Solution	Marks	Remarks
1. $(1+x)^n+(1+2x)^n$		
$\boxed{=1+{}_nC_1x+{}_nC_2x^2+\cdots+1+{}_nC_1(2x)+{}_nC_2(2x)^2+\cdots}$	either	
Coefficient of $x^2=({}_nC_2+4\,{}_nC_2)x^2$ pp-1	1A+1A	
$=5\,{}_nC_2$ $\quad 1+nx+\frac{n(n-1)}{2}x^2+\cdots$	both	
$5\,{}_nC_2=75$ $\quad 1+2nx+2n(n-1)x^2$		
${}_nC_2=15$		
$\frac{n(n-1)}{2}=15$ accept	1M	For ${}_nC_2=\frac{n(n-1)}{2}$ ✓ (can be omitted)
$n^2-n-30=0$		
$n=6$ or -5 (rejected)		
$n=6$	1A	
	4	
2. $y=(x-1)^4+4$		
$\frac{dy}{dx}=4(x-1)^3$	1A	
Slope of the line $y=4x+8$ is 4.		
$4(x-1)^3=4$	1M	For setting $\frac{dy}{dx}=4$
$(x-1)^3=1$ $x=2$ Put $x=2$, $y=(2-1)^4+4$ $=5$ Equation of tangent is	1M	For finding x, y and equation of tangent
$\frac{y-5}{x-2}=4$		
$y=4x-3$	1A	
	4	

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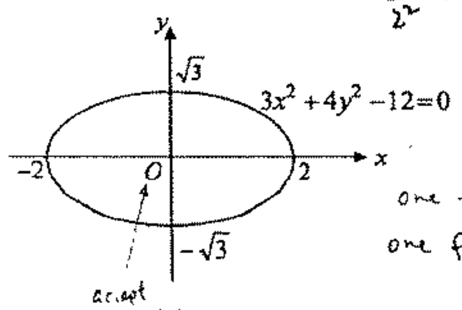
Solution	Marks	Remarks
3. $x\sin y = 2002$		
$\sin y + x\cos y\frac{dy}{dx} = 0$ $\boxed{\text{OR } \sin y\frac{dx}{dy} + x\cos y = 0}$ 1A	1M+1A+1A	1M for product rule, 1A for $\frac{d}{dx}\sin y = \cos y\frac{dy}{dx}$, 1A for $\frac{d}{dx}(2002) = 0$
$\frac{dy}{dx} = \frac{-\tan y}{x}$	1A	Accept $\frac{dy}{dx} = \frac{-\sin y}{x\cos y}$
Alternative solution (1)		
$\sin y = \frac{2002}{x}$	1A	
$\cos y\frac{dy}{dx} = \frac{-2002}{x^2}$	1M+1A	1M for chain rule 1A for RHS
$\frac{dy}{dx} = \frac{-2002}{x^2\cos y} = \frac{-2002}{\cos y}\cdot\frac{\sin y}{2002}\cdot\frac{1}{x} = \frac{-\tan y}{x}$	1A	
Alternative solution (2)		
$x = \frac{2002}{\sin y}$	1A	
$\frac{dx}{dy} = -2002\csc y\cot y = \frac{-2002\cos y}{\sin^2 y}$	1M+1A	1M for finding $\frac{dx}{dy}$ 1A for RHS
$\frac{dy}{dx} = \frac{-\sin y\tan y}{2002}$	1A	
	4	
4. Let $x = \sin\theta$. $dx = \cos\theta\, d\theta$	1A	
$\int_0^{\frac{1}{2}}\frac{1}{\sqrt{1-x^2}}dx = \int_0^{\frac{\pi}{6}}\frac{1}{\sqrt{1-\sin^2\theta}}\cos\theta\, d\theta$	1A+1M	1A for limits, 1M for integrand
$= \int_0^{\frac{\pi}{6}} d\theta$		
$= [\theta]_0^{\frac{\pi}{6}}$		
$= \frac{\pi}{6}$	1A	Omit dx, $d\theta$ in most cases (pp−1)
	4	

Solution	Marks	Remarks
5. (a) $\lvert x-4\rvert = 2\sqrt{(x-1)^2+y^2}$	1M+1A (either one / all)	1M for distance formula Accept omitting absolute sign
$x^2-8x+16 = 4(x-1)^2+4y^2$ $= 4x^2-8x+4+4y^2$		
$3x^2+4y^2-12=0$	1	
(b) $\frac{x^2}{2^2}+\frac{y^2}{\sqrt{3}^2}=1$ one for x one for y	1A+1A	1A for shape Axes not labeled (pp−1)
	5	
6. $\boxed{\begin{cases} y=\sin x \\ y=\cos x \end{cases} \quad \sin x = \cos x}$	1M	(can be omitted)
$x=\frac{\pi}{4}$ or 45° (accept)		
Area $= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$ accept $\int_0^{\frac{\pi}{4}} (\sin x - \cos x)\,dx$	1M+1A	1M for $= \int_a^b (y_2-y_1)\,dx$
$= \left[\sin x + \cos x\right]_0^{\frac{\pi}{4}}$	1M	for $\int \cos x\,dx = \sin x$ and $\int \sin x\,dx = -\cos x$
$= \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-1$ $= \sqrt{2}-1$	1A	
	5	

