香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2004年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2004

附加數學
ADDITIONAL MATHEMATICS

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。
After the examinations, marking schemes will be available for reference at the teachers' centre.

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative solutions. In general, a correct alternative solution merits <u>all the marks</u> allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol (pp–1) should be used to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u–1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable).

   Note the following points:

   (a) At most deduct 1 mark for wrong/no units for the whole paper.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the <u>net</u> total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles ⬚ ,

   whereas alternative answers are enclosed by solid rectangles ▭ .

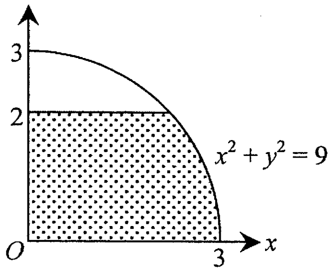
8. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

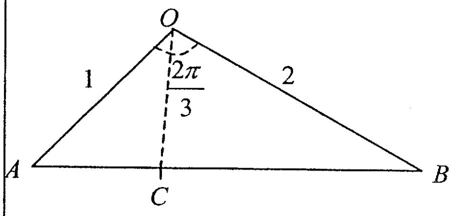
   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted. For answers with an excess degree of accuracy, deduct 1 mark for the first time if happened. In any case, do not deduct any marks for excess degree of accuracy in those steps where candidates could not score any marks.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

10. Unless the form of answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

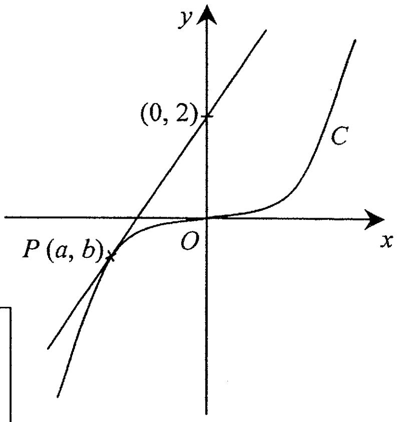
Solution	Marks	Remarks
1. (a) $\int \cos(3x+1)\,dx$		
$= \frac{1}{3}\sin(3x+1)+c$, where c is a constant.	1M+1A	1M for $\int \cos u\,du = \sin u$ Withhold 1A if c was omitted
(b) $\int (2-x)^{2004}\,dx$		
$= -\frac{(2-x)^{2005}}{2005}+c$, where c is a constant.	1M+1A	1M for $\int u^n\,du = \frac{u^{n+1}}{n+1}$
OR $= \frac{(x-2)^{2005}}{2005}+c$		
	4	
2. (a) $(1+2x)^6 = 1 + {}_6C_1(2x) + {}_6C_2(2x)^2 + {}_6C_3(2x)^3 + \cdots$	1M	${}_6C_1 = 6$, ${}_6C_2 = 15$, ${}_6C_3 = 20$ (pp−1) if dots were omitted in all cases.
$= 1+12x+60x^2+160x^3+\cdots$	1A	
(b) $(1-\frac{1}{x}+\frac{1}{x^2})(1+2x)^6$		
$= (1-\frac{1}{x}+\frac{1}{x^2})(1+12x+60x^2+160x^3+\cdots)$		
Constant term		
$= 1(1)-1(12)+1(60)$	1M	
$= 49$	1A	
	4	
3. $\frac{dy}{dx} = 3x^2+1$		
$y = \int (3x^2+1)\,dx$	1M	
$y = x^3+x+k$, where k is a constant.	1A	Award even if k was omitted.
Put $x=1$, $y=0$:		
$0 = 1^3+1+k$	1M	
$k = -2$		
$\therefore$ the equation of C is $y = x^3+x-2$.	1A	
	4	

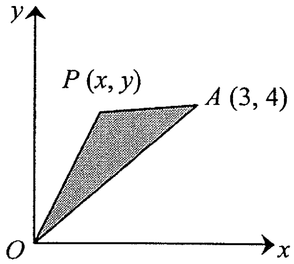
Solution	Marks	Remarks
4. (a) 		
Volume $= \int_0^2 \pi x^2 \, \mathrm{d}y$	1M	For $V = \int_a^b \pi x^2 \, \mathrm{d}y$
$= \int_0^2 \pi(9-y^2)\, \mathrm{d}y$	1A	
$= \pi [9y - \frac{1}{3}y^3]_0^2$	1M	For primitive function
$= \pi(18 - \frac{8}{3})$		
$= \frac{46\pi}{3}$	1A	
	4	
5. $\sin 3x + \sin x = \cos x$ $2\sin 2x \cos x = \cos x$ $\cos x\,(2\sin 2x - 1) = 0$	1A	For LHS
Alternative Solution $\sin 3x + \sin x = \cos x$ $3\sin x - 4\sin^3 x + \sin x = \cos x$ $4\sin x \cos^2 x = \cos x$ $\cos x\,(4\sin x \cos x - 1) = 0$	1A	
$\cos x = 0$ or $\sin 2x = \frac{1}{2}$	1A+1A	
$2x = m\pi + (-1)^m \frac{\pi}{6}$	1M	1M for the correct form of a general solution
$x = 2n\pi \pm \frac{\pi}{2}$ $x = \frac{m\pi}{2} + (-1)^m \frac{\pi}{12}$, where m, n are integers. $(360n° \pm 90°)$ $(90m° + (-1)^m(15°))$ OR $x = n\pi + \frac{\pi}{2}$, $x = (2n+1)\frac{\pi}{2}$	1A	
	5	

