB$.	1	Follow through
	(5)	

Solution	Marks	Remarks
8. (a) $\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$	1A	OR $\overrightarrow{BC}=\overrightarrow{BO}+\overrightarrow{OC}$
$=k(6\mathbf{i}+3\mathbf{j})-(2\mathbf{i}+6\mathbf{j})$		
$=(6k-2)\mathbf{i}+(3k-6)\mathbf{j}$	1A	
(b) $\because BC\perp OA,\quad \therefore \overrightarrow{BC}\cdot\overrightarrow{OA}=0$	1M	Accept $\overrightarrow{BC}\cdot\overrightarrow{OC}=0$
$[(6k-2)\mathbf{i}+(3k-6)\mathbf{j}]\cdot(6\mathbf{i}+3\mathbf{j})=0$		
$(6k-2)(6)+(3k-6)(3)=0$	1M	
$k=\dfrac{2}{3}$	1A	
Alternative solution		
$OA^2=45$, $OB^2=40$, $OC^2=45k^2$, $BC^2=(6k-2)^2+(3k-6)^2$	1M	For finding 2 of the lengths
Since ΔOBC is a right-angled triangle,		
so $45k^2+(36k^2-24k+4)+(9k^2-36k+36)=40$ (Pythagoras theorem)	1M	
$90k^2-60k=0$		
$k=\dfrac{2}{3}$ or 0 (rej.)	1A	(pp-1) if arrow sign was omitted in most cases
	(5)	
9. (a) By Pythagoras Theorem, $\dfrac{x^2}{4}+y^2=25$	1A	
(b) $\therefore \dfrac{x}{2}\cdot\dfrac{\mathrm{d}x}{\mathrm{d}t}+2y\cdot\dfrac{\mathrm{d}y}{\mathrm{d}t}=0$	1M+1A	1M for differentiation
$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{-4y}{x}\cdot\dfrac{\mathrm{d}y}{\mathrm{d}t}$		
By (a), when $y=3$, $x=8$	1M	For finding x
$\therefore \left.\dfrac{\mathrm{d}x}{\mathrm{d}t}\right\vert_{y=3}=\dfrac{-4\cdot 3}{8}(-2)=3$		
Alternative solution		
$\dfrac{x}{2}+2y\cdot\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	1M	1M for differentiation
$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-x}{4y}$		
$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}y}{\mathrm{d}x}}=\dfrac{-4y}{x}\cdot\dfrac{\mathrm{d}y}{\mathrm{d}t}$	1A	
By (a), when $y=3$, $x=8$	1M	For finding x
$\therefore \left.\dfrac{\mathrm{d}x}{\mathrm{d}t}\right\vert_{y=3}=\dfrac{-2}{\left(\frac{-8}{4\cdot 3}\right)}=3$		
Hence the rate of change of the distance between A and B is $3\ \mathrm{m\,s^{-1}}$.	1A	Accept $\left.\dfrac{\mathrm{d}x}{\mathrm{d}t}\right\vert_{y=3}=3\ \mathrm{m\,s^{-1}}$ (u-1) if $\mathrm{m\,s^{-1}}$ was omitted
	(5)	

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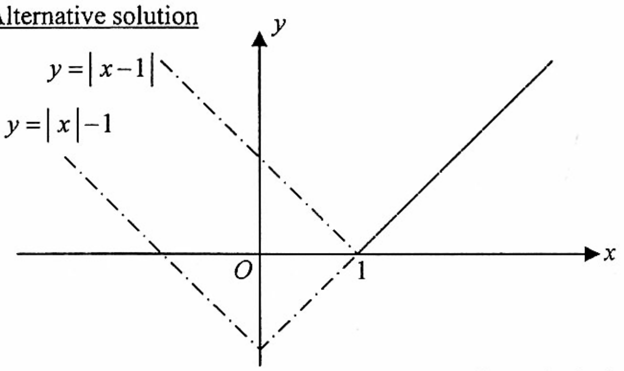
Solution	Marks	Remarks
10. (a) The equation of the given line is $\frac{x}{4}+\frac{y}{2}=1$.	1M	For any straight line form
$\therefore f'(x)=y=2-\frac{x}{2}$		
Hence the slope of the tangent at $x=1$ is $f'(1)=\frac{3}{2}$.	1M+1A	1M for substituting $x=1$
(b) There is only one turning point, with x-coordinate 4.	1A	(pp-1) for $(4,0)$
It is a maximum point since the slope changes from positive to negative.	1A	OR since $f''(x)=\frac{-1}{2}<0$
	(5)	
11. (a) For $0\le x\le 1$, $\lvert x-1\rvert=\lvert x\rvert-1$ becomes		
$1-x=x-1$	1A	
$x=1$	1A	
(b) For $x<0$, $\lvert x-1\rvert=\lvert x\rvert-1$ becomes		
$1-x=-x-1$		
Hence there is no solution in this case.	1A	For considering BOTH $x<0$ and $x>1$
For $x>1$, $\lvert x-1\rvert=\lvert x\rvert-1$ becomes	1M	
$x-1=x-1$		
which is true for all real x		
Hence the solution in this case is $x>1$.		
For either one		
Combining all three cases, the overall solution is $x\ge 1$.	1A	
Alternative solution		
	1M	For attempting to use graphical method
	1A	For either graph
From the graph, we see that the condition for $\lvert x-1\rvert=\lvert x\rvert-1$ is $x\ge 1$.	1A	
	(5)	
12. $(1-2x+x^2)^n=(1-x)^{2n}$	1M	For $1-2x+x^2=(1-x)^2$
$=1-{}_{2n}C_1\,x+{}_{2n}C_2\,x^2-{}_{2n}C_3\,x^3+\cdots$	1M	For bin. expansion up to x^2
OR General term $={}_{2n}C_r(-1)^r x^r$	1M	
$\therefore \frac{2n(2n-1)}{2}=66$	1M	For ${}_{2n}C_2=66$
$2n^2-n-66=0$		
$n=6$ or $\frac{-11}{2}$ (rej.)	1A	
Hence the coefficient of $x^3=-{}_{12}C_3$	1M	
$=-220$	1A	