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Solution	Marks	Remarks
7. (a) $\lvert x-1\rvert > 2$		
$x-1>2$ or $x-1<-2$	1A	
$x>3$ or $x<-1$	1A	
Alternative solution (1)		
$\lvert x-1\rvert > 2$		
$(x-1)^2 > 4$	1A	
$x^2-2x-3>0$		
$(x+1)(x-3)>0$		
$x>3$ or $x<-1$	1A	
Alternative solution (2)		
Consider the cases (1) $x \geq 1$, (2) $x<1$.		Accept including "=" sign
Case 1 : $x \geq 1$		
The inequality becomes		
$x-1>2$	1A	
$x>3$		
Since $x>1$, $x>3$.		
Case 2 : $x<1$ and		
The inequality becomes		
$-x+1>2$		
$x<-1$		
Since $x<1$, $x<-1$.		
Combining the 2 cases, $x>3$ or $x<-1$.	1A	
(b) $\lvert\, \lvert y\rvert -1 \,\rvert > 2$		
Using the result in (a),		
$\lvert y\rvert > 3$ or $\lvert y\rvert < -1$ (no solution)	1M+1M	1M for using (a), 1M for 2nd term having no solution
$y>3$ or $y<-3$	1A	
Alternative solution (1)		
$\lvert\, \lvert y\rvert -1 \,\rvert > 2$		
$\lvert y\rvert -1 > 2$ or $\lvert y\rvert -1 < -2$	1A	
$\lvert y\rvert > 3$ or $\lvert y\rvert < -1$ (no solution)	1M	For 2nd term having no solution
$y>3$ or $y<-3$	1A	
Alternative solution (2)		
Consider the cases (1) $\lvert y\rvert > 1$, (2) $\lvert y\rvert < 1$.		
Case 1 : $\lvert y\rvert > 1$ ($y>1$ or $y<-1$)		
The inequality becomes		
$\lvert y\rvert -1 > 2$	1A	
$\lvert y\rvert > 3$		
$y>3$ or $y<-3$		
Since $\lvert y\rvert > 1$, $y>3$ or $y<-3$.		
Case 2 : $\lvert y\rvert < 1$ ($-1<y<1$)		
The inequality becomes		
$-\lvert y\rvert +1 > 2$		
$\lvert y\rvert < -1$ (no solution)	1M	
Combining the 2 cases, $y>3$ or $y<-3$.	1A	
	5	

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Solution	Marks	Remarks
8. $\dfrac{\tan x-\sin^2 x}{\tan x+\sin^2 x}=\dfrac{\frac{\sin x}{\cos x}-\sin^2 x}{\frac{\sin x}{\cos x}+\sin^2 x}$		
$=\dfrac{\sin x-\sin^2 x\cos x}{\sin x+\sin^2 x\cos x}$		
$=\dfrac{1-\sin x\cos x}{1+\sin x\cos x}$	1A	
$=\dfrac{1-\frac{1}{2}\sin 2x}{1+\frac{1}{2}\sin 2x}$	1M	For $\sin x\cos x=\frac{1}{2}\sin 2x$
$=\dfrac{2-\sin 2x}{2+\sin 2x}$ $\boxed{=\dfrac{4-(2+\sin 2x)}{2+\sin 2x}}$		
$=\dfrac{4}{2+\sin 2x}-1$	1	
Alternative solution		
$\dfrac{4}{2+\sin 2x}-1$		
$=\dfrac{4}{2+2\sin x\cos x}-1$	1M	For $\sin 2x=2\sin x\cos x$
$=\dfrac{2}{1+\sin x\cos x}-1$		
$=\dfrac{1-\sin x\cos x}{1+\sin x\cos x}$	1A	
$=\dfrac{\tan x(1-\sin x\cos x)}{\tan x(1+\sin x\cos x)}$		
$=\dfrac{\tan x-\sin^2 x}{\tan x+\sin^2 x}$	1	
$\dfrac{\tan x-\sin^2 x}{\tan x+\sin^2 x}$ is the least when $(2+\sin 2x)$ is the greatest, i.e. when $\sin 2x=1$. $\boxed{x=\frac{\pi}{4}}$	1M	(can be omitted)
$\therefore$ Least value $=\dfrac{4}{2+1}-1$		
$=\dfrac{1}{3}$	1A	
	5	

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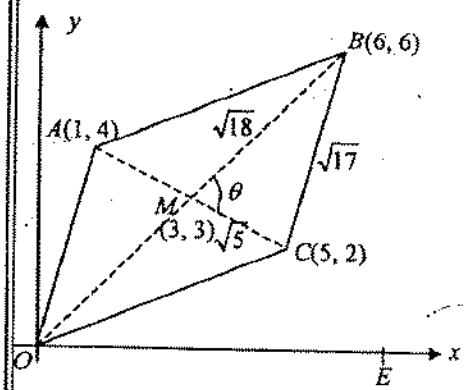
Solution	Marks	Remarks
9. $z^2+z+1=0$		
$z=\dfrac{-1\pm\sqrt{1-4}}{2}$		
$=-\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$	1M	
120° -120°		
α, β are $\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}$, $\cos(-\dfrac{2\pi}{3})+i\sin(-\dfrac{2\pi}{3})$.	1A+1A	Accept degrees, Accept cis $\dfrac{4\pi}{3}$
$\alpha^6+\beta^6=(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3})^6+[\cos(-\dfrac{2\pi}{3})+i\sin(-\dfrac{2\pi}{3})]^6$		
$=\cos 4\pi+i\sin 4\pi+\cos(-4\pi)+i\sin(-4\pi)$		
$=2\cos 4\pi$	1M	For De Moivre Theorem
$=2$	1A	
<u>Alternative solution</u>		
$z^2+z+1=0$		
$(z-1)(z^2+z+1)=0$		
$z^3-1=0$		
As α, β are the roots of $z^3-1=0$,		
$\alpha^3=\beta^3=1$		
$\alpha^6+\beta^6=(\alpha^3)^2+(\beta^3)^2$	1M	
$=1^2+1^2$		
$=2$		