Solution	Marks	Remarks
6. (a) $\overrightarrow{OC}=\frac{2\overrightarrow{OA}+\overrightarrow{OB}}{1+2}$		
$=\frac{2\vec{a}+\vec{b}}{3}$	1A	
(b) $\lvert\overrightarrow{OC}\rvert^2=\overrightarrow{OC}\cdot\overrightarrow{OC}$		
$=(\frac{2\vec{a}+\vec{b}}{3})\cdot(\frac{2\vec{a}+\vec{b}}{3})$	1M	
$=\frac{1}{9}[4\vec{a}\cdot\vec{a}+4\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}]$		
$=\frac{1}{9}[4(1)^2+4(1)(2)\cos\frac{2\pi}{3}+(2)^2]$	1A+1A	1A for $\vec{a}\cdot\vec{a}=1$ and $\vec{b}\cdot\vec{b}=4$
$=\frac{4}{9}$		1A for $\vec{a}\cdot\vec{b}=(1)(2)\cos\frac{2\pi}{3}$
$\therefore \lvert\overrightarrow{OC}\rvert=\frac{2}{3}$	1A	Omit vector sign / dot sign in most cases (pp–1)
Alternative Solution		
$AB^2=OA^2+OB^2-2(OA)(OB)\cos\angle AOB$		
$=1^2+2^2-2(1)(2)\cos\frac{2\pi}{3}$	1M	
$AB=\sqrt{7}$		
$AC=\frac{1}{3}AB=\frac{\sqrt{7}}{3}$ $BC=\frac{2\sqrt{7}}{3}$		
$\cos\angle OAB=\frac{OA^2+AB^2-OB^2}{2(OA)(AB)}$ $\cos\angle OBA$		OR $\frac{OB}{\sin\angle OAB}=\frac{AB}{\sin\angle AOB}$ $\frac{2}{\sin\angle OAB}=\frac{\sqrt{7}}{\sin\frac{2\pi}{3}}$ $\sin\angle OAB=\frac{\sqrt{3}}{\sqrt{7}}$
$=\frac{1^2+(\sqrt{7})^2-2^2}{2(1)(\sqrt{7})}$ $=\frac{2^2+(\sqrt{7})^2-1^2}{2(2)(\sqrt{7})}$	1M	
$=\frac{2}{\sqrt{7}}$ $=\frac{5}{2\sqrt{7}}$		
$OC^2=OA^2+AC^2-2(OA)(AC)\cos\angle OAC$		
$=1^2+(\frac{\sqrt{7}}{3})^2-2(1)(\frac{\sqrt{7}}{3})(\frac{2}{\sqrt{7}})$	1M	
$=\frac{4}{9}$		
$\therefore \lvert\overrightarrow{OC}\rvert=\frac{2}{3}$	1A	
	5	

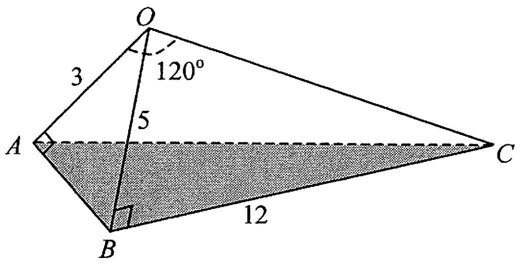
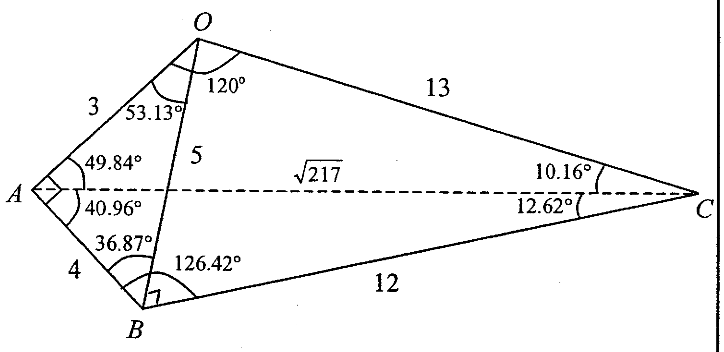
Solution	Marks	Remarks
7. For $n=1$		
$9^n-1=9-1=8$, which is divisible by 8.	1	
∴ the statement is true for $n=1$. Assume 9^k-1 is divisible by 8, where k is a positive integer. (OR $9^k-1=8m$, where k and m are positive integers.)	1	
$9^{k+1}-1=9(9^k)-1$ $=9(8m+1)-1$ $=72m+8$ $=8(9m+1)$ OR $=9^{k+1}-9+8$ $=9(9^k-1)+8$ $=9(8m)+8$ $=8(9m+1)$	1	OR $=8(9^k)+9^k-1$ $=8(9^k)+8m$ $=8(9^k+m)$
∴ $9^{k+1}-1$ is also divisible by 8. ∴ the statement is also true for $n=k+1$ if it is true for $n=k$.	1	
∴ the statement is also true for $n=k+1$ if it is true for $n=k$. By the principle of mathematical induction, the statement is true for all positive integers n.	1	Not awarded if anyone of the above marks was withheld
Alternative Solution (1)		
Consider $x^n-y^n=(x-y)(x^{n-1}+x^{n-2}y+x^{n-3}y^2+\cdots+xy^{n-2}+y^{n-1})$	1M + 1A	1M for considering x^n-y^n
Put $x=9$, $y=1$: $9^n-1=(9-1)(9^{n-1}+9^{n-2}+9^{n-3}+\cdots+9+1)$	1M + 1A	
$=8m$, where $m=9^{n-1}+9^{n-2}+\cdots+1$ is an integer.		
∴ 9^n-1 is divisible by 8 for all positive integers n.	1	
Alternative Solution (2)		
Consider $f(x)=x^n-1$, where n is a positive integer. $f(1)=1^n-1=0$	1M	
∴ $(x-1)$ is a factor of $f(x)$. $f(x)=(x-1)(x^{n-1}+x^{n-2}+\cdots+x+1)$	1A	
Put $x=9$. $9^n-1=(9-1)(9^{n-1}+9^{n-2}+9^{n-3}+\cdots+9+1)$	1M	
$9-1=8$ is a factor of 9^n-1.	1A	
∴ 9^n-1 is divisible by 8 for all positive integers n.	1	
Alternative solution (3)		
$9^n-1=(1+8)^n-1$	2A	
$=1+{}_nC_1(8)+{}_nC_2(8)^2+\cdots+8^n-1$	1M	
$=8[{}_nC_1+{}_nC_2(8)+\cdots+8^{n-1}]$	1A	
∴ 9^n-1 is divisible by 8 for all positive integers n.	1	
	5	