Solution	Marks	Remarks
Alternative solution		
$(1-2x+x^2)^n = [1+(-2x+x^2)]^n$	1M	For grouping the trinomial expression into binomial
$= 1 + {}_nC_1(-2x+x^2) + {}_nC_2(-2x+x^2)^2 + {}_nC_3(-2x+x^2)^3 + \cdots$	1M	For binomial expansion up to $(-2x+x^2)^2$
$= 1 + {}_nC_1(-2x+x^2) + {}_nC_2(4x^2-4x^3+\cdots) + {}_nC_3(-8x^3+\cdots) + \cdots$		
$= 1 - 2{}_nC_1x + ({}_nC_1 + 4{}_nC_2)x^2 + (-4{}_nC_2 - 8{}_nC_3)x^3 + \cdots$		
$\therefore n + 4\dfrac{n(n-1)}{2} = 66$	1M	For ${}_nC_1 + 4{}_nC_2 = 66$
$2n^2 - n - 66 = 0$		
$(n-6)(2n+11) = 0$		
$n = 6$ or $\dfrac{-11}{2}$ (rej.)	1A	
Hence the coefficient of $x^3 = -4{}_6C_2 - 8{}_6C_3$	1M	
$= -220$	1A	(pp-1) if dots were omitted in most cases
	(6)	
13. (a) $\begin{cases} y = x^2 \\ y = mx - 2m \end{cases}$		
$\therefore x^2 - mx + 2m = 0$ ---------- (*)	1M	For eliminating y
Since C intersects L at 2 distinct points, so $\Delta > 0$		
i.e. $(-m)^2 - 4(1)(2m) > 0$	1M	Accept $\Delta \ge 0$
Alternative solution		
$\therefore y^2 + m(4-m)y + 4m^2 = 0$	1M	For eliminating x
Since C intersects L at 2 distinct points, so $\Delta > 0$		
i.e. $m^2(4-m)^2 - 4(1)(4m^2) > 0$	1M	Accept $\Delta \ge 0$
$m(m-8) > 0$		
$m < 0$ or $m > 8$	1A	
(b) (i) Let A, B and M be (x_1, y_1), (x_2, y_2) and (p, q) respectively.		
Since x_1 and x_2 are roots of (*), so $x_1 + x_2 = m$.	1M	
Alternative solution		
$x_1 = \dfrac{m+\sqrt{m^2-8m}}{2}$ and $x_2 = \dfrac{m-\sqrt{m^2-8m}}{2}$ (or vice versa); $\therefore x_1 + x_2 = m$	1M	
$\therefore p = \dfrac{x_1+x_2}{2} = \dfrac{m}{2}$		
$\therefore q = m\left(\dfrac{m}{2}\right) - 2m = \dfrac{m^2-4m}{2}$		
i.e. M is $\left(\dfrac{m}{2}, \dfrac{m^2-4m}{2}\right)$	1	
(ii) If AB is bisected by the straight line $x + y = 5$, then		
$\dfrac{m}{2} + \dfrac{m^2-4m}{2} = 5$	1A	
$m^2 - 3m - 10 = 0$		
$m = -2$ or 5 (rej. by (a))		
i.e. $m = -2$	1A	
	(7)	

