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FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

****

Section A (62 marks)
Answer **ALL** questions in this section and write your answers in the spaces provided in this Question-Answer Book.

1. Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\sin(2x+1)}{x}\right]$.

(3 marks)

2. Prove the identity $\cos^2 x - \cos^2 y \equiv -\sin(x+y)\sin(x-y)$.

(3 marks)

3. It is given that

$$(1-2x+3x^2)^n = 1 - 10x + kx^2 + \text{ terms involving higher powers of } x,$$

where n is a positive integer and k is a constant. Find the values of n and k.

(5 marks)

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4. If $kx^2 + x + k > 0$ for all real values of x, where $k \neq 0$, find the range of possible values of k.

(4 marks)

5. The straight line $y = x + k$ intersects the curve $y = x^2$ at two points P and Q. It is known that the locus of the mid-point of PQ, as k varies, lies on a straight line L. Find the equation of L.

(4 marks)

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6. Solve $x|x|+5x+6=0$.

(4 marks)

7. Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors such that $|\mathbf{a}|=\sqrt{3}$, $|\mathbf{b}|=2$ and the angle between them is $150°$.

(a) Find $\mathbf{a}\cdot\mathbf{b}$.

(b) Find $|\mathbf{a}+2\mathbf{b}|$.

(5 marks)

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8. Prove that $n^3 - n + 3$ is divisible by 3 for all positive integers n.

(5 marks)

9. (a) Express $\cos\theta - \sqrt{3}\sin\theta$ in the form $r\cos(\theta + \alpha)$, where $r > 0$ and $0° < \alpha < 90°$.

(b) Find the general solution of the equation $\cos 2x - \sqrt{3}\sin 2x = 1$.

(6 marks)

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10. The slope at any point (x, y) of a curve is given by $\frac{dy}{dx} = 3 + 2\cos 2x$. If the curve passes through the point $(\frac{\pi}{4}, \frac{3\pi}{4})$, find its equation.

(5 marks)

11. Let L_1 be the straight line $y = 2x - 5$. L_2 and L_3 are two straight lines passing through the origin and each makes an angle of $45°$ with L_1 (see Figure 1).

(a) Find the equations of L_2 and L_3.

(b) Find the area of the triangle bounded by L_1, L_2 and L_3.

(6 marks)

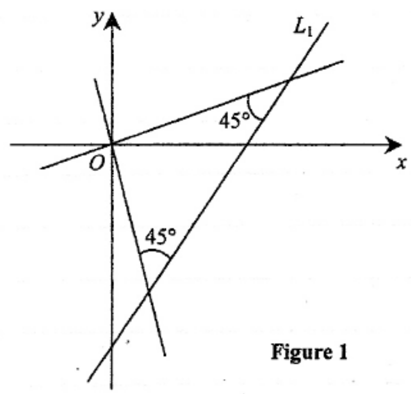

Figure 1

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12. (a) Let $x^2 - xy + y^2 = 7$. Find $\frac{dy}{dx}$.

(b) Find the equation of the normal to the curve $x^2 - xy + y^2 = 7$ at the point $(1, 3)$.

(5 marks)

13. Let $f(x)$ be a polynomial. Figure 2 shows a sketch of the curve $y = f'(x)$, where $-2 \le x \le 6$. The curve cuts the x-axis at the origin and $(a, 0)$, where $0 < a < 6$. It is known that the areas of the shaded regions R_1 and R_2 as shown in Figure 2 are 3 and 1 respectively.

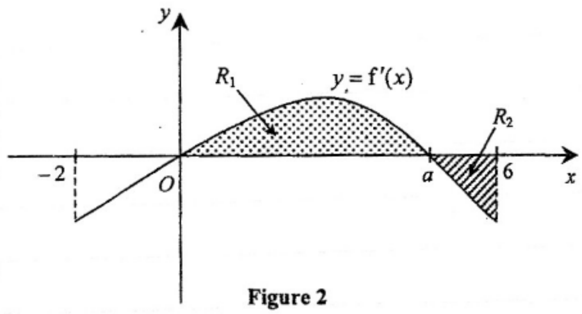

Figure 2

(a) Write down the x-coordinates of the maximum and minimum points of the curve $y = f(x)$ for $-2 < x < 6$.

