2004-CE
A MATH

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2004

ADDITIONAL MATHEMATICS

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

1. Answer **ALL** questions in Section A and any **FOUR** questions in Section B.
2. Write your answers in the answer book provided. **For Section A, there is no need to start each question on a fresh page.**
3. All working must be clearly shown.
4. Unless otherwise specified, numerical answers must be **exact**.
5. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
6. The diagrams in the paper are not necessarily drawn to scale.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (62 marks)
Answer **ALL** questions in this section.

1. Find

(a) $\int \cos(3x+1)\,dx$,

(b) $\int (2-x)^{2004}\,dx$.

(4 marks)

2. (a) Expand $(1+2x)^6$ in ascending powers of x up to the term x^3.

(b) Find the constant term in the expansion of $(1-\frac{1}{x}+\frac{1}{x^2})(1+2x)^6$.

(4 marks)

3. The slope at any point (x, y) of a curve C is given by $\frac{dy}{dx} = 3x^2 + 1$. If the x-intercept of C is 1, find the equation of C.

(4 marks)

4.

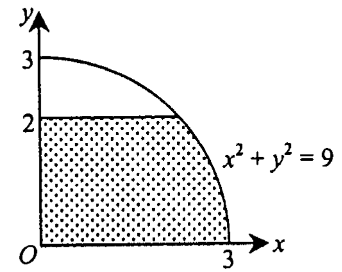

Figure 1

In Figure 1, the shaded region is bounded by the circle $x^2 + y^2 = 9$, the x-axis, the y-axis and the line $y = 2$. Find the volume of the solid generated by revolving the region about the y-axis.

(4 marks)

5. Find the general solution of the equation

$$\sin 3x + \sin x = \cos x .$$

(5 marks)

6.

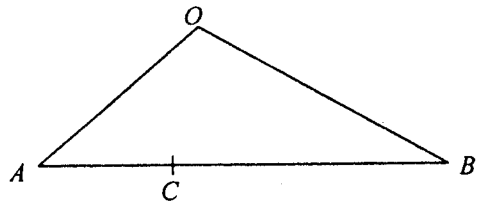

Figure 2

In Figure 2, OAB is a triangle. C is a point on AB such that $AC : CB = 1 : 2$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) If $|\mathbf{a}| = 1$, $|\mathbf{b}| = 2$ and $\angle AOB = \frac{2\pi}{3}$, find $|\overrightarrow{OC}|$.

(5 marks)

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7. Prove that $9^n - 1$ is divisible by 8 for all positive integers n.

(5 marks)

8. Solve the following equations :

(a) $|x-3| = 1$.

(b) $|x-1| = |x^2 - 4x + 3|$.

(6 marks)

9.

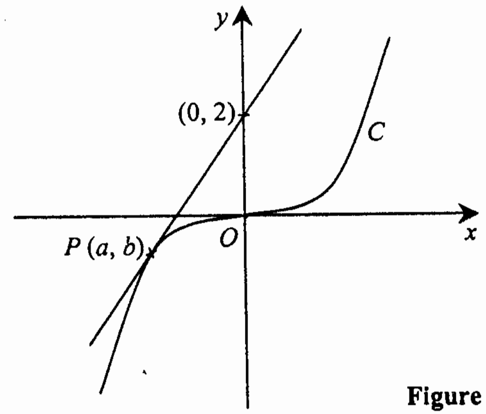

Figure 3

In Figure 3, $P(a, b)$ is a point on the curve $C: y = x^3$. The tangent to C at P passes through the point (0, 2).

(a) Show that $b = 3a^3 + 2$.

(b) Find the values of a and b.

(6 marks)

10. Let O be the origin and A be the point (3, 4). P is a variable point such that the area of ΔOPA is always equal to 2.

Show that the locus of P is a pair of parallel lines.

Find the distance between these two lines.

(6 marks)

11.