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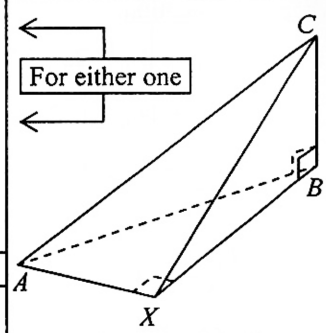
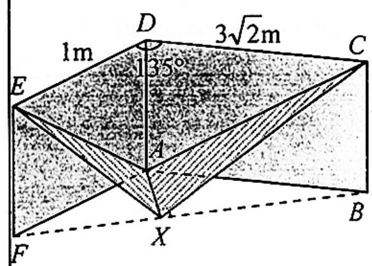
Solution	Marks	Remarks
14. (a) $\because AX \perp XB$, $\therefore AX^2 + XB^2 = AB^2$ (Pythagoras Theorem)	1M	For either one
$AX^2 + XB^2 + BC^2 = AB^2 + BC^2$	1M	
$\because XB^2 + BC^2 = XC^2$ and $AB^2 + BC^2 = AC^2$ (Pythagoras Theorem)		
$\therefore AX^2 + XC^2 = AC^2$		
Hence $AX \perp XC$ (Converse of Pythagoras Theorem)	1	
	(3)	
(b) (i) $FB = \sqrt{(1)^2 + (3\sqrt{2})^2 - 2(1)(3\sqrt{2})\cos 135^\circ} = 5$	1M	For cosine formula
$\frac{AF}{\sin \angle ABF} = \frac{FB}{\sin \angle FAB}$ $\frac{1}{\sin \angle ABF} = \frac{5}{\sin 135^\circ}$	1M	
$\sin \angle ABF = \frac{1}{5\sqrt{2}}$		
$\therefore \cos \angle ABF = \frac{\sqrt{(5\sqrt{2})^2 - 1^2}}{5\sqrt{2}} = \frac{7}{5\sqrt{2}}$	1A	
Hence $XB = AB \cos \angle ABF$ $= (3\sqrt{2})\left(\frac{7}{5\sqrt{2}}\right) = \frac{21}{5}$ m	1	
Alternative solution (1)		
$FB = \sqrt{(1)^2 + (3\sqrt{2})^2 - 2(1)(3\sqrt{2})\cos 135^\circ} = 5$	1M	For cosine formula
The area of $\Delta AFB = \frac{1}{2}(1)(3\sqrt{2})\sin 135^\circ$ $= \frac{3}{2}$	1A	
Hence $\frac{1}{2}(5)(AX) = \frac{3}{2}$ $AX = \frac{3}{5}$	1M	
Therefore $XB = \sqrt{(3\sqrt{2})^2 - \left(\frac{3}{5}\right)^2} = \frac{21}{5}$ m	1	
Alternative solution (2)		
$FB = \sqrt{(1)^2 + (3\sqrt{2})^2 - 2(1)(3\sqrt{2})\cos 135^\circ} = 5$	1M	For cosine formula
Let $XB = x$ and therefore $FX = 5 - x$		
In ΔAXB , $AX^2 = (3\sqrt{2})^2 - x^2$ In ΔAXF , $AX^2 = (1)^2 - (5-x)^2$	1A	
$\therefore 18 - x^2 = 1 - 25 + 10x - x^2$	1M	
$x = \frac{21}{5}$ m	1	

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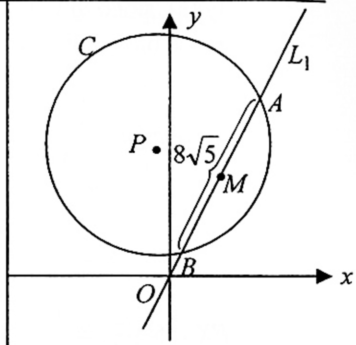
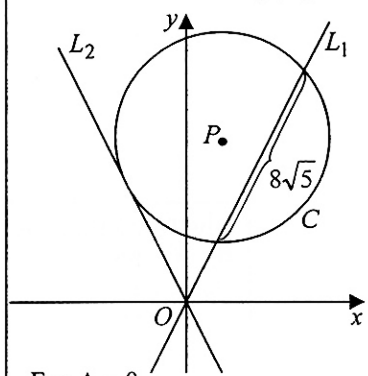
Solution	Marks	Remarks
(ii) By (a), since $AX \perp XB$, so $AX \perp XC$; and since $AX \perp XF$, so $AX \perp XE$. Hence the required angle (θ) is $\angle CXE$.	1A	
$\tan \angle CXB = \dfrac{\left(\frac{7}{5}\right)}{\left(\frac{21}{5}\right)}$		
$= \dfrac{1}{3}$	1A	Either one
$FX = 5 - \dfrac{21}{5} = \dfrac{4}{5}$		
$\tan \angle EXF = \dfrac{\left(\frac{7}{5}\right)}{\left(\frac{4}{5}\right)}$		
$= \dfrac{7}{4}$		
$\therefore \tan\theta = \tan(180° - \angle CXB - \angle EXF)$	1M	For $\theta = 180° - \angle CXB - \angle EXF$
$= -\tan(\angle CXB + \angle EXF)$		
$= -\dfrac{\frac{1}{3}+\frac{7}{4}}{1-\frac{1}{3}\cdot\frac{7}{4}}$	1M	
$= -5$	1A	
<u>Alternative solution</u>		
By (a), since $AX \perp XB$, so $AX \perp XC$; and since $AX \perp XF$, so $AX \perp XE$. Hence the required angle (θ) is $\angle CXE$.	1A	
$CX = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{21}{5}\right)^2}$		
$= \dfrac{7\sqrt{10}}{5}$	1A	Either one
$FX = 5 - \dfrac{21}{5} = \dfrac{4}{5}$		
$EX = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$		
$= \dfrac{\sqrt{65}}{5}$		
$\therefore EC^2 = CX^2 + EX^2 - 2(CX)(EX)\cos\theta$		
$5^2 = \left(\frac{7\sqrt{10}}{5}\right)^2 + \left(\frac{\sqrt{65}}{5}\right)^2 - 2\left(\frac{7\sqrt{10}}{5}\right)\left(\frac{\sqrt{65}}{5}\right)\cos\theta$	1M	
$\cos\theta = \dfrac{-1}{\sqrt{26}}$		
$\therefore \tan\theta = \dfrac{\sqrt{26-1^2}}{-1}$	1M	
$= -5$	1A	
	(9)	