Solution	Marks	Remarks
8. (a) $\lvert x-3\rvert=1$		
$x-3=1$ or $x-3=-1$	1M	
$x=4$ or 2	1A	
Alternative Solution		
$\lvert x-3\rvert=1$		
$(x-3)^2=1$	1M	
$x^2-6x+8=0$		
$x=4$ or 2	1A	
(b) $\lvert x-1\rvert=\lvert x^2-4x+3\rvert$		
$\lvert x-1\rvert=\lvert(x-1)(x-3)\rvert$		
$\lvert x-1\rvert(\lvert x-3\rvert-1)=0$	1M	
$\lvert x-1\rvert=0$ or $\lvert x-3\rvert=1$	1A+1A	
$x=1$ or $x=4$ or 2 (using (a))		
$x=1,2$ or 4	1A	
Alternative Solution (1)		
$\lvert x-1\rvert=\lvert x^2-4x+3\rvert$		
$x-1=x^2-4x+3$ or $x-1=-(x^2-4x+3)$	1M	
$x^2-5x+4=0$ $\quad x^2-3x+2=0$	1A+1A	
$(x-1)(x-4)=0$ $\quad (x-1)(x-2)=0$		
$x=1$ or 4 $\quad x=1$ or 2		
$\therefore x=1,2$ or 4	1A	
Alternative Solution (2)		
$\lvert x-1\rvert=\lvert x^2-4x+3\rvert$		
$(x-1)^2=(x^2-4x+3)^2$	1M	
$(x-1)^2-(x-1)^2(x-3)^2=0$		
$(x-1)^2[(x-3)^2-1]=0$		
$(x-1)^2=0$ or $(x-3)^2=1$	1A+1A	
$x=1$ or $x-3=1$ or -1		
$x=4$ or 2		
$x=1,2$ or 4	1A	
Alternative Solution (3)		
$\lvert x-1\rvert=\lvert x^2-4x+3\rvert$		
$\lvert x-1\rvert=\lvert(x-1)(x-3)\rvert$		
Consider the three cases:		
(1) $x\le 1$, (2) $1<x\le 3$, (3) $x>3$	1M	Accept the equality signs omitted
Case 1:		
$-(x-1)=(x-1)(x-3)$ ....(4)		
$x=1$ or 2		
Since $x\le 1$, $x=1$		
Case 2:		
$x-1=-(x-1)(x-3)$ ....(5)		
$x=1$ or 2		
Since $1<x\le 3$, $x=2$		
Case 3:		
$x-1=(x-1)(x-3)$ ....(6)	2A	Award 1A if only one or two of the equations (4), (5), (6) was/were correct
$x=1$ or 4		
Since $x>3$, $x=4$		
Combining the 3 cases, $x=1$, 2 or 4.	1A	
	6	

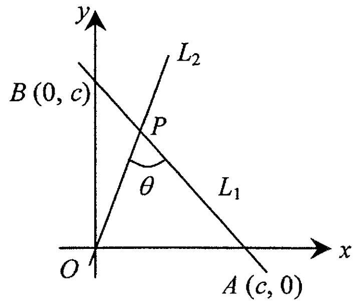
Solution	Marks	Remarks
9. (a) $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2$		
At $P(a, b)$, $\frac{\mathrm{d}y}{\mathrm{d}x}=3a^2$.	1M	
Since the tangent to C at P passes through (0, 2), $\frac{b-2}{a-0}=3a^2$	1M	
$b=3a^3+2$	1	
Alternative Solution $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2$ At $P(a, b)$, $\frac{\mathrm{d}y}{\mathrm{d}x}=3a^2$.	1M	
Equation of the tangent $y-b=3a^2(x-a)$ Put $x=0$, $y=2$: $2-b=3a^2(0-a)$	1M	OR $y=3a^2x+2$ Put $x=a$, $y=b$:
$b=3a^3+2$	1	$b=3a^3+2$
(b) Since P lies on C, $b=a^3$.	1A	
$\begin{cases} b=3a^3+2 \\ b=a^3 \end{cases}$ $3a^3+2=a^3$	1M	
OR $\begin{cases} y=3ax^2+2 \\ y=x^3 \end{cases}$ $3ax^2+2=x^3$	1M	
Substitute $x=a$: $3a^3+2=a^3$	1A	
$a=-1$ $b=a^3=-1$ $\therefore$ $a=b=-1$	1A	For both a and b
	6	

Solution	Marks	Remarks
10. 		
Let the coordinates of P be $(x,\ y)$. Area of $\Delta OAP = \left\|\frac{1}{2}\begin{vmatrix}0&0\\3&4\\x&y\\0&0\end{vmatrix}\right\|$ $= \left\|\frac{1}{2}(3y-4x)\right\|$	1M	Accept omitting absolute values
$\frac{1}{2}\|3y-4x\|=2$ $3y-4x=4$ or $3y-4x=-4$ $4x-3y+4=0$ $\quad 4x-3y-4=0$	1M	
The slopes of the two lines are equal. $\therefore$ the locus of P is a pair of parallel lines.	1	
Alternative Solution Since the area of ΔOPA is a constant, the distance from P to OA is a constant.	2M	
$\therefore$ the locus of P is a pair of parallel lines which are of equal distance from OA.	1	
Consider the line $4x-3y+4=0$: Put $y=0$, $x=-1$. $\therefore (-1,\ 0)$ is a point on the line.	1M	
Distance between the two lines $= \left\|\frac{4(-1)-3(0)-4}{\sqrt{4^2+(-3)^2}}\right\|$	1M	
$=\frac{8}{5}$	1A	
Alternative Solution (1) Distance from O to the line $4x-3y+4=0$ $=\frac{4}{\sqrt{4^2+(-3)^2}}$	1M	
$=\frac{4}{5}$ Distance between the lines $=2\times\frac{4}{5}$	1M	
$=\frac{8}{5}$	1A	

Solution	Marks	Remarks
Alternative Solution (2)		
Distance between the two lines		
$= \left\lvert \dfrac{4-(-4)}{\sqrt{4^2+(-3)^2}} \right\rvert$	2M	Accept omitting absolute values
$= \dfrac{8}{5}$	1A	
Alternative Solution (3)		
Let h be the distance from P to OA.		
Since area of $\Delta OPA = 2$,		
$\dfrac{h(OA)}{2} = 2$	1M	
$h\sqrt{3^2+4^2} = 4$		
$h = \dfrac{4}{5}$		
Distance between the two lines $= 2h$	1M	
$= \dfrac{8}{5}$	1A	
	6	

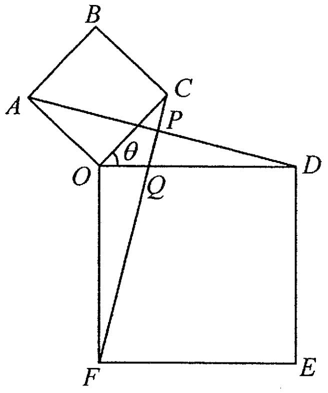
Solution	Marks	Remarks
11. 		
(a) Consider ΔOBC: $OC^2 = OB^2 + BC^2$ $= 5^2 + 12^2$	1M	
$OC = 13$ Consider ΔOAC: By cosine law, $AC^2 = OA^2 + OC^2 - 2(OA)(OC)\cos\angle AOC$ $= 3^2 + 13^2 - 2(3)(13)\cos 120°$	1M	
$= 217$ $AC = \sqrt{217}$	1A	
(b) Marking guideline: 1M : considering $\angle ABC$ 1M : correct method of working 1A : correctly arriving at the conclusion		
Consider ΔABC: $AB^2 = 5^2 - 3^2$ $AB = 4$ $AB^2 + BC^2 = 4^2 + 12^2$	1M	
$= 160 \neq AC^2 \ (= 217)$ $\therefore \ \angle ABC \neq 90°$	1M	For considering $\angle ABC$
OR $\cos\angle ABC = \dfrac{4^2 + 12^2 - (\sqrt{217})^2}{2(4)(12)} = -0.5938$	1M	
$\angle ABC \boxed{\approx 126.4°} \neq 90°$	1M	For considering $\angle ABC$
$\therefore$ $\angle OBA$ does not represent the angle between the planes. The student is incorrect.	1A	
	6	
		