Solution	Marks	Remarks
10. (a) $\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}$		
$=\overrightarrow{OA}+\overrightarrow{OC}$		
$=(\vec{i}+4\vec{j})+(5\vec{i}+2\vec{j})$	1M	For finding $\overrightarrow{OB}$ or $\overrightarrow{AC}$
$=6\vec{i}+6\vec{j}$	1A	
$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}$		
$=(5\vec{i}+2\vec{j})-(\vec{i}+4\vec{j})$		
$=4\vec{i}-2\vec{j}$	1A	
(b) $\overrightarrow{OB}\cdot\overrightarrow{AC}=\lvert\overrightarrow{OB}\rvert\,\lvert\overrightarrow{AC}\rvert\cos\theta$		
$(6\vec{i}+6\vec{j})\cdot(4\vec{i}-2\vec{j})=\sqrt{6^2+6^2}\,\sqrt{4^2+(-2)^2}\cos\theta$	1M	
$24-12=\sqrt{72}\,\sqrt{20}\cos\theta$	1M	For LHS
$\cos\theta=\dfrac{1}{\sqrt{10}}$		
$\theta=72°$ (correct to the nearest degree)	1A	Omit vector sign or dot product sign in most cases (pp-1)
Alternative solution (1)		
$mOB=1$		
$mAC=-\dfrac{1}{2}$	1M	
$\tan\theta=\left\lvert\dfrac{m_1-m_2}{1+m_1m_2}\right\rvert$ accept absolute value		
$=\dfrac{1-(-\frac{1}{2})}{1+1(-\frac{1}{2})}$	1M	
$=3$		
$\theta=72°$ (correct to the nearest degree)	1A	
Alternative solution (2)		
$BC=\sqrt{(6-5)^2+(6-2)^2}=\sqrt{17}$		
$CM=\sqrt{(5-3)^2+(2-3)^2}=\sqrt{5}$	1M	
$MB=\sqrt{(6-3)^2+(6-3)^2}=\sqrt{18}$		
$\cos\theta=\dfrac{5+18-17}{2\sqrt{5}\,\sqrt{18}}$	1M	
$\theta=72°$ (correct to the nearest degree)	1A	
Alternative solution (3)		
Slope of $OB=1$		
$\therefore\ \angle BOE=\tan^{-1}(1)=45°$	1M	
Slope of $AC=\dfrac{4-2}{1-5}=-\dfrac{1}{2}$		
$\therefore\ \ \ =180°+\tan^{-1}(-\frac{1}{2})=153.4°$		
$\theta=\ \ \ -\angle BOE$	1M	
$=180°-(153.4°-45°)$		
$=72°$ (correct to the nearest degree)	1A	
	6	



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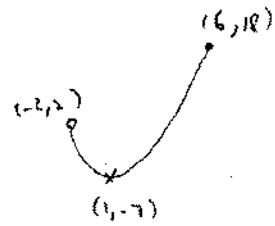
Solution	Marks	Remarks
11. (a) $\begin{cases} y = x^2 - 2x - 6 \\ y = 2x + 6 \end{cases}$		
$x^2 - 2x - 6 = 2x + 6$		
$x^2 - 4x - 12 = 0$	1M	
$x = -2$ or 6		
When $x = -2$, $y = 2$		
When $x = 6$, $y = 18$		
$\therefore$ the coordinates of A and B are $(-2, 2)$ and $(6, 18)$ respectively.	1A	
$f(x) = x^2 - 2x - 6$		
$= (x-1)^2 - 1 - 6 = (x-1)^2 - 7$	1M	
$\therefore$ the coordinates of C are $(1, -7)$.	1A	
(b) The range of values of x such that $f(x) \le g(x)$ is $-2 \le x \le 6$.	1A	
From Figure 3, $f(x) = k$ has only one real root when $2 < k \le 18$ OR $k = -7$.	1A+1A	
	7	

$x^2 - 2x - 6 = 0$

$(x-1)^2 = 7$

$x = 1 \pm \sqrt{7}$

Solution	Marks	Remarks
12. (a) For $n=1$, LHS $=2(2)=4$		
RHS $=1(2^{1+1})=4=$ LHS	1	
$\therefore$ the statement is true for $n=1$.		
Assume $2(2)+3(2)^2+\cdots+(k+1)(2^k)=k(2^{k+1})$ for any positive integer k.	1	
Then $2(2)+3(2)^2+4(2)^3+\cdots+(k+1)(2^k)+(k+2)(2^{k+1})$		
$=k(2^{k+1})+(k+2)(2^{k+1})$	1	
$=2^{k+1}(k+k+2)$		
$=(k+1)2^{k+2}$	1	
The statement is also true for $n=k+1$ if it is true for $n=k$. By the principle of mathematical induction,		
the statement is true for all positive integers n.	1	Not awarded if any one of the above marks was withheld.
(b) $1(2)+2(2^2)+3(2^3)+\cdots+98(2^{98})$	1M	
$=2[1+2(2)+3(2^2)+\cdots+98(2^{97})]$		
$=2+2[2(2)+3(2^2)+\cdots+98(2^{97})]$		
$=2+2(97)(2^{98})$	1M	For using (a)
$=2+97(2^{99})$	1	
Alternative solution (1)		
Put $n=97$:		
$2(2)+3(2^2)+4(2^3)+\cdots+98(2^{97})=97(2^{98})$	1M	For using (a)
Add 1 to both sides:		
$1+2(2)+3(2^2)+\cdots+98(2^{97})=97(2^{98})+1$	1M	
Multiply both sides by 2:		
$2+2(2^2)+3(2^3)+\cdots+98(2^{98})=97(2^{99})+2$	1	
Alternative solution (2)		
$1(2)+2(2^2)+3(2^3)+\cdots+98(2^{98})$		
$=(1+1)(2)+(2+1)(2^2)+(3+1)(2^3)+\cdots+(98+1)(2^{98})-(2+2^2+\cdots+2^{98})$	1M	
$=2(2)+3(2^2)+4(2^3)+\cdots+(99)(2^{98})-(2+2^2+2^3+\cdots 2^{98})$		For using (a)
$=98(2^{98+1})-\dfrac{2(2^{98}-1)}{2-1}$	√1M	
$=98(2^{99})-2^{99}+2$		
$=97(2^{99})+2$	1	
	8	