Solution	Marks	Remarks
15. (a) The distance from P to L_1 is $\left\lvert \frac{2a-b}{\sqrt{2^2+(-1)^2}} \right\rvert$ ---------------------------- (*).	1A	
Let A and B be the intersections of C and L_1, and M be the foot of perpendicular of P to AB. Then $AM = \frac{1}{2}AB$ (line from centre $\perp$ chord bisects chord).		
Hence $\left(\frac{2a-b}{\sqrt{5}}\right)^2 + \left(4\sqrt{5}\right)^2 = r^2$ (Pythagoras theorem)	1M	
i.e. $r^2 = \frac{4a^2-4ab+b^2+400}{5}$	1	Withhold if the absolute sign of (*) is missed
	(3)	
(b) (i) The distance from P to L_2 is $\left\lvert \frac{2a+b}{\sqrt{2^2+1^2}} \right\rvert$.	1A	Withhold 1A if the absolute sign was omitted OR (x, y) was used instead of (a, b)
Since C touches L_2, so $r = \left\lvert \frac{2a+b}{\sqrt{5}} \right\rvert$.	1M	
$\therefore r^2 = \frac{4a^2+4ab+b^2}{5}$	1A	
Alternative Solution Solving $C: (x-a)^2+(y-b)^2 = r^2$ and $L_2: y=-2x$, we have $x^2-2ax+a^2+4x^2+4bx+b^2 = r^2$.		
$5x^2-2(a-2b)x+a^2+b^2-r^2 = 0$	1A	
$\Delta = 4(a-2b)^2 - 4\cdot 5\left(a^2+b^2-r^2\right) = 0$	1M	For $\Delta = 0$
$a^2-4ab+4b^2 = 5\left(a^2+b^2-r^2\right)$		
$\therefore r^2 = \frac{4a^2+4ab+b^2}{5}$	1A	
By (a), $\frac{4a^2-4ab+b^2+400}{5} = \frac{4a^2+4ab+b^2}{5}$	1M	
i.e. $ab = 50$ Hence the equation of the locus of P is $xy = 50$.	1A	
(ii) C is smallest when the chord is in fact the diameter of C, i.e. when P lies on L_1.	1M	Can be omitted
Hence P satisfies $xy = 50$ (from (b)(i)) and $L_1: y = 2x$	1M	
Solving, $(x, y) = (-5, -10)$ or $(5, 10)$	1A	

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Solution	Marks	Remarks
Alternative Solution		
From (a) and (b)(i), $r^2 = \frac{1}{5}\left[4a^2 - 4(50) + \left(\frac{50}{a}\right)^2 + 400\right]$	1M	
$= \frac{4}{5}a^2 + 40 + \frac{500}{a^2}$		
$\therefore 2r\frac{\mathrm{d}r}{\mathrm{d}a} = \frac{8a}{5} - \frac{1000}{a^3}$ -- (*)	1M	
When $\frac{\mathrm{d}r}{\mathrm{d}a} = 0$, $a^4 = 625$ which gives $a = \pm 5$.		
From (*), $r\frac{\mathrm{d}^2r}{\mathrm{d}a^2} + \left(\frac{\mathrm{d}r}{\mathrm{d}a}\right)^2 = \frac{4}{5} + \frac{1500}{a^4}$		
$\left.\frac{\mathrm{d}^2r}{\mathrm{d}a^2}\right\|_{a=\pm5} = \frac{1}{r}\left[\frac{4}{5} + \frac{1500}{(\pm 5)^4}\right] > 0$		Accept using sign test
Hence when C is smallest, r is minimum and that occurs when $a = \pm 5$		
Therefore the centre is $(a,b) = (-5,-10)$ or $(5,10)$	1A	Withhold 1A if the condition of minimum was unchecked
$\therefore$ by (a), $r^2 = 80$		
Hence C is $(x+5)^2 + (y+10)^2 = 80$ or $(x-5)^2 + (y-10)^2 = 80$	1A	
i.e. $x^2 + y^2 \pm 10x \pm 20y + 45 = 0$		
	(9)	

ALTERNATIVE SOLUTION.