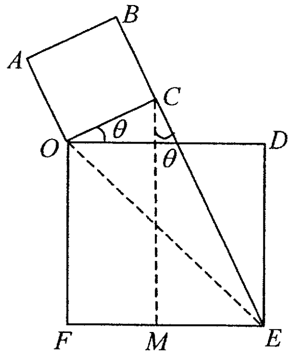
Solution	Marks	Remarks
12. (a)		

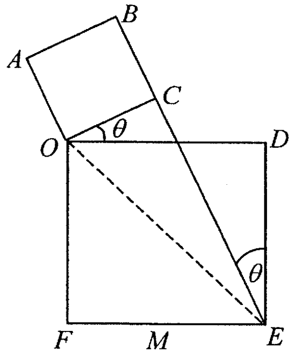


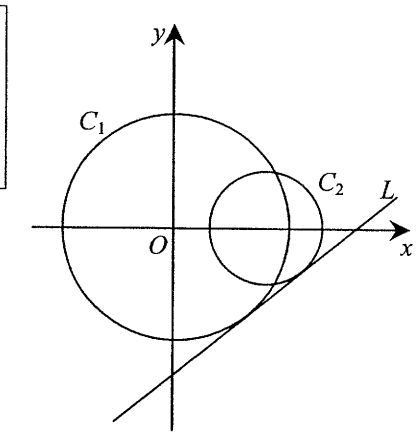
Solution	Marks	Remarks
Slope of $L_1 = -1$ Slope of $L_2 = 2$	$\}$ 1A	
$\tan\theta = \left\lvert \frac{m_2 - m_1}{1 + m_1 m_2} \right\rvert$ $= \left\lvert \frac{2-(-1)}{1+2(-1)} \right\rvert$	1M	Accept omitting absolute sign
$= 3$	1A	
(b) The coordinates of A and B are $(c, 0)$ and $(0, c)$ respectively.	1A	
$\begin{cases} y = 2x \\ y = -x + c \end{cases}$ $2x = -x + c$	1M	
$x = \frac{c}{3}$		
$y = \frac{2c}{3}$ $\therefore$ the coordinates of P are $(\frac{c}{3}, \frac{2c}{3})$.		
Let $AP : PB = r : 1$. $\frac{1(c) + r(0)}{1+r} = \frac{c}{3}$ $r = 2$ OR $\frac{1(0) + r(c)}{1+r} = \frac{2c}{3}$ $r = 2$	1M	
$\therefore AP : PB = 2 : 1$	1A	
OR $AP : PB =$ area of ΔOAP : area of ΔOBP $= \frac{1}{2}(c)(\frac{2c}{3}) : \frac{1}{2}(c)(\frac{c}{3})$	1M	
$= 2 : 1$	1A	

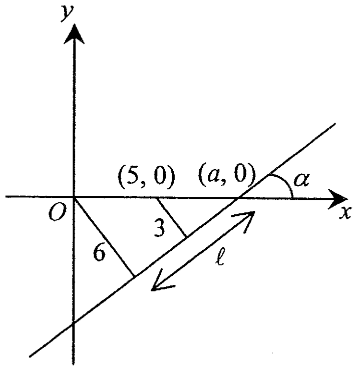
Solution	Marks	Remarks
Alternative Solution (1)		
The coordinates of A and B are $(c, 0)$ and $(0, c)$ respectively.	1A	
Let $AP : PB = r : 1$ and coordinates of P be (u, v).		
$u = \frac{1(c)+r(0)}{1+r}$ $= \frac{c}{1+r}$ $v = \frac{1(0)+r(c)}{1+r}$ $= \frac{rc}{1+r}$	1M	
Since P lies on L_2, $\frac{rc}{1+r} = 2(\frac{c}{1+r})$ $r = 2$	1M	
$\therefore AP : PB = 2 : 1$	1A	
Alternative Solution (2)		
The coordinates of A and B are $(c, 0)$ and $(0, c)$ respectively.	1A	
$\begin{cases} y = 2x \\ y = -x + c \end{cases}$ $2x = -x + c$ $x = \frac{c}{3}$ $y = \frac{2c}{3}$ $\therefore$ the coordinates of P are $(\frac{c}{3}, \frac{2c}{3})$.	1M	
$AP = \sqrt{(c-\frac{c}{3})^2 + (\frac{-2c}{3})^2} = \frac{\sqrt{8}c}{3}$ $PB = \sqrt{(\frac{c}{3})^2 + (\frac{2c}{3}-c)^2} = \frac{\sqrt{2}c}{3}$ $AP : PB = \frac{\sqrt{8}c}{3} : \frac{\sqrt{2}c}{3}$	1M	
$= 2:1$	1A	
Alternative Solution (3)		
Consider ΔOPA: $\frac{AP}{\sin(135°-\theta)} = \frac{OP}{\sin 45°}$ Consider ΔOPB: $\frac{PB}{\sin(\theta-45°)} = \frac{OP}{\sin 45°}$	1M + 1A	1M for using sine law, 1A if at least one was correct
$\frac{AP}{\sin(135°-\theta)} = \frac{PB}{\sin(\theta-45°)}$	1M	
$\frac{AP}{PB} = \frac{\sin(135°-\theta)}{\sin(\theta-45°)} = \frac{1}{\tan(\theta-45°)}$ $= \frac{1+\tan\theta}{\tan\theta-1}$ $= \frac{1+3}{3-1}$ $= 2$	1A	
	7	