$1(2)+2(2^2)+\cdots+n2^n=(n-1)2^{n+1}+2$

$n=1$: L.H.S $=2$, R.H.S $=2$

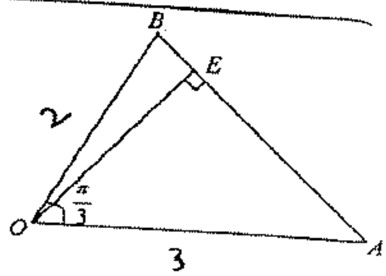
Assume $1(2)+2(2^2)+\cdots+k2^k=(k-1)2^{k+1}+2$

$1(2)+2(2^2)+\cdots+k2^k+(k+1)2^{k+1}$

$=(k-1)2^{k+1}+2+(k+1)2^{k+1}$

$=k2^{k+2}+2$

Set $n=98$

Solution	Marks	Remarks
13. (a) $\vec{a}\cdot\vec{b}=\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\angle AOB$		
$=(3)(2)\cos\frac{\pi}{3}$	1M	
$=3$	1A	
	2	
(b) $\overrightarrow{OE}=\dfrac{t\overrightarrow{OA}+(1-t)\overrightarrow{OB}}{t+(1-t)}$		
$=t\vec{a}+(1-t)\vec{b}$	1A	
Since $OE\perp AB$,		
$\overrightarrow{OE}\cdot\overrightarrow{AB}=0$		
$[t\vec{a}+(1-t)\vec{b}]\cdot(\vec{b}-\vec{a})=0$		
$t\,\vec{a}\cdot\vec{b}-t\,\vec{a}\cdot\vec{a}+(1-t)\,\vec{b}\cdot\vec{b}-(1-t)\,\vec{a}\cdot\vec{b}=0$	1M	For distributive law
$3t-9t+4(1-t)-3(1-t)=0$	1M	For $\vec{a}\cdot\vec{a}=9$ or $\vec{b}\cdot\vec{b}=4$
$1-7t=0$	1A	
$t=\frac{1}{7}$		
$\therefore\overrightarrow{OE}=\frac{1}{7}\vec{a}+\frac{6}{7}\vec{b}$	1A	
$\overrightarrow{BE}=\overrightarrow{OE}-\overrightarrow{OB}$ $=\frac{1}{7}\vec{a}-\frac{1}{7}\vec{b}$	5	
(c) $\overrightarrow{BA}\cdot\overrightarrow{BF}$		
$=\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BF}\rvert\cos\angle ABF$	1A	
$=\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BE}\rvert$	1A	
$=$ a constant since $\lvert\overrightarrow{BA}\rvert$ and $\lvert\overrightarrow{BE}\rvert$ are constants		
$\therefore$ the student is correct.	1	
By Cosine Law,		
$\lvert\overrightarrow{BA}\rvert^2=\lvert\overrightarrow{OA}\rvert^2+\lvert\overrightarrow{OB}\rvert^2-2\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OB}\rvert\cos\angle AOB$		
$=3^2+2^2-2(3)(2)\cos\frac{\pi}{3}$		
$=7$		
$\lvert\overrightarrow{BA}\rvert=\sqrt{7}$		
$\vdots$		
$\lvert\overrightarrow{BE}\rvert=\frac{\sqrt{7}}{7}$	1M	For finding $\lvert\overrightarrow{BA}\rvert$ and $\lvert\overrightarrow{BE}\rvert$
From (b), $\overrightarrow{BE}=\frac{1}{7}\overrightarrow{BA}$		
$\lvert\overrightarrow{BE}\rvert=\frac{1}{7}\lvert\overrightarrow{BA}\rvert$		
$=\frac{\sqrt{7}}{7}$		
$\therefore\overrightarrow{BA}\cdot\overrightarrow{BF}=\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BE}\rvert$		
$=\sqrt{7}\left(\frac{\sqrt{7}}{7}\right)=1$	1A	

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Solution	Marks	Remarks
Alternative solution for finding $\vert\overrightarrow{BA}\vert\vert\overrightarrow{BE}\vert$		
$\overrightarrow{BA}\cdot\overrightarrow{BE}$		
$=(\overrightarrow{OA}-\overrightarrow{OB})\cdot(\overrightarrow{OE}-\overrightarrow{OB})$	1M	
$=(\vec{a}-\vec{b})\cdot(\frac{1}{7}\vec{a}+\frac{6}{7}\vec{b}-\vec{b})$		
$=\frac{1}{7}(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$		
$=\frac{1}{7}(\vec{a}\cdot\vec{a}-2\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b})$		
$=\frac{1}{7}[3^2-2(3)+2^2]$		
$=1$	1A	
$\therefore\ \overrightarrow{BA}\cdot\overrightarrow{BF}=1$		
Alternative solution		
(c) Let $\overrightarrow{OF}=k\overrightarrow{OE}$. ($k$ is a real no.) $=k(\frac{1}{7}\vec{a}+\frac{6}{7}\vec{b})$	1A	
$\overrightarrow{BF}=\overrightarrow{OF}-\overrightarrow{OB}$	1M	
$=k\overrightarrow{OE}-\overrightarrow{OB}$		
$=k(\frac{1}{7}\vec{a}+\frac{6}{7}\vec{b})-\vec{b}$		
$\overrightarrow{BA}\cdot\overrightarrow{BF}$		
$=(\vec{a}-\vec{b})\cdot[\frac{k}{7}\vec{a}+(\frac{6k}{7}-1)\vec{b}]$	1M !	
$=\frac{k}{7}\vec{a}\cdot\vec{a}+(\frac{6k}{7}-1)\vec{a}\cdot\vec{b}-\frac{k}{7}\vec{a}\cdot\vec{b}-(\frac{6k}{7}-1)\vec{b}\cdot\vec{b}$		
$=\frac{k}{7}(9)+(\frac{6k}{7}-1)(3)-\frac{k}{7}(3)-(\frac{6k}{7}-1)(4)$		
$=1$	1A	
$\therefore\ \overrightarrow{BA}\cdot\overrightarrow{BF}$ is constant and the student is correct.	1	
	5	Omit vector sign or dot product sign in most cases (pp−1)