15 (a) Consider area of $\triangle APB$

$$\frac{1}{2}r^2\sin 2\angle APM = \frac{1}{2}AB \times PM$$

$$r^2(2\sin\angle APM\cos\angle APM) = AB \times PM$$

1A

$$2r^2\,\frac{4\sqrt{5}}{r}\sqrt{1-\left(\frac{4\sqrt{5}}{r}\right)^2} = 8\sqrt{5}\boxed{\left|\frac{2a-b}{\sqrt{5}}\right|} \qquad \longleftarrow 1M.$$

$$r\sqrt{1-\frac{80}{r^2}} = \left|\frac{2a-b}{\sqrt{5}}\right|$$

$$r(r^2-80) = (2a-b)^2$$

$$\text{i.e. } r^2 = \frac{4a^2-4ab+b^2+400}{5} \qquad \longleftarrow 1$$

(b) (i) Solving $C: (x-a)^2+(y-b)^2=r^2$ and $L_2: y=-2x$, we have

$$x^2-2ax+a^2+4x^2+4bx+b^2=r^2$$

from (a) $$5x^2-2(a-2b)x+a^2+b^2-\frac{4a^2-4ab+b^2+400}{5}=0 \qquad \longleftarrow 1M\ (r^2 \text{ from (a)}) \quad \longleftarrow 1A\ (\text{for } Ax^2+Bx+C)$$

$$5x^2-2(a-2b)x+\frac{a^2+4ab+4b^2-400}{5}=0. \qquad \longleftarrow 1A\ (\text{for correct}$$

$$\Delta = 4(a-2b)^2-4\cdot5\cdot\left(\frac{a^2+4ab+4b^2-400}{5}\right)=0. \qquad \longleftarrow 1M$$

$$a^2-4ab+4b^2-(a^2+4ab+4b^2-400)=0$$

$$8ab=400$$

$$ab=50 \qquad \longleftarrow 1A.$$

(ii). Area of C attains its least when the length of diameter $= 8\sqrt{5}$

$$\text{i.e. } 2r = 8\sqrt{5}. \qquad \longleftarrow 1M$$

$$r^2=80.$$

from (a) $$\frac{4a^2-4ab+b^2+400}{5}=80$$

$$(2a-b)^2=0$$

$$2a-b=0$$

$$2x-y=0.$$

Solution	Marks	Remarks
16. (a) (i) The area of the minor segment enclosed by $\overset{\frown}{PR}$ and PR		
$=\frac{1}{2}(1)^2(2\theta)-\frac{1}{2}(1)(1)(\sin 2\theta)$	1A	
$=\theta-\frac{1}{2}\sin 2\theta$		
$\therefore A=\pi(1)^2-\pi(\cos\theta)^2-\left(\theta-\frac{1}{2}\sin 2\theta\right)$	1M	For $A=C_1-C_2-$ segment
$=\pi\sin^2\theta-\theta+\frac{1}{2}\sin 2\theta$	1	
(ii) $\frac{\mathrm{d}A}{\mathrm{d}\theta}=\pi(2\sin\theta\cos\theta)-1+\frac{1}{2}(\cos 2\theta)(2)$	1A	
$=\pi\sin 2\theta-1+\left(1-2\sin^2\theta\right)$		
$=\pi\sin 2\theta-2\sin\theta\cos\theta\tan\theta$		
$=(\pi-\tan\theta)\sin 2\theta$	1	Follow through
	(5)	
(b) Since $0<2\theta<\pi$, so $\sin 2\theta>0$.	1M	
$\therefore \frac{\mathrm{d}A}{\mathrm{d}\theta}=0$ when $\tan\theta=\pi$	1M	For $\frac{\mathrm{d}A}{\mathrm{d}\theta}=0$
$\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2}=(\pi-\tan\theta)(2\cos 2\theta)+\left(-\sec^2\theta\right)\sin 2\theta$		
$=2(\pi-\tan\theta)\cos 2\theta-2\tan\theta$		
$\left.\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2}\right\vert_{\tan\theta=\pi}=0-2\pi<0$		
Alternative Solution: $\because \frac{\mathrm{d}A}{\mathrm{d}\theta}>0$ when $0<\tan\theta<\pi$, and $\frac{\mathrm{d}A}{\mathrm{d}\theta}<0$ when $\tan\theta>\pi$,		
Therefore A attains its greatest value when $\tan\theta=\pi$.	1A	
	(3)	
(c) The perimeter is $s=\overset{\frown}{PQR}+PR+$ circumference of C_2		
$=2\pi-2\theta+2\sin\theta+2\pi\cos\theta$	1A	
When A is max., $\tan\theta=\pi$ which gives $\cos\theta=\frac{1}{\sqrt{1+\pi^2}}$ and $\sin\theta=\frac{\pi}{\sqrt{1+\pi^2}}$		
$\frac{\mathrm{d}s}{\mathrm{d}\theta}=-2+2\cos\theta-2\pi\sin\theta$	1M	
$\therefore \left.\frac{\mathrm{d}s}{\mathrm{d}\theta}\right\vert_{\tan\theta=\pi}=-2+\frac{2}{\sqrt{1+\pi^2}}-\frac{2\pi^2}{\sqrt{1+\pi^2}}$		OR $\frac{\mathrm{d}s}{\mathrm{d}\theta}=-2(1-\cos\theta+\pi\sin\theta)$
$\approx -7.38 \neq 0$	1M	<0 for any $0<\theta<\frac{\pi}{2}$
Alternative solution: At $\theta=\tan^{-1}\pi\approx 1.26$, $s\approx 2\pi-2(1.26)+2\sin(1.26)+2\pi\cos(1.26)$ ≈ 7.57	1M	
At $\theta=1.2$, $s=2\pi-2(1.2)+2\sin(1.2)+2\pi\cos(1.2)$ $\approx 8.02>7.57$	1M	Can use any θ for $0<\theta<1.26$
Hence s will not attain its greatest value when A attains its greatest value. The student is incorrect.	1A	Follow through
	(4)	