(b) It is known that $f(-2) = 2$ and $f(0) = 1$.

(i) By considering $\int_0^a f'(x)\,dx$, find the value of $f(a)$.

(ii) In Figure 3, sketch the curve $y = f(x)$ for $-2 \le x \le 6$.

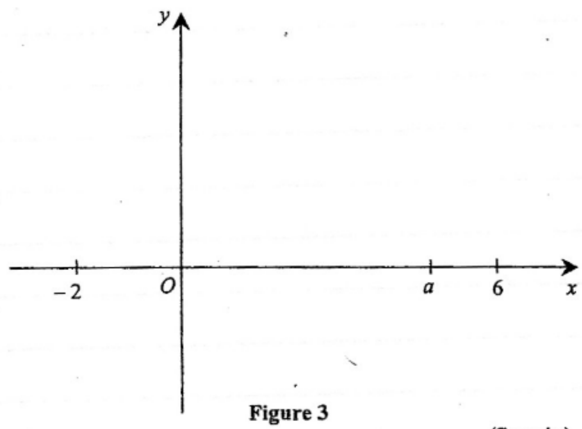

Figure 3

(7 marks)

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SECTION B (48 marks)
Answer any **FOUR** questions in this section. Each question carries 12 marks.
Write your answers in the CE(B) answer book.

14. Let J be the circle $x^2 + y^2 = r^2$, where $r > 0$.

(a) Suppose that the straight line $L: y = mx + c$ is a tangent to J.

(i) Show that $c^2 = r^2(m^2 + 1)$.

(ii) If L passes through a point (h, k), show that $(k - mh)^2 = r^2(m^2 + 1)$.

(4 marks)

(b)

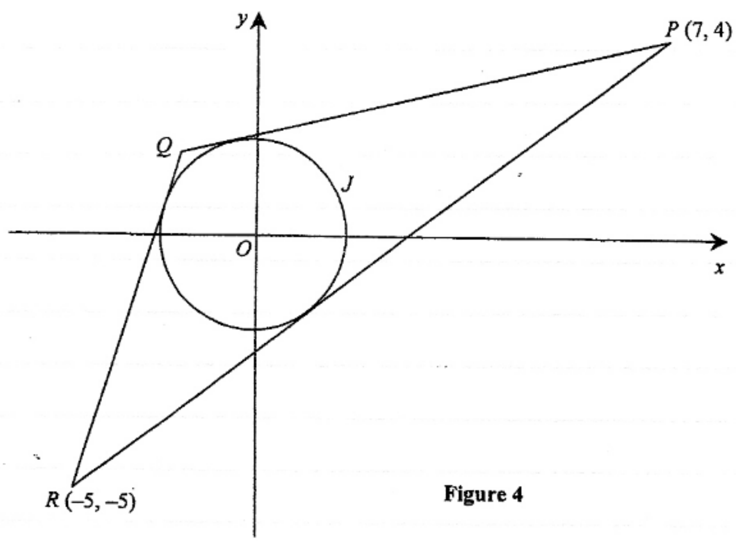

Figure 4

J is inscribed in a triangle PQR (see Figure 4). The coordinates of P and R are $(7, 4)$ and $(-5, -5)$ respectively.

(i) Find the radius of J.

(ii) Using (a) (ii), or otherwise, find the slope of PQ.

(iii) Find the coordinates of Q.

(8 marks)

15.