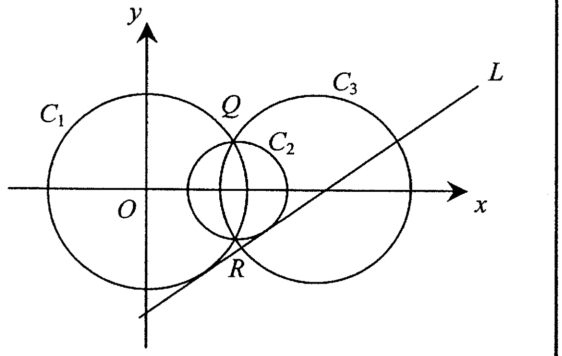
Solution	Marks	Remarks
13. (a) (i) $\overrightarrow{OC}=\cos\theta\,\vec{i}+\sin\theta\,\vec{j}$	1A	
$\overrightarrow{OA}=\cos(90^\circ+\theta)\,\vec{i}+\sin(90^\circ+\theta)\,\vec{j}$ $=-\sin\theta\,\vec{i}+\cos\theta\,\vec{j}$	1A	$-\cos(90^\circ-\theta)\,\vec{i}+\sin(90^\circ-\theta)\vec{j}$
(ii) $\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}$ $=2\vec{i}-(-\sin\theta\,\vec{i}+\cos\theta\vec{j})$	1M	
$=(2+\sin\theta)\,\vec{i}-\cos\theta\,\vec{j}$	1 4	
(b) $\overrightarrow{FC}=\overrightarrow{OC}-\overrightarrow{OF}$ $=\cos\theta\,\vec{i}+\sin\theta\,\vec{j}-(-2\vec{j})$	1M	
$=\cos\theta\,\vec{i}+(\sin\theta+2)\vec{j}$ $\overrightarrow{AD}\cdot\overrightarrow{FC}$ $=[(2+\sin\theta)\,\vec{i}-\cos\theta\,\vec{j}\,]\cdot[\cos\theta\,\vec{i}+(\sin\theta+2)\,\vec{j}\,]$	1M	
$=(2+\sin\theta)\cos\theta-\cos\theta(\sin\theta+2)$ $=0$	1M	
$\overrightarrow{AD}$ is always perpendicular to $\overrightarrow{FC}$.	1	
Alternative Solution Let P be the point of intersection of $\overrightarrow{AD}$ and $\overrightarrow{FC}$. Consider ΔAOD and ΔCOF : $OA=OC$ $OD=OF$ $\angle AOD=90^\circ+\theta=\angle COF$ $\Delta AOD\cong\Delta COF$ (S.A.S.)	1M + 1	1M for attempting to prove $\Delta AOD\cong\Delta COF$
$\angle ADO=\angle CFO$ $\therefore$ O, P, D and F are concyclic.	1	OR $\angle PQD=\angle OQF$ $\angle PDQ+\angle PQD=\angle OFQ+\angle OQF$ $=90^\circ$
$\angle FPD=\angle FOD$ $\angle FPD=90^\circ$ $\therefore$ $\overrightarrow{AD}$ is always perpendicular to $\overrightarrow{FC}$.	1	
	4	

Solution	Marks	Remarks
(c) $\overrightarrow{BC}=\overrightarrow{AO}$ $=-(-\sin\theta\,\vec{i}+\cos\theta\,\vec{j})$ $=\sin\theta\,\vec{i}-\cos\theta\,\vec{j}$ $\overrightarrow{CE}=\overrightarrow{OE}-\overrightarrow{OC}$ $=2\vec{i}-2\vec{j}-(\cos\theta\,\vec{i}+\sin\theta\,\vec{j})$ $=(2-\cos\theta)\vec{i}-(2+\sin\theta)\vec{j}$	1M	For finding $\overrightarrow{BC}$, $\overrightarrow{CE}$, or $\overrightarrow{BE}$
If B, C and E are collinear, $\dfrac{\sin\theta}{2-\cos\theta}=\dfrac{\cos\theta}{2+\sin\theta}$	1M	
OR $\overrightarrow{CE}=\overrightarrow{OE}-\overrightarrow{OC}$ $=2\vec{i}-2\vec{j}-(\cos\theta\,\vec{i}+\sin\theta\,\vec{j})$ $=(2-\cos\theta)\vec{i}-(2+\sin\theta)\vec{j}$	1M	
If B, C and E are collinear, $\overrightarrow{OC}\cdot\overrightarrow{CE}=0$ $(\cos\theta\,\vec{i}+\sin\theta\,\vec{j})\cdot[\,(2-\cos\theta)\,\vec{i}-(2+\sin\theta)\,\vec{j}\,]=0$ $\cos\theta(2-\cos\theta)-\sin\theta(2+\sin\theta)=0$	1M	
OR If B, C and E are collinear, $\angle ECM=\theta$ $CM=2+\sin\theta$ $ME=2-\cos\theta$ Consider ΔCME:	1M	$\left\lvert\overrightarrow{CE}\right\rvert^2=(2-\cos\theta)^2+(2+\sin\theta)^2$ $\left\lvert\overrightarrow{OC}\right\rvert^2+\left\lvert\overrightarrow{CE}\right\rvert^2=\left\lvert\overrightarrow{OE}\right\rvert^2$ $1+[(2-\cos\theta)^2+(2+\sin\theta)^2]=8$
$\tan\theta=\dfrac{2-\cos\theta}{2+\sin\theta}$	1M	
$\cos\theta-\sin\theta=\dfrac{1}{2}$ $\dfrac{1}{\sqrt{2}}\cos\theta-\dfrac{1}{\sqrt{2}}\sin\theta=\dfrac{1}{2\sqrt{2}}$		
$\cos(\theta+45°)=\dfrac{1}{2\sqrt{2}}$ $\boxed{\sin(45°-\theta)=\dfrac{1}{2\sqrt{2}}}$ $\theta+45°\approx 69.3°$ $\boxed{45°-\theta\approx 20.7°}$	1M	
$\theta=24°$ (correct to the nearest degree)	1A	

Solution	Marks	Remarks
OR $\cos\theta - \sin\theta = \frac{1}{2}$ $\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = \frac{1}{4}$	1M	
$\sin 2\theta = \frac{3}{4}$ $2\theta \approx 48.6°$ or $131.4°$ $\theta \approx 24.3°$ or $65.7°$ (rejected) $\theta = 24°$ (correct to the nearest degree)	1A	
OR $\cos\theta - \sin\theta = \frac{1}{2}$ Let $t = \tan\frac{\theta}{2}$. Then $\cos\theta = \frac{1-t^2}{1+t^2}$ and $\sin\theta = \frac{2t}{1-t^2}$ $\therefore \frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2} = \frac{1}{2}$	1M	
$3t^2 + 4t - 1 = 0$ $t = \frac{-2 \pm \sqrt{7}}{3}$ $\because\ 0° < \theta < 90°,\ \therefore\ t = \frac{-2+\sqrt{7}}{3} (\approx 0.2153)$ $\theta = 24°$ (correct to the nearest degree)	1A	
Alternative Solution Consider ΔOCE: $\sin\angle OEC = \frac{OC}{OE}$ $= \frac{1}{\sqrt{2^2+2^2}} = \frac{1}{\sqrt{8}}$	1M	
$\angle OEC \approx 20.7°$ $\theta = 45° - \angle OEC$ $\approx 45° - 20.7°$	2M	
$= 24°$ (correct to the nearest degree)	1A	
	4	Omit vector sign/dot sign in most cases (pp–1)