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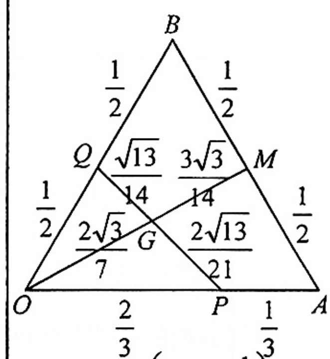
Solution	Marks	Remarks
17. (a) $\overrightarrow{OM}=\dfrac{\mathbf{a}+\mathbf{b}}{2}$	1A	
	(1)	
(b) (i) $\overrightarrow{OP}=\dfrac{2}{3}\mathbf{a}$ and $\overrightarrow{OQ}=k\mathbf{b}$		
$\therefore \overrightarrow{OG}=\dfrac{3\left(\frac{2}{3}\mathbf{a}\right)+4(k\mathbf{b})}{3+4}$	1M	
$=\dfrac{2\mathbf{a}+4k\mathbf{b}}{7}$	1A	
(ii) Since O, G and M are collinear, so		
$\dfrac{\left(\frac{2}{7}\right)}{\left(\frac{1}{2}\right)}=\dfrac{\left(\frac{4k}{7}\right)}{\left(\frac{1}{2}\right)}$	1M	
<u>Alternative Solution</u> Since O, G and M are collinear, so $\overrightarrow{OG}\cdot\overrightarrow{AB}=0$		
$\dfrac{2\mathbf{a}+4k\mathbf{b}}{7}\cdot(\mathbf{b}-\mathbf{a})=0$	1M	
$4k\lvert\mathbf{b}\rvert^2+(2-4k)\mathbf{a}\cdot\mathbf{b}-2\lvert\mathbf{a}\rvert^2=0$ $\therefore \mathbf{a}\cdot\mathbf{b}=(1)(1)\cos 60^\circ=\dfrac{1}{2}$ $\therefore 4k(1)^2+(2-4k)\left(\dfrac{1}{2}\right)-2(1)^2=0$		
$k=\dfrac{1}{2}$ $\therefore \overrightarrow{OQ}=\dfrac{1}{2}\mathbf{b}$		
$\therefore \overrightarrow{PQ}=\dfrac{1}{2}\mathbf{b}-\dfrac{2}{3}\mathbf{a}$	1	
	(4)	
(c) (i) $\mathbf{a}\cdot\mathbf{b}=(1)(1)\cos 60^\circ=\dfrac{1}{2}$	1A	
$\lvert\overrightarrow{PQ}\rvert^2=\left(\dfrac{1}{2}\mathbf{b}-\dfrac{2}{3}\mathbf{a}\right)\cdot\left(\dfrac{1}{2}\mathbf{b}-\dfrac{2}{3}\mathbf{a}\right)$	1M	
$=\dfrac{1}{4}\lvert\mathbf{b}\rvert^2-\dfrac{2}{3}\mathbf{a}\cdot\mathbf{b}+\dfrac{4}{9}\lvert\mathbf{a}\rvert^2$ $=\dfrac{1}{4}(1)^2-\dfrac{2}{3}\left(\dfrac{1}{2}\right)+\dfrac{4}{9}(1)^2$ $=\dfrac{13}{36}$		
$\therefore \lvert\overrightarrow{PQ}\rvert=\dfrac{\sqrt{13}}{6}$	1A	