$\overrightarrow{BA}\cdot\overrightarrow{BF}=(\vec{a}-\vec{b})\cdot(\overrightarrow{OF}-\overrightarrow{OB})$

$=(\vec{a}-\vec{b})\cdot\overrightarrow{OF}-(\vec{a}-\vec{b})\cdot\vec{b}$

$=0-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}$

$=-3+2^2$

$=1$

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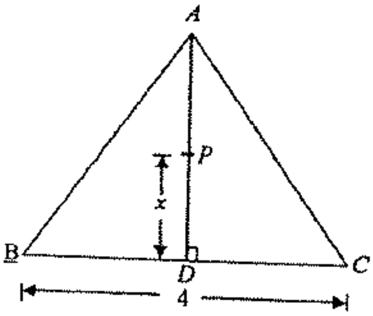
Solution	Marks	Remarks
14. (a) 		
(i) $PB = PC = \sqrt{x^2+4}$		
$PA = (3-x)$		
$r = PA + PB + PC$		
$= 2\sqrt{x^2+4} + (3-x)$	1A	
$\frac{dr}{dx} = 2(\frac{1}{2})\frac{2x}{\sqrt{x^2+4}} - 1$		
$= \frac{2x}{\sqrt{x^2+4}} - 1$	1	
(ii) (1) $\frac{dr}{dx} \geq 0$		Accept $\frac{dr}{dx} > 0$
$\frac{2x}{\sqrt{x^2+4}} - 1 \geq 0$	1M	
$2x \geq \sqrt{x^2+4}$		
$4x^2 \geq x^2 + 4$		
$x \geq \frac{2}{\sqrt{3}}$		
$\therefore r$ is increasing on $3 \geq$ $x \geq \frac{2}{\sqrt{3}}$.	1A	Accept $x > \frac{2}{\sqrt{3}}$
(2) $\frac{dr}{dx} \leq 0$ $\quad x \geq \frac{2\sqrt{3}}{3}$		
$x \leq \frac{2}{\sqrt{3}}$		
$\therefore r$ is decreasing on $0 \leq$ $x \leq \frac{2}{\sqrt{3}}$. $\frac{2\sqrt{3}}{3}$	1A	Accept $x < \frac{2}{\sqrt{3}}$
r is the least at $x = \frac{2}{\sqrt{3}}$. $\frac{2\sqrt{3}}{3}$	1M	
Least value of $r = 2\sqrt{(\frac{2}{\sqrt{3}})^2 + 4} + (3 - \frac{2}{\sqrt{3}})$		
$= 2\sqrt{3} + 3$	1A	

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Solution	Marks	Remarks
Alternative solution		
(ii) $\frac{dr}{dx}=\frac{2x}{\sqrt{x^2+4}}-1=0$	1M	
$x=\frac{2}{\sqrt{3}}$		
$\frac{d^2r}{dx^2}=2(x^2+4)^{-\frac{1}{2}}-x(x^2+4)^{-\frac{3}{2}}(2x)$		
$=\frac{8}{(x^2+4)^{\frac{3}{2}}}$	1M	For checking
$\left.\frac{d^2r}{dx^2}\right\rvert_{x=\frac{2}{\sqrt{3}}}\boxed{=\frac{3\sqrt{3}}{8}}>0$		
$\therefore r$ attains a minimum at $x=\frac{2}{\sqrt{3}}$.		
r is increasing on $\boxed{3\geq}\ x\geq\frac{2}{\sqrt{3}}$.	1A	Not awarded if checking was incomplete.
and decreasing on $\boxed{0\leq}\ x\leq\frac{2}{\sqrt{3}}$.	1A	
Least value of $r=2\sqrt{(\frac{2}{\sqrt{3}})^2+4}+(3-\frac{2}{\sqrt{3}})$		
$=2\sqrt{3}+3$	1A	
(iii) The greatest value of r occurs at the end-points.		
At $x=0$, $r=2\sqrt{0+4}+(3-0)=7$	1M	
At $x=3$, $r=2\sqrt{3^2+4}+(3-3)=2\sqrt{13}$		
$\therefore$ the greatest value of r is $2\sqrt{13}$.	1A	when $x=\frac{2}{\sqrt{3}}$
	9	
(b) $r=2\sqrt{x^2+4}+(1-x)$		$r_{least}=1+2\sqrt{3}$
$\boxed{\frac{dr}{dx}=\frac{2x}{\sqrt{x^2+4}}-1}$	1A	
From (a), r is decreasing on $0\leq x\leq 1$.		
r is the least at $x=1$.	1M	
Least value $=2\sqrt{1+4}+(1-1)$		
$=2\sqrt{5}$	1A	
	3	