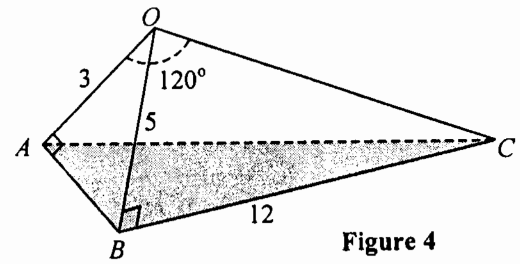

Figure 4

In Figure 4, $OABC$ is a pyramid such that $OA = 3$, $OB = 5$, $BC = 12$, $\angle AOC = 120°$ and $\angle OAB = \angle OBC = 90°$.

(a) Find AC.

(b) A student says that the angle between the planes OBC and ABC can be represented by $\angle OBA$.

Determine whether the student is correct or not.

(6 marks)

12.

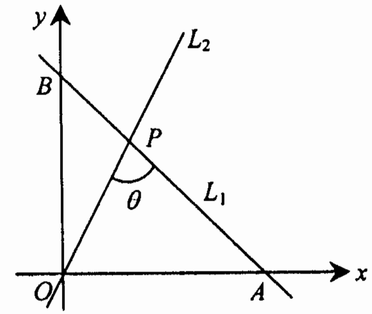

Figure 5

Figure 5 shows two lines $L_1 : y = -x + c$ and $L_2 : y = 2x$, where $c > 0$. The two lines intersect at point P.

(a) Let θ be the acute angle between L_1 and L_2. Find $\tan\theta$.

(b) L_1 intersects the x- and y-axes at the points A and B respectively. Find $AP : PB$.

(7 marks)

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Section B (48 marks)
Answer any **FOUR** questions in this section.
Each question carries 12 marks.

13.

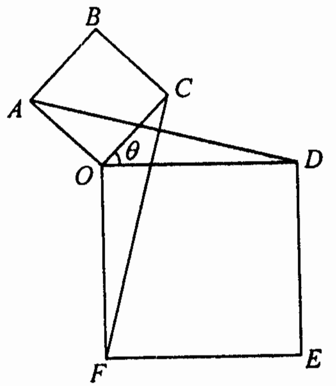

Figure 6

In Figure 6, $OABC$ and $ODEF$ are two squares such that $OA = 1$, $OF = 2$ and $\angle COD = \theta$, where $0° < \theta < 90°$. Let $\overrightarrow{OD} = 2\mathbf{i}$ and $\overrightarrow{OF} = -2\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are two perpendicular unit vectors.

(a) (i) Express $\overrightarrow{OC}$ and $\overrightarrow{OA}$ in terms of θ, $\mathbf{i}$ and $\mathbf{j}$.

(ii) Show that $\overrightarrow{AD} = (2 + \sin\theta)\mathbf{i} - \cos\theta\,\mathbf{j}$.

(4 marks)

(b) Show that $\overrightarrow{AD}$ is always perpendicular to $\overrightarrow{FC}$.

(4 marks)

(c) Find the value(s) of θ such that points B, C and E are collinear. Give your answer(s) correct to the nearest degree.

(4 marks)

14. C_1 and C_2 are the circles $x^2 + y^2 = 36$ and $x^2 + y^2 - 10x + 16 = 0$ respectively.

(a) (i) Show that, for all values of θ, the variable point $P(6\cos\theta, 6\sin\theta)$ always lies on C_1.

(ii) Find, in terms of θ, the equation of the tangent to C_1 at $P(6\cos\theta, 6\sin\theta)$.

(3 marks)

(b)

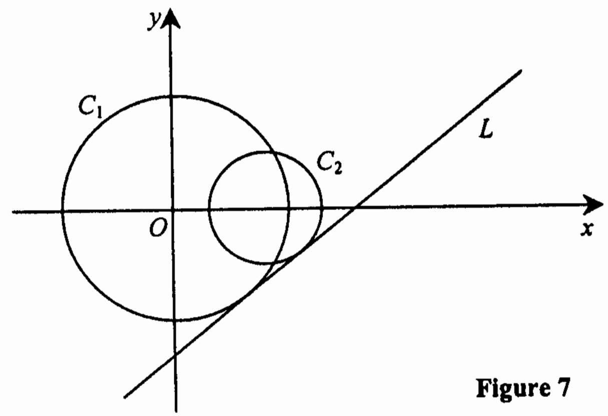

Figure 7

Let L be the common tangent to C_1 and C_2 with a positive slope (see Figure 7).