Solution	Marks	Remarks
(ii) $\left\|\overrightarrow{OM}\right\| = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$	1A	
$\overrightarrow{PQ} \cdot \overrightarrow{OM} = \left\|\overrightarrow{PQ}\right\|\left\|\overrightarrow{OM}\right\| \cos \angle QGM$		
$\left(\frac{1}{2}\mathbf{b} - \frac{2}{3}\mathbf{a}\right) \cdot \left(\frac{\mathbf{a}+\mathbf{b}}{2}\right) = \left(\frac{\sqrt{13}}{6}\right)\left(\frac{\sqrt{3}}{2}\right) \cos \angle QGM$	1M	
$\frac{\sqrt{39}}{12} \cos \angle QGM = \frac{1}{4}\|\mathbf{b}\|^2 - \frac{1}{12}\mathbf{a} \cdot \mathbf{b} - \frac{1}{3}\|\mathbf{a}\|^2$		
$= \frac{1}{4}(1)^2 - \frac{1}{12}\left(\frac{1}{2}\right) - \frac{1}{3}(1)^2$		
$= \frac{-1}{8}$	1A	
$\therefore \cos \angle QGM = \frac{-3}{2\sqrt{39}}$		
$\therefore \angle QGM = 104°$ (correct to the nearest degree)	1A	
Alternative Solution		$\left(QM = \frac{1}{2}\right)$
$OQ = \left\|\frac{1}{2}\mathbf{b}\right\| = \frac{1}{2}$		
$\frac{OQ}{\sin \angle OPQ} = \frac{QP}{\sin \angle QOP}$		
$\frac{\frac{1}{2}}{\sin \angle OPQ} = \frac{\frac{\sqrt{13}}{6}}{\sin 60°}$	1M	
$\sin \angle OPQ = \frac{3\sqrt{3}}{2\sqrt{13}}$	1A	
Alternative Solution		**Marking Criteria** 1M for any trigo. method 1A for any ONE relevant side / angle correctly found 1A for ALL relevant sides / angles correctly found 1A for $\angle QGM$
$OP = 1 \cdot \left(\frac{2}{2+1}\right) = \frac{2}{3}$		
$\because OQ^2 = OP^2 + PQ^2 - 2(OP)(PQ)\cos \angle OPQ$		
$\therefore \left(\frac{1}{2}\right)^2 = \left(\frac{2}{3}\right)^2 + \left(\frac{\sqrt{13}}{6}\right)^2 - 2\left(\frac{2}{3}\right)\left(\frac{\sqrt{13}}{6}\right)\cos \angle OPQ$	1M	
$\cos \angle OPQ = \frac{5}{2\sqrt{13}}$	1A	
$\angle OPQ \approx 46.1°$ (since $\angle OPQ$ is acute from the figure)		
Since M is the mid-point of AB, so $\angle AOM = \angle BOM = 30°$	1A	
$\therefore \angle OGP \approx 180° - 30° - 46.1° = 103.9°$		
Hence $\angle QGM = \angle OGP = 104°$ (correct to the nearest degree)	1A	
	(7)	

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Solution	Marks	Remarks
18. (a) $y = 2\sqrt{x} - x$		
$\frac{dy}{dx} = \frac{1}{\sqrt{x}} - 1$	1A	
For horizontal tangent, $\left.\frac{dy}{dx}\right\|_{x=r} = 0$.		
$\therefore \frac{1}{\sqrt{r}} - 1 = 0$ which gives $r = 1$	1	
	(2)	
(b) (i) Area under C_1 is $\int_0^1 (2\sqrt{x} - x)dx$	1A	
$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^1$		
$= \frac{5}{6}$	1A	
Area under C_2 is $\int_2^3 \left[2\sqrt{3-x} - (3-x)\right]dx$	1M	
$= \left[2\cdot\frac{2}{-3}(3-x)^{\frac{3}{2}} - 3x + \frac{1}{2}x^2\right]_2^3$		
$= \frac{5}{6}$		
Alternative Solution Since C_2 is the reflection of C_1 about the line $x = \frac{3}{2}$, so the area under C_2 equals the area under C_1.	1M	
So the area of $S = 4\left(\frac{5}{6}\right) + 2(1\cdot 1)$		
$= \frac{16}{3}$	1A	
(ii) Volume $= \frac{1}{2}\left\{\pi\int_0^1 (2\sqrt{x} - x)^2 dx + \pi(1)^2(1) + \pi\int_2^3 \left[2\sqrt{3-x} - (3-x)\right]^2 dx\right\}$	1M+1M	1M for $V_1 = \pi\int_a^b y^2 dx$ 1M for $V = \frac{1}{2}\left[V_1 + \pi(1)^2(1) + V_2\right]$
$= \frac{\pi}{2}\int_0^1 \left(4x - 4x^{\frac{3}{2}} + x^2\right)dx + \frac{\pi}{2} + \frac{\pi}{2}\int_2^3 \left[4(3-x) - 4(3-x)^{\frac{3}{2}} + (3-x)^2\right]dx$		
$= \frac{\pi}{2}\left[2x^2 - \frac{8}{5}x^{\frac{5}{2}} + \frac{1}{3}x^3\right]_0^1 + \frac{\pi}{2} + \frac{\pi}{2}\left[-2(3-x)^2 + \frac{8}{5}(3-x)^{\frac{5}{2}} - \frac{1}{3}(3-x)^3\right]_2^3$	1A	
$= \frac{37}{30}\pi$	1	