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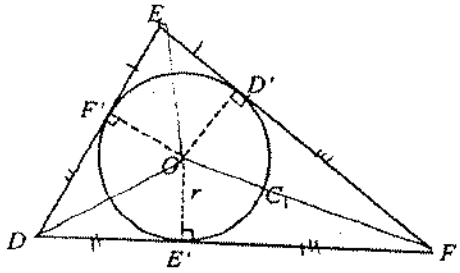
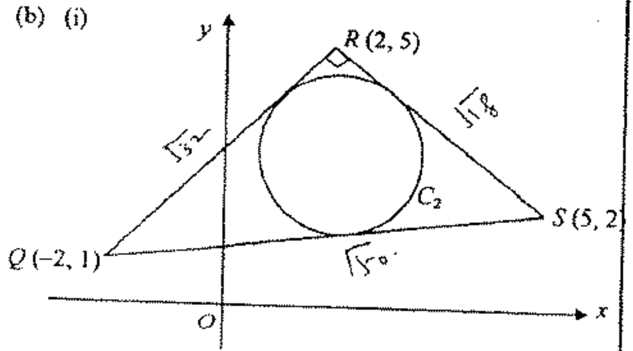
Solution	Marks	Remarks
15. (a)		
		
A = Area of ΔODE + area of ΔOEF + area of ΔODF (O is centre of C_1)		
$=\frac{1}{2}(DE)(r)+\frac{1}{2}(EF)(r)+\frac{1}{2}(DF)(r)$	2A	
$=\frac{1}{2}(DE+EF+FD)(r)$		
$A=\frac{1}{2}pr$	1M+1	1M for $p=DE+EF+FD$
Alternative solution		
A = Area of $OD'EF'$ + area of $\Delta OE'FD'$ + area of $\Delta OF'DE'$		
$=2(\frac{1}{2})(ED')(r)+2(\frac{1}{2})(FE')(r)+2(\frac{1}{2})(DF')(r)$	2A	
$=\frac{1}{2}(2ED'+2FE'+2DF')(r)$		
$A=\frac{1}{2}pr$	1M+1	1M for $p=2(ED'+FE'+DF')$
	4	
(b) (i)		
		
$QR=\sqrt{(-2-2)^2+(1-5)^2}=\sqrt{32}$		
$RS=\sqrt{(2-5)^2+(5-2)^2}=\sqrt{18}$		
$SQ=\sqrt{(-2-5)^2+(1-2)^2}=\sqrt{50}$		
$p=\sqrt{32}+\sqrt{18}+\sqrt{50}$		
$=12\sqrt{2}$		
Area of $\Delta QRS=\frac{1}{2}(QR)(RS)$		
$=\frac{1}{2}(\sqrt{32})(\sqrt{18})$	1M	
$=12$		

Solution	Marks	Remarks
Alternative solution		
Area of $\Delta QRS = \dfrac{1}{2}\begin{vmatrix} -2 & 1 \\ 5 & 2 \\ 2 & 5 \\ -2 & 1 \end{vmatrix}$		
$= \dfrac{1}{2}(-4+25+2-5-4+10)$	1M	
$= 12$		
Using (a), $A = \dfrac{1}{2}pr$		
$12 = \dfrac{1}{2}(12\sqrt{2})\,r$	1M	
$r = \sqrt{2}$	1A	
(ii) Let (h, k) be the centre of C_2. From (ii), slope of $QR = 1$ and slope of $RS = -1$, so the angle bisector of QR and RS is a vertical line.	1M	
As the centre of C_2 lies on the angle bisector, so $h = 2$.	1A	
Distance between R and the centre $= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$ $= 2$	1M	
$k = 5 - 2 = 3$	1M	
$\therefore$ the coordinates of the centre of C_2 are $(2, 3)$.		
The equation of C_2 is $(x-2)^2 + (y-3)^2 = 2$.	1A	OR $x^2 + y^2 - 4x - 6y + 11 = 0$
Alternative solution (1)		
Equation of QR: $\dfrac{y-5}{x-2} = \dfrac{5-1}{2-(-2)}$	1M	
$x - y + 3 = 0$		
Let (h, k) be the centre of C_2.		
$\left(\dfrac{h-k+3}{\sqrt{2}}\right) = \sqrt{2}$	1M	For distance formula
$h - k + 1 = 0$ -----(1)	1A	For either (1), (2) or (3)
Equation of RS: $\dfrac{y-5}{x-2} = \dfrac{5-2}{2-5}$		1 M for eqth of QR, RS and QS 1M
$x + y - 7 = 0$		
$-\left(\dfrac{h+k-7}{\sqrt{2}}\right) = \sqrt{2}$		
$h + k - 5 = 0$ -----(2)		
Solve (1) and (2), $h = 2$, $k = 3$.	1M	For solving (1) and (2)
$\therefore$ the equation of C_2 is $(x-2)^2 + (y-3)^2 = 2$. 1A	1A	
Equation of QS: $x - 7y + 9 = 0$		
$\left\lvert \dfrac{h-7k+9}{-\sqrt{50}} \right\rvert = \sqrt{2}$	1A	
$h - 7k + 19 = 0$ -----(3)	1M	

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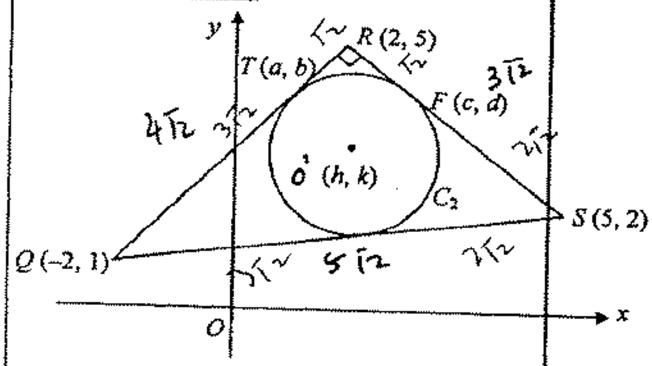
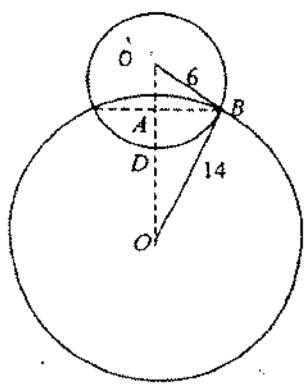
Solution	Marks	Remarks
Alternative solution (2)		
		