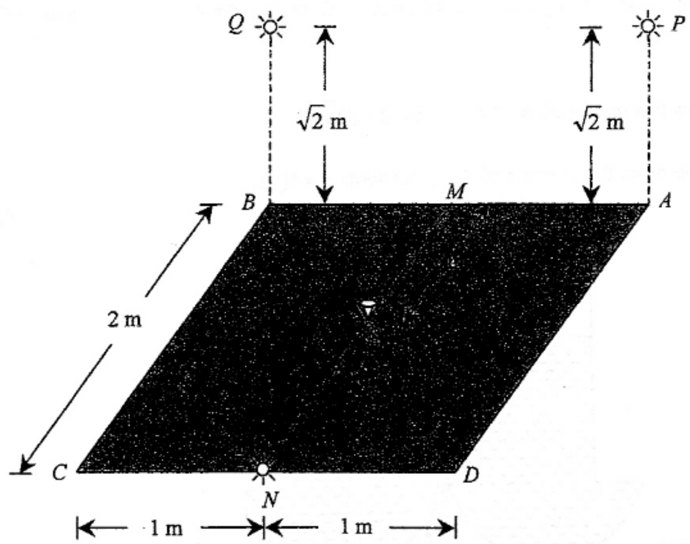

Figure 5

In Figure 5, $ABCD$ is a horizontal square board of side 2 m for displaying diamonds. Let M, N be the mid-points of BA and CD respectively. Three identical small bulbs are located at points N, P and Q respectively for illumination purpose, where P and Q are at a height $\sqrt{2}$ m vertically above A and B respectively. A diamond is placed at a point S along MN and $MS = x$ m, where $0 \le x \le \frac{3}{2}$. Let $PS + QS + NS = \ell$ m.

(a) Express ℓ in terms of x.

Hence show that $\frac{d\ell}{dx} = \frac{2x}{\sqrt{x^2+3}} - 1$.

(2 marks)

(b) Find the values of x at which ℓ attains

(i) the least value, and

(ii) the greatest value.

(6 marks)

(c) Suppose that the intensity of light energy received by the diamond from each bulb varies inversely as the square of the distance of the bulb from the diamond, with k (> 0, in suitable unit) being the variation constant. Let E (in suitable unit) be the total intensity of light energy received by the diamond from the three bulbs.

(i) Express E in terms of k and x.

(ii) A student guesses that when ℓ attains its least value, E will attain its greatest value. Explain whether the student's guess is correct or not.

(4 marks)

16. Let C be the curve $y=\frac{1}{3}x^2-\frac{4}{3}x+1$. C_1 is a part of C with $0 \le x \le 1$ and C_2 is a part of C with $3 \le x \le 4$.

(a) (i) Show that the equation of C_1 is $x=2-\sqrt{3y+1}$.

(ii) Write down the equation of C_2 in the form $x=\mathrm{f}(y)$.

(2 marks)

(b)

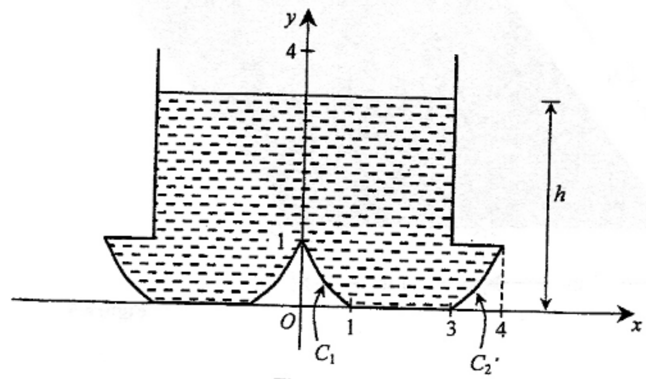

Figure 6

A container is formed by revolving C_1, the line segment $y=0$ (for $1 \le x \le 3$), C_2, the line segment $y=1$ (for $3 \le x \le 4$) and the line segment $x=3$ (for $1 \le y \le 4$) about the y-axis (see Figure 6). Starting from time $t=0$, water is poured into the container at a constant rate of 8π cubic units per minute. Let the volume and depth of water in the container at time t minutes be V cubic units and h units respectively.

(i) Consider $0 < h < 1$.

(1) Show that $V=\frac{16\pi}{9}[(3h+1)^{\frac{3}{2}}-1]$.

(2) Find $\frac{\mathrm{d}h}{\mathrm{d}t}$ in terms of h.

(ii) Consider $1 < h < 4$. Find $\frac{\mathrm{d}h}{\mathrm{d}t}$.

(iii) It is known that $h=1$ at $t=t_1$ and $h=4$ at $t=t_2$. Sketch a graph to show how h varies with t for $0 \le t \le t_2$. (You are not required to find the values of t_1 and t_2.)

(10 marks)

17. (a)

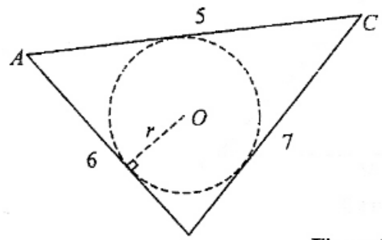

Figure 7

ABC is a triangle with $AB = 6$, $BC = 7$ and $CA = 5$. A circle is inscribed in the triangle (see Figure 7). Let O be the centre of the circle and r be its radius.