Solution	Marks	Remarks
14. (a) (i) $x^2+y^2=(6\cos\theta)^2+(6\sin\theta)^2$ $=36$ $\therefore$ P always lies on C_1.	1	
(ii) $x^2+y^2=36$ $2x+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ At P, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-6\cos\theta}{6\sin\theta}=\dfrac{-\cos\theta}{\sin\theta}$ Equation of tangent to C at P is $y-6\sin\theta=\dfrac{-\cos\theta}{\sin\theta}(x-6\cos\theta)$	$\left.\right\}$ 1M	
$y\sin\theta-6\sin^2\theta=-x\cos\theta+6\cos^2\theta$		
$x\cos\theta+y\sin\theta-6=0$	1A	$y=(-\cot\theta)x+6\sin\theta+6\cos\theta\cot\theta$
Alternative Solution Using the formula $xx_1+yy_1=36$, the equation of the tangent to C_1 at P is		
$x(6\cos\theta)+y(6\sin\theta)=36$	1A	
$x\cos\theta+y\sin\theta-6=0$	1A	
	3	
(b) (i) $x^2+y^2-10x+16=0$ $(x-5)^2+y^2=9$ Centre of C_2 is (5, 0), radius = 3	$\left.\right\}$ 1M	
If L is a common tangent to C_1 and C_2, $\left\lvert\dfrac{5\cos\theta+0(\sin\theta)-6}{\sqrt{\cos^2\theta+\sin^2\theta}}\right\rvert=3$	1M+1M	1M for distance formula Accept omitting absolute sign
$\lvert 5\cos\theta-6\rvert=3$ $5\cos\theta-6=3$ or $5\cos\theta-6=-3$		
$\cos\theta=\dfrac{9}{5}$ (rejected) or $\cos\theta=\dfrac{3}{5}$	1A	
OR $\begin{cases}x\cos\theta+y\sin\theta-6=0 & \text{------(1)}\\ x^2+y^2-10x+16=0 & \text{------(2)}\end{cases}$ From (1) : $y=\dfrac{6-x\cos\theta}{\sin\theta}$ Substitute into (2) :		
$x^2+(\dfrac{6-x\cos\theta}{\sin\theta})^2-10x+16=0$	1M	
$x^2-(12\cos\theta+10\sin^2\theta)x+36+16\sin^2\theta=0$		
$\Delta=(12\cos\theta+10\sin^2\theta)^2-4(36+16\sin^2\theta)=0$	1M	
$25(1-\cos^2\theta)+60\cos\theta-52=0$	1M	
$25\cos^2\theta-60\cos\theta+27=0$		
$\cos\theta=\dfrac{9}{5}$ (rejected) or $\cos\theta=\dfrac{3}{5}$	1A	

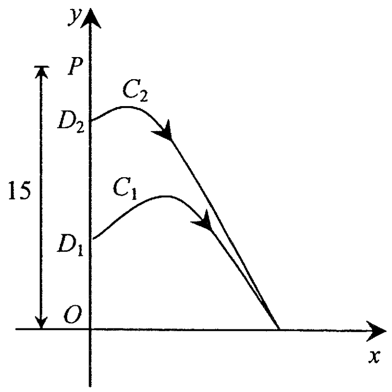
Solution	Marks	Remarks
When $\cos\theta=\frac{3}{5}$, $\sin\theta=\frac{4}{5}$ (rejected) or $-\frac{4}{5}$		
Equation of *L* is $\frac{3}{5}x+\frac{4}{5}y-6=0$ (rejected) or $\frac{3}{5}x-\frac{4}{5}y-6=0$ $3x-4y-30=0$	1A	
Alternative Solution $x^2+y^2-10x+16=0$ $(x-5)^2+y^2=9$ Centre of C_2 is (5, 0), radius = 3	1M	
$\frac{a-5}{a}=\frac{3}{6}$ $a=10$	1M	
$\ell=\sqrt{10^2-6^2}=8$ Slope of the tangent $=\tan\alpha=\frac{3}{4}$	1M + 1A	
The equation of the tangent is $y=\frac{3}{4}(x-10)$	1A	
(ii)		
Equation of the family of circles passing through *Q* and *R* is $x^2+y^2-10x+16+k(x^2+y^2-36)=0$ (*k* is a constant)	1M	OR $x^2+y^2-36+k_1(x^2+y^2-10x+16)=0$
$(1+k)(x^2+y^2)-10x+16-36k=0$		OR $x^2+y^2-36+k_2(5x-26)=0$
The centre of the circle is $(\frac{5}{1+k},0)$.	1M	OR $(\frac{5k_1}{1+k_1},0)$ OR $(-\frac{5k_2}{2},0)$
The centre of C_3 lies on *L*. $3(\frac{5}{1+k})-4(0)-30=0$	1M	
$k=-\frac{1}{2}$		OR $k_1=-2$ OR $k_2=-4$
The equation of C_3 is $x^2+y^2-10x+16-\frac{1}{2}(x^2+y^2-36)=0$		
$x^2+y^2-20x+68=0$	1A	(OR $(x-10)^2+y^2=32$)

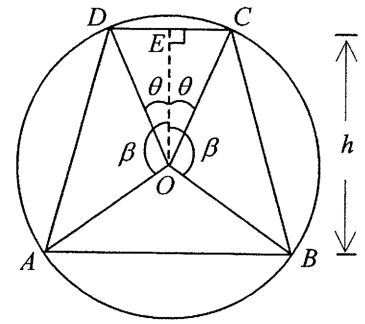


Solution	Marks	Remarks
Alternative Solution $\begin{cases} x^2+y^2-10x+16=0 \\ x^2+y^2=36 \end{cases}$		$\begin{cases} x^2+y^2-10x+16=0 \\ 5x-26=0 \end{cases}$
$x^2+(36-x^2)-10x+16=0$	1M	
$x=\frac{26}{5}$ $y^2=36-(\frac{26}{5})^2$ $y=\pm\frac{4\sqrt{14}}{5}$ $\therefore$ the coordinates of Q are $(\frac{26}{5}, \frac{4\sqrt{14}}{5})$. Centre of C_3 is at the point where L cuts the x-axis.		
$\therefore$ the coordinates of the centre of C_3 are $(10, 0)$.	1M	
Radius of C_3 $=\sqrt{(10-\frac{26}{5})^2+(0-\frac{4\sqrt{14}}{5})^2}=\sqrt{32}$	1M	
The equation of C_3 is $(x-10)^2+y^2=32$	1A	
	9	