$\dfrac{RT}{TQ}=\dfrac{\sqrt{2}}{3\sqrt{2}}=\dfrac{1}{3}$	1M	
$a=\dfrac{-2+6}{1+3}=1$, $b=\dfrac{1+15}{1+3}=4$	1M	
$\dfrac{RF}{FS}=\dfrac{\sqrt{2}}{2\sqrt{2}}=\dfrac{1}{2}$		
$c=\dfrac{5+4}{1+2}=3$, $d=\dfrac{2+10}{1+2}=4$	1A	For a, b, c, d
$\dfrac{h+2}{2}=\dfrac{1+3}{2}$ $\therefore h=2$	1M	
$\dfrac{k+5}{2}=\dfrac{4+4}{2}$ $\therefore k=3$		
$\therefore$ the equation of C_2 is $(x-2)^2+(y-3)^2=2$.	1A	
$x^2+y^2-4x-6y+11=0$		

Solution	Marks	Remarks
16. (a) Volume $=\int_h^r \pi x^2 \mathrm{d}y$ accept wrong upper lower limit	1M	For $V=\pi\int_a^b x^2\mathrm{d}y$,
$=\pi\int_h^r (r^2-y^2)\mathrm{d}y$	1A	
$=\pi\left[r^2y-\frac{1}{3}y^3\right]_h^r$	1A	For primitive function
$=\pi\left[r^3-\frac{1}{3}r^3-r^2h+\frac{1}{3}h^3\right]$		
$=\frac{\pi}{3}(2r^3-3r^2h+h^3)$	1	
	4	
(b) (i) Using the result in (a), substitute $r=14$, $h=13$: may be wrong correct		
Capacity of the pot		
$=\frac{4}{3}\pi(14)^3-\frac{\pi}{3}[2(14)^3-3(14)^2(13)+(13)^3]$	1M	
$=3645\pi$	1A	
Alternative solution		
Capacity of the pot		
$=\frac{4}{3}\pi(14)^3-\pi\int_{13}^{14}(196-y^2)\mathrm{d}y$	1M	OR $V=\pi\int_{-14}^{13}(196-y^2)\mathrm{d}y$ correct may be wrong
$=\frac{4}{3}\pi(14)^3-\pi\left[196y-\frac{1}{3}y^3\right]_{13}^{14}$		$\pi\int_0^{13}((196-y^2)\mathrm{d}y$ $=1815\frac{2}{3}\pi$
$=\frac{10976\pi}{3}-\frac{41\pi}{3}$		$\pi\int_{-14}^{0}((196-y^2)\mathrm{d}y$ $=1829\frac{1}{3}\pi$.
$=3645\pi$	1A	

(ii)



Solution	Marks	Remarks
Let C be the centre of the stopper.		
$OB=14$, $BC=6$, $OA=13$		**Alternative solution**
$AB=\sqrt{OB^2-OA^2}$	1M	Subs. $y=13$ into $x^2+y^2=196$, $x=\sqrt{27}$
$=\sqrt{14^2-13^2}=\sqrt{27}$		
$AC=\sqrt{BC^2-AB^2}$	1M	Subs. $x=\sqrt{27}$ into $x^2+y^2=36$, $y=3$
$=\sqrt{6^2-27}=3$		
$AD=6-3=3$	1A	

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Solution	Marks	Remarks
Capacity of the perfume bottle = Capacity of pot found in (i) − Volume of portion of the stopper lying inside the pot	1M	
$=3645\pi-\frac{\pi}{3}[2(6)^3-3(6)^2(3)+(3)^3]$	1M	For 2nd term
$=3645\pi-45\pi$		
$=3600\pi$	1A	
	8	

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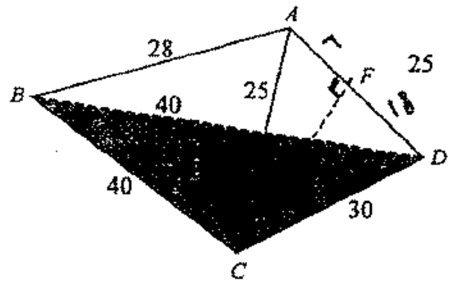
Solution	Marks	Remarks

17. (a)

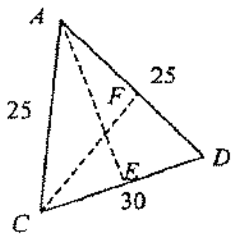
Marking Criteria:

Find CF	2M+1A
Find BF	3M
Find $\angle BFC$	1M+1A



Consider ΔACD:

Let E be the foot of perpendicular from A to CD.