(i) Find the area of ΔABC.

(ii) By considering the areas of ΔAOB, ΔBOC and ΔCOA, show that $r = \frac{2\sqrt{6}}{3}$.

(4 marks)

(b)

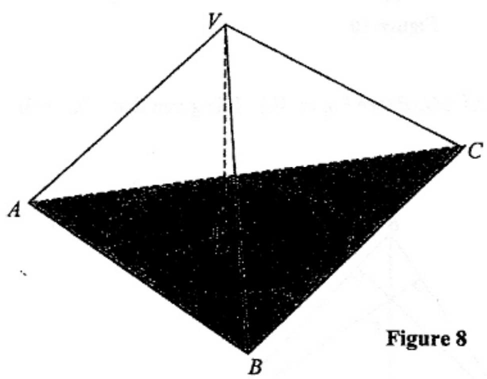

Figure 8

$VABC$ is a tetrahedron with the ΔABC described in (a) as the base (see Figure 8). Furthermore, point O is the foot of perpendicular from V to the plane ABC. It is given that the angle between the planes VAB and ABC is $60°$.

(i) Find the volume of the tetrahedron $VABC$.

(ii) Find the area of ΔVBC.

(iii) Find the angle between the side AB and the plane VBC, giving your answer correct to the nearest degree.

(8 marks)

18.

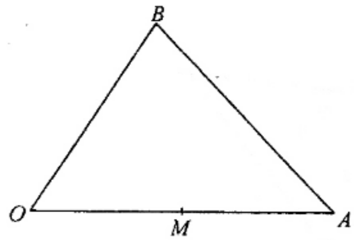

Figure 9

Figure 9 shows a triangle OAB. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and M be the mid-point of OA.

(a)

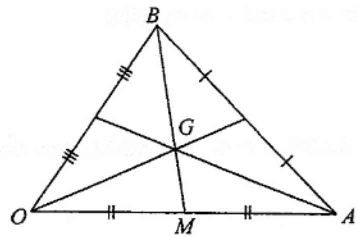

Figure 10

Let G be the centroid of ΔOAB (see Figure 10). It is given that $BG : GM = 2 : 1$. Express $\overrightarrow{OG}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(1 mark)

(b)

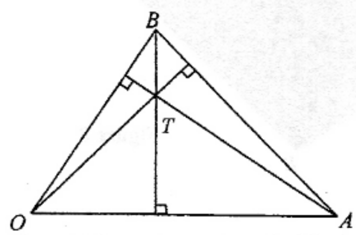

Figure 11

Let T be the orthocentre of ΔOAB (see Figure 11). Show that $\overrightarrow{OT} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{a} = 0$ and write down the value of $\overrightarrow{OT} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{b}$.

(3 marks)

(c)

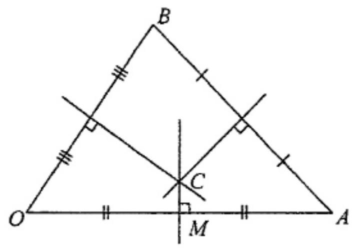

Figure 12

Let C be the circumcentre of ΔOAB (see Figure 12). Show that $2\overrightarrow{OC}\cdot\mathbf{a}=|\mathbf{a}|^2$ and find $\overrightarrow{OC}\cdot\mathbf{b}$ in terms of $|\mathbf{b}|$.

(3 marks)

(d) Consider the points G, T and C described in (a), (b) and (c) respectively.

(i) Using the above results, find the values of $(\overrightarrow{GT}-2\overrightarrow{CG})\cdot\mathbf{a}$ and $(\overrightarrow{GT}-2\overrightarrow{CG})\cdot\mathbf{b}$.

(ii) Show that G, T and C are collinear.

Note : You may use the following property for vectors in the two-dimensional space : If $\mathbf{w}\cdot\mathbf{u}=\mathbf{w}\cdot\mathbf{v}=0$, where $\mathbf{u}$ and $\mathbf{v}$ are non-parallel, then $\mathbf{w}=\mathbf{0}$.

(5 marks)

END OF PAPER