Solution	Marks	Remarks
Alternative Solution		
Volume $= \frac{1}{2}\left[2\cdot\pi\int_0^1\left(2\sqrt{x}-x\right)^2 dx + \pi(1)^2(1)\right]$	1M+1M	1M for $V_1 = \pi\int_a^b y^2 dx$ 1M for $V = \frac{1}{2}\left[2V_1 + \pi(1)^2(1)\right]$
$= \pi\int_0^1\left(4x - 4x^{\frac{3}{2}} + x^2\right)dx + \frac{\pi}{2}$		
$= \pi\left[2x^2 - \frac{8}{5}x^{\frac{5}{2}} + \frac{1}{3}x^3\right]_0^1 + \frac{\pi}{2}$	1A	
$= \frac{37}{30}\pi$	1	
(iii) The volume of the middle part is $\frac{37}{90}\pi$ (by (ii)).		
Area of S_1 is $\frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$		
Hence the length of the middle part is $\left(\frac{37\pi}{90}\right)\Big/\left(\frac{\pi}{2}\right) = \frac{37}{45}$	1M	
$\therefore OQ = \frac{1}{2}\left(3 - \frac{37}{45}\right) = \frac{49}{45}$		
$\therefore OQ : OP = \frac{49}{45} : 3 = 49 : 135$	1A	
Alternative Solution		
The volume of the first part is $\frac{37}{90}\pi$ (by (ii)).		
By (ii), the volume formed by C_1 is $\pi\int_0^1\left(4x - 4x^{\frac{3}{2}} + x^2\right)dx = \frac{11\pi}{30}$.		
Area of S_1 is $\frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$		
Hence $OQ - 1 = \left(\frac{37\pi}{90} - \frac{11\pi}{30}\right)\Big/\frac{\pi}{2}$	1M	For RHS
$\therefore OQ = \frac{49}{45}$		
$\therefore OQ : OP = \frac{49}{45} : 3 = 49 : 135$	1A	
	(10)	

SUPPLEMENTARY NOTES (1)

1.

Marking Scheme	
$x=\sqrt{2}$	1A

Sample 1

$x=\sqrt{2}$	✓	1A
$=1.414$		

Sample 2

$x=1.414$	✗	0A

2.

Marking Scheme		
(a)	⋮	1M
	$x=\sqrt{2}$	1A
(b)	Substitute: $x=\sqrt{2}$	1M
	⋮	
	$y=4.3$	1A

Sample 3

(a)	⋮	✓	1M
	$x=1.414$	✗	0A
(b)	Substitute: $x=1.414$	✓	1M
	⋮		
	$y=4.3$	✓	1A

Sample 4

(a)	⋮	✓	1M
	$x=\sqrt{3}$	✗	0A
(b)	Substitute: $x=\sqrt{3}$	✓	1M
	⋮		
	$y=4.3$	✗	0A

3.

Marking Scheme		
(a)	⋮	1M
	$x=2$	1A
(b)	⋮	1M
	$y=4$	1A

Sample 5

(a)	……	✗	0M
	……	✗	0A
(b)	⋮	✓	1M
	$x=2$	✗	0A
	⋮	✓	1M
	$y=4$	✓	1A

Sample 6

⋮	✓	1M
$x=2$	✓	1A
⋮	✓	1M
$y=4$	✓	1A

---

4. **Final Answer**

Like terms are not collected		**Answers were not simplified**	
$12x-3y+2=10x+1$ $x=2n\pi+\dfrac{\pi}{2}-\dfrac{\pi}{4}$ $y-6\sin\theta=(\cos\theta)(x-2\tan\theta)$	} 0A	$\dfrac{dy}{dx}=\dfrac{x\cos(x^2+1)}{x^2}$ $x=\dfrac{4}{8}$ $20x-30y=0$	} (pp −1)

SUPPLEMENTARY NOTES (2)

Section A Question 5

Part I

1. $n=1$, $\frac{1}{a-1}-\frac{1}{a}-\frac{1}{a^2}-\cdots-\frac{1}{a^n}=\frac{1}{a^n(a-1)}$.

   The statement is true for $n=1$. ✕

2. Assume / Let the statement be true for $n=1$. ✕

3. Assume / Let the statement be true for $n=1$. ✓ (pp–1)

   $\frac{1}{a-1}-\frac{1}{a}=\frac{1}{a(a-1)}$

   The statement is true for $n=1$.

Part II

4. $\frac{1}{a-1}-\frac{1}{a}-\frac{1}{a^2}-\cdots-\frac{1}{a^k}=\frac{1}{a^k(a-1)}$ (NOT an assumption) ✕

5. Assume (OR let / when / if / suppose) the statement is true for $n=k$. (k NOT defined) ✓

6. Assume the statement is true for **some / any / all positive integer(s).** ✓

   Assume the statement is true for **some / any / all integer(s).** ✓ (pp–1)

   Assume the statement is true for **some / any / all integer(s)** $\boldsymbol{k}$. ✓ (pp–1)

   Assume the statement is true for **some / any / all integer(s)** $\boldsymbol{n}$. ✓

   Assume the statement is true for $n=k$, where k is **real / a constant.** ✕

7. Assume $n=k$. / Assume $n=k$ is true. / Assume n is true for k . ✓ (pp–1)

8. Assume $n=k$, / Assume $n=k$ is true, / Assume n is true for k ,

   $\frac{1}{a-1}-\frac{1}{a}-\frac{1}{a^2}-\cdots-\frac{1}{a^k}=\frac{1}{a^k(a-1)}$. ✓

Part III

9. The statement is true for **all real numbers /all integers / any integers.** ✕

10. The statement is true for **all** $\boldsymbol{n}$ **/all integers** $\boldsymbol{n}$. ✓