(i) Using (a), or otherwise, find the equation of L.

(ii) It is known that C_1 and C_2 intersect at two distinct points Q and R. A circle C_3, passing through Q and R, is bisected by L. Find the equation of C_3.

(9 marks)

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15. Given two curves $C_1 : y = f(x)$, where $f(x)$ is a quadratic function, and

$$C_2 : y = -\frac{1}{5}x^2 - (\frac{h-20}{10})x + h .$$

C_1 has the vertex (4, 9) and passes through the point (10, 0).

(a) Show that $f(x) = -\frac{1}{4}x^2 + 2x + 5$.

(3 marks)

(b) (i) Show that C_2 also passes through the point (10, 0).

(ii) If C_1 and C_2 meet at two points, find, in terms of h, the x-coordinate of the point other than (10, 0).

(5 marks)

(c) Figure 8 shows a fountain. A vertical water pipe OP of height 15 units is installed on the horizontal ground. Two streams of water are ejected continuously from two small holes D_1 and D_2 in the pipe, with D_2 above D_1. The two streams of water lie in the same vertical plane. A rectangular coordinate system is introduced in this plane, with O as the origin and OP on the positive y-axis. The fountain is designed such that the stream of water ejected from D_1 lies on the curve C_1, and that ejected from D_2 lies on C_2.

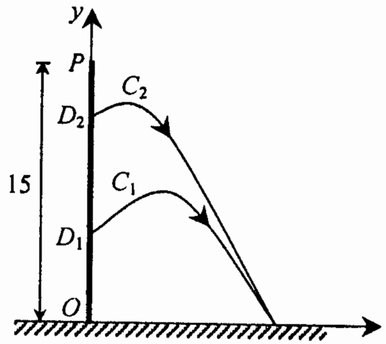

Figure 8

(i) Find OD_1.

(ii) If the two streams of water do not cross each other in the air before meeting at the same point on the ground, find the range of possible values of OD_2.

(4 marks)

16.

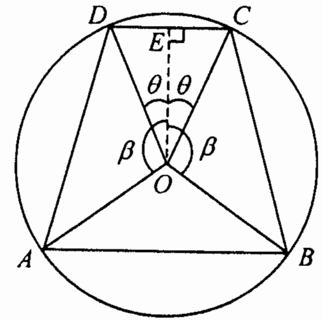

Figure 9

In Figure 9, $ABCD$ is a quadrilateral inscribed in a circle centred at O and with radius r, such that $AB \mathbin{//} DC$ and O lies inside the quadrilateral.

Let $\angle COD = 2\theta$ and reflex $\angle AOB = 2\beta$, where $0 < \theta < \frac{\pi}{2} < \beta < \pi$. Point E denotes the foot of perpendicular from O to DC. Let S be the area of $ABCD$.

(a) Show that $S = \frac{r^2}{2}[\sin 2\theta - \sin 2\beta + 2\sin(\beta - \theta)]$.

(3 marks)

(b) Suppose β is fixed. Let S_β be the greatest value of S as θ varies.

Show that $S_\beta = 2r^2 \sin^3(\frac{2\beta}{3})$ and the corresponding value of θ is $\frac{\beta}{3}$.

[Hint : You may use the identity $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$.]

(6 marks)

(c) A student says :

Among all possible values of β, the quadrilateral $ABCD$ becomes a square when S_β in (b) attains its greatest value.

Determine whether the student is correct or not.

(3 marks)

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17. (a) Let $y = (x - \pi)\sin x + \cos x$.