Solution	Marks	Remarks
15. (a) Let $f(x) = a(x-4)^2 + 9$.	1A	
Put $f(10) = 0$:		
$0 = a(10-4)^2 + 9$	1M	
$a = -\frac{1}{4}$		
$\therefore f(x) = -\frac{1}{4}(x-4)^2 + 9$		
$= -\frac{1}{4}x^2 + 2x + 5$	1	
Alternative Solution (1)		
Let the equation of C_1 be $y = ax^2 + bx + c$.		
Since C_1 passes through (4, 9) and (10, 0)		
$16a + 4b + c = 9$ - - - - - (1) $100a + 10b + c = 0$ - - - - - (2)	$\left.\right\}$ 1A	OR $c - \frac{b^2}{4a} = 9$
Since the vertex of C_1 is (4, 9),		
$\frac{dy}{dx} = 2ax + b$		OR $-\frac{b}{2a} = 4$
$2a(4) + b = 0$	1M	
$b = -8a$ - - - - - (3)		
Solve (1), (2) and (3),		
$a = -\frac{1}{4}$, $b = 2$, $c = 5$		
$\therefore f(x) = -\frac{1}{4}x^2 + 2x + 5$	1	
Alternative Solution (2)		
Consider $g(x) = -\frac{1}{4}x^2 + 2x + 5$.		
$g(x) = -\frac{1}{4}(x^2 - 8x) + 5$		
$= -\frac{1}{4}(x^2 - 8x + 16 - 16) + 5$	1M	For completing square
$= -\frac{1}{4}(x-4)^2 + 9$	1A	
The vertex of $y = g(x)$ is (4, 9).		
$g(10) = -\frac{1}{4}(10)^2 + 2(10) + 5$		
$= 0$		
$g(x)$ passes through the point (10, 0).		
$\therefore f(x) = -\frac{1}{4}x^2 + 2x + 5$	1	
	3	

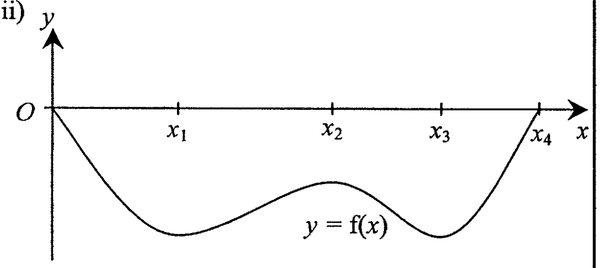
Solution	Marks	Remarks
(b) (i) $y=-\frac{1}{5}x^2-(\frac{h-20}{10})x+h$		
Put $x=10$,		
$y=-\frac{1}{5}(10)^2-(\frac{h-20}{10})(10)+h$		
$=-20-(h-20)+h=0$	1	
$\therefore$ C_2 also passes through the point (10, 0).		
(ii) $\begin{cases} y=-\frac{1}{4}x^2+2x+5 \\ y=-\frac{1}{5}x^2-(\frac{h-20}{10})x+h \end{cases}$		
$-\frac{1}{4}x^2+2x+5=-\frac{1}{5}x^2-(\frac{h-20}{10})x+h$	1M	
$\frac{1}{20}x^2-\frac{h}{10}x+h-5=0$	1A	
$x^2-2hx+20h-100=0$ ------ (*)		
From (a) and (b) (i), $x=10$ is a root of (*). Let α be the other root of the equation. Sum of roots $=2h$		
$\alpha+10=2h$	1M	OR $\alpha(10)=20h-100$
$\alpha=2h-10$	1A	$\alpha=2h-10$
$\therefore$ the x-coordinate of the point of intersection other than (10, 0) is $2h-10$.		
OR $(x-10)[x-(2h-10)]=0$	1M	
$x=10$ or $2h-10$		
$\therefore$ the x-coordinate of the point of intersection other than (10, 0) is $2h-10$.	1A	
	5	

Solution	Marks	Remarks
(c) (i) $C_1: y=-\frac{1}{4}x^2+2x+5$ Put $x=0$, $y=5$ $\therefore OD_1=5$	1A	
(ii) $C_2: y=-\frac{1}{5}x^2-(\frac{h-20}{10})x+h$ Put $x=0$, $y=h$ $\therefore OD_2=h$ As D_2 is above D_1, $5<h\leq 15$ ------(1)	1M	
From (b) (ii), C_1 and C_2 intersect at the points where $x=10$ and $x=2h-10$. In order for the two streams of water not to cross each other before reaching the ground, $\boxed{\begin{array}{l}2h-10<0 \text{ or} \\ \quad h<5 \text{ or}\end{array}}$ $\begin{array}{l}2h-10\geq 10 \\ h\geq 10 \text{ ------ (2)}\end{array}$	1M	
Combining (1) and (2), $10\leq OD_2\leq 15$.	1A 4	

Solution	Marks	Remarks
16. (a) S = area of ΔOCD + area of ΔOBC + area of ΔOAB + area of ΔOAD		
$= \frac{r^2}{2}\sin 2\theta + \frac{r^2}{2}\sin(\beta-\theta) + \frac{r^2}{2}\sin(2\pi-2\beta) + \frac{r^2}{2}\sin(\beta-\theta)$	1M+1A	
$= \frac{r^2}{2}[\sin 2\theta - \sin 2\beta + 2\sin(\beta-\theta)]$	1	
Alternative Solution $S = \frac{1}{2}(AB+CD)h$		
$= \frac{1}{2}[2r\sin(\pi-\beta) + 2r\sin\theta]\,[r\cos\theta + r\cos(\pi-\beta)]$	1M+1A	
$= r^2(\sin\beta + \sin\theta)(\cos\theta - \cos\beta)$ $= r^2(\sin\beta\cos\theta - \sin\beta\cos\beta + \sin\theta\cos\theta - \sin\theta\cos\beta)$		
$= \frac{r^2}{2}[\sin 2\theta - \sin 2\beta + 2\sin(\beta-\theta)]$	1	
	3	
(b) $S = \frac{r^2}{2}[\sin 2\theta - \sin 2\beta + 2\sin(\beta-\theta)]$		
$\frac{\mathrm{d}S}{\mathrm{d}\theta} = \frac{r^2}{2}[2\cos 2\theta - 2\cos(\beta-\theta)]$	1M	
$\boxed{\frac{\mathrm{d}S}{\mathrm{d}\theta} = 0}$ $\cos 2\theta = \cos(\beta-\theta)$	1A	OR $\cos 2\theta - \cos(\beta-\theta) = 0$ $-2\sin\frac{\theta+\beta}{2}\sin\frac{3\theta-\beta}{2} = 0$ $\theta = \frac{\beta}{3}$
$2\theta = 2n\pi \pm (\beta-\theta)$ $2\theta = \beta-\theta$ $\theta = \frac{\beta}{3}$ $\boxed{2\theta = 2\pi + (\beta-\theta) \quad 2\theta = 2\pi - (\beta-\theta) \quad \theta = \frac{2\pi}{3} + \frac{\beta}{3} \text{ (rejected)} \quad \theta = 2\pi - \beta \text{ (rejected)}}$	1	
$\frac{\mathrm{d}^2S}{\mathrm{d}\theta^2} = -r^2[2\sin 2\theta + \sin(\beta-\theta)]$ At $\theta = \frac{\beta}{3}$, $\frac{\mathrm{d}^2S}{\mathrm{d}\theta^2} = -r^2[2\sin\frac{2\beta}{3} + \sin(\beta - \frac{\beta}{3})]$ $= -3r^2\sin\frac{2\beta}{3} < 0$	1M	For checking
S attains a maximum at $\theta = \frac{\beta}{3}$. As S has only one turning point, S attains the greatest value at $\theta = \frac{\beta}{3}$.		
$S_\beta = \frac{r^2}{2}[\sin\frac{2\beta}{3} - \sin 2\beta + 2\sin(\beta - \frac{\beta}{3})]$ $= \frac{r^2}{2}(3\sin\frac{2\beta}{3} - \sin 2\beta)$		
$= \frac{r^2}{2}[3\sin\frac{2\beta}{3} - (3\sin\frac{2\beta}{3} - 4\sin^3\frac{2\beta}{3})]$	1M	OR $= \frac{r^2}{2}(4\sin^3\frac{2\beta}{3})$, OR $= \frac{r^2}{2}[3\sin\frac{2\beta}{3} - \sin 3(\frac{2\beta}{3})]$
$= 2r^2\sin^3(\frac{2\beta}{3})$	1	
	6	

Solution	Marks	Remarks
(c) Among all possible values of β, S_β attains the greatest value when $\sin(\frac{2\beta}{3}) = 1$,	1A	
i.e. $\frac{2\beta}{3} = \frac{\pi}{2}$ $\beta = \frac{3\pi}{4}$		
$\theta = \frac{\beta}{3} = \frac{\pi}{4}$	1M	
$\angle COD = 2\theta = \frac{\pi}{2}$ $\angle BOC \boxed{= \angle AOD} = \beta - \theta = \frac{3\pi}{4} - \frac{\pi}{4}$ $= \frac{\pi}{2}$ $\boxed{CA \text{ and } BD \text{ are straight lines and } CA \perp BD.}$		
$\therefore ABCD$ is a square $\boxed{\text{(diagonals } \perp \text{ and bisect each other).}}$ $\therefore$ the student is correct.	1	
	3	

Solution	Marks	Remarks
17. (a) (i) $y=(x-\pi)\sin x+\cos x$		
$\frac{dy}{dx}=(x-\pi)\frac{d}{dx}\sin x+\sin x\frac{d}{dx}(x-\pi)+\frac{d}{dx}\cos x$	1M	For product rule
$=(x-\pi)\cos x+\sin x-\sin x$ $=(x-\pi)\cos x$	1	
$\int(x-\pi)\cos x\,dx$ $=(x-\pi)\sin x+\cos x+c$, where c is a constant.	1A	withhold if c was omitted
(ii) (1) Area of R_1 $=\int_{\frac{\pi}{2}}^{\pi}(x-\pi)\cos x\,dx$	1A	can be awarded in finding R_2
$=\left[(x-\pi)\sin x+\cos x\right]_{\frac{\pi}{2}}^{\pi}$ $=(0-1)-(-\frac{\pi}{2}+0)$ $=\frac{\pi}{2}-1$ Area of R_2 $=-\int_{\pi}^{\frac{3\pi}{2}}(x-\pi)\cos x\,dx$ $=-\left[(x-\pi)\sin x+\cos x\right]_{\pi}^{\frac{3\pi}{2}}$ $=-(-\frac{\pi}{2}+0)+(0-1)$		
$=\frac{\pi}{2}-1$	1A	For area of R_1 and area of R_2
(2) $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(x-\pi)\cos x\,dx=\int_{\frac{\pi}{2}}^{\pi}(x-\pi)\cos x\,dx+\int_{\pi}^{\frac{3\pi}{2}}(x-\pi)\cos x\,dx$ $=$ Area of R_1 $-$ Area of R_2 $=(\frac{\pi}{2}-1)-(\frac{\pi}{2}-1)$	1M	(can be omitted)
$=0$	1A	
Alternative Solution $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(x-\pi)\cos x\,dx=\left[(x-\pi)\sin x+\cos x\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$		
$=(-\frac{\pi}{2}+0)-(-\frac{\pi}{2}+0)$	1M	
$=0$	1A	
	7	

Solution	Marks	Remarks
(b) (i) $\int_{x_1}^{x_3} \mathrm{f}'(x)\,\mathrm{d}x = 0$	1A	
$[\mathrm{f}(x)]_{x_1}^{x_3} = 0$		
$\mathrm{f}(x_1) = \mathrm{f}(x_3)$	1	
Alternative Solution Area of S_1 = Area of S_2 $\int_{x_1}^{x_2} \mathrm{f}'(x)\,\mathrm{d}x = -\int_{x_2}^{x_3} \mathrm{f}'(x)\,\mathrm{d}x$ $[\mathrm{f}(x)]_{x_1}^{x_2} = -[\mathrm{f}(x)]_{x_2}^{x_3}$	1A	
$\mathrm{f}(x_2) - \mathrm{f}(x_1) = -[\mathrm{f}(x_3) - \mathrm{f}(x_2)]$		
$\mathrm{f}(x_1) = \mathrm{f}(x_3)$	1	
(ii) 	1A+1A	1A for W-shape, 1A for x-coordinates of the turning points
	1A	All correct
	5	