(i) Show that $\dfrac{dy}{dx} = (x - \pi)\cos x$.

Hence find $\displaystyle\int (x - \pi)\cos x \, dx$.

(ii) Figure 10 shows the graph of $y = (x - \pi)\cos x$ for $0 \le x \le \dfrac{3\pi}{2}$.

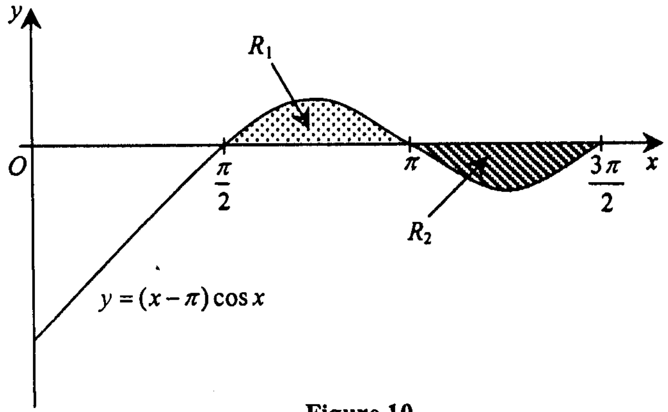

Figure 10

(1) Find the areas of the two shaded regions R_1 and R_2 as shown in Figure 10.

(2) Find $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (x - \pi)\cos x \, dx$.

(7 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

If you attempt Question 17, fill in the first three boxes above and tie this sheet to your answer book.

(b)

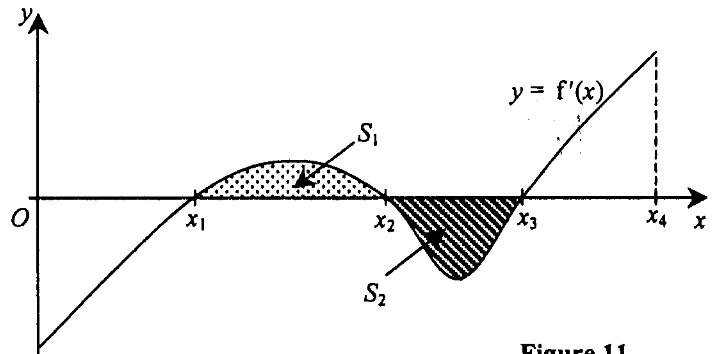

Figure 11

Let $f(x)$ be a continuous function. Figure 11 shows a sketch of the graph of $y = f'(x)$ for $0 \le x \le x_4$. It is known that the areas of the shaded regions S_1 and S_2 as shown in Figure 11 are equal.

(i) Show that $f(x_1) = f(x_3)$.

(ii) Furthermore, $f(0) = f(x_4) = 0$ and $f(x) \ne 0$ for $0 < x < x_4$. In Figure 12, draw a sketch of the graph of $y = f(x)$ for $0 \le x \le x_4$.

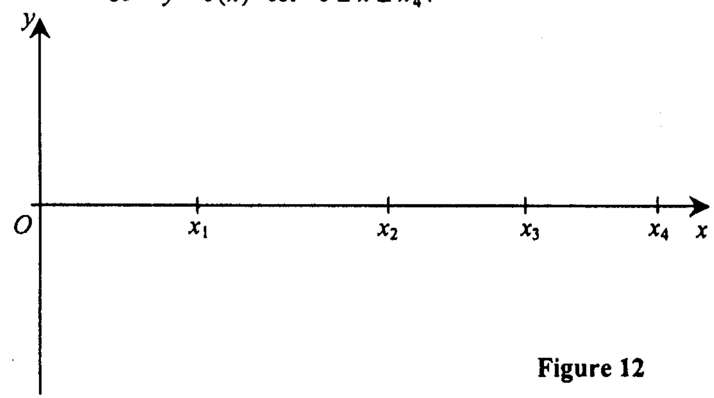

Figure 12

(5 marks)

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