$\cos\angle ADE = \frac{DE}{AD} = \frac{15}{25} = \frac{3}{5}$ — 1M

Remarks: $\angle ACD = \angle ADC = 53.13°$, $\angle CAD = 73.74°$

$CF = CD \sin\angle ADE$

$= 30\left(\frac{\sqrt{5^2 - 3^2}}{5}\right)$ (accept round up to 24) — 1M

$= 24$ — 1A

Alternative solution

$AE = \sqrt{25^2 - 15^2}$ — 1M

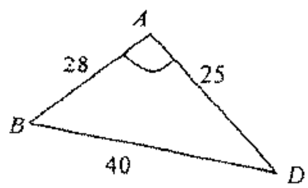
$= 20$

$\frac{1}{2}(CD)(AE) = \frac{1}{2}(AD)(CF)$

$\frac{1}{2}(30)(20) = \frac{1}{2}(25)(CF)$ — 1M

$CF = 24$ — 1A

Consider ΔABD:



$\cos\angle BAD = \frac{28^2 + 25^2 - 40^2}{2(28)(25)}$ — 1M

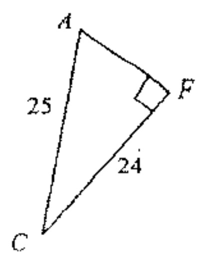
$= -\frac{191}{1400}$

Remarks: $\angle BAD = 97.84°$, $\angle ADB = 43.90°$, $\angle ABD = 38.26°$

$625 - AF^2 = 30^2 - (25 - AF)^2$

$AF = 7$

Consider ΔACF:

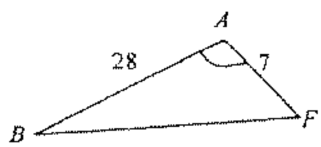


$AF^2 = AC^2 - CF^2$

$= 25^2 - 24^2 = 49$ — 1M

$AF = 7$

Consider ΔABF:



$BF^2 = AB^2 + AF^2 - 2(AB)(AF)\cos\angle BAF$

$= 28^2 + 7^2 - 2(28)(7)\left(-\frac{191}{1400}\right)$ — 1M

$BF = \sqrt{886.48}$ (≈ 29.77)

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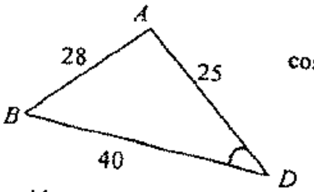
Solution	Marks	Remarks

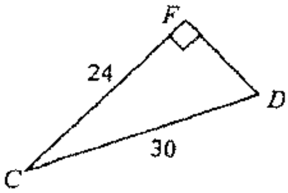
Alternative solution

Consider ΔABD :



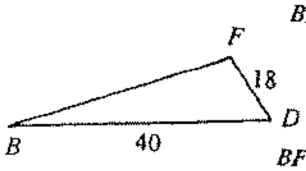
$$\cos\angle BDA = \frac{40^2+25^2-28^2}{2(40)(25)} = \frac{1441}{2000}$$ 1M

Consider ΔCDF :



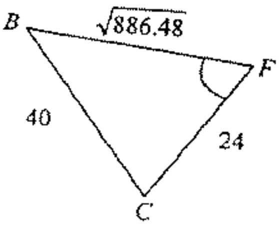
$$DF^2 = CD^2 - CF^2 = 30^2 - 24^2$$
$$DF = 18$$ 1M

Consider ΔBDF :



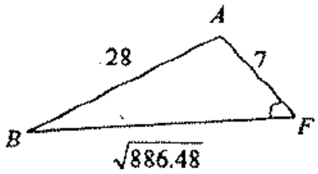
$$BF^2 = BD^2 + DF^2 - 2(BD)(DF)\cos\angle BDF$$
$$= 40^2 + 18^2 - 2(40)(18)\left(\frac{1441}{2000}\right)$$ 1M
$$BF = \sqrt{886.48} \quad (\approx 29.77)$$

Consider ΔBCF :



$$\cos\angle BFC = \frac{BF^2+CF^2-BC^2}{2(BF)(CF)}$$
$$= \frac{886.48+24^2-40^2}{2(\sqrt{886.48})(24)}$$ 1M
$$= -0.096$$
$\angle BFC = 96°$ (correct to the nearest degree) 1A

8

Solution	Marks	Remarks
(b) Consider ΔABF: $BF^2 + FA^2 = 886.48 + 49$ $= 935.48$ $\neq AB^2$ $\therefore \angle AFB \neq 90^\circ$	1M+1M+1A	1M for considering $\angle AFB$ (OR $\angle BFD$)
		
Alternative solution (1) $\cos\angle AFB$ $= \dfrac{7^2 + 886.48 - 28^2}{2(7)(\sqrt{886.48})}$ $= 0.363 \neq 0$ $\therefore \angle AFB \neq 90^\circ$ $\angle AFB \approx 68.72^\circ$	1M+1M+1A	1M for considering ΔAFB
Alternative solution (2) From (a), $\cos\angle BAD = \dfrac{-191}{1400}$. $\angle BAD > 90^\circ$, i.e. $\angle BAF > 90^\circ$. $\therefore \angle AFB < 90^\circ$	1M+1M+1A	1M for considering ΔAFB
From (a), $\angle BFC = 96^\circ$. Since $CF \perp AD$ but BF is not perpendicular to AD, $\angle BFC$ does not represent the angle between the two planes. The student is incorrect.	1A	
	4	

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Solution	Marks	Remarks
18. (a) $z^2 = \cos 2\theta + i\sin 2\theta$ $(\cos^2\theta - \sin^2\theta) + 2i\sin\theta\cos\theta$	1A	$z^2+1 = (\cos^2\theta - \sin^2\theta) + 2i\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta$
$z^2+1 = (\cos 2\theta + 1) + i\sin 2\theta$		
$\lvert z^2+1\rvert^2 = (\cos 2\theta + 1)^2 + \sin^2 2\theta$	1M	$= 2\cos^2\theta + 2i\sin\theta\cos\theta$
$= \cos^2 2\theta + 2\cos 2\theta + 1 + \sin^2 2\theta$		
$= 2(1+\cos 2\theta)$	1	$= 2\cos\theta(\cos\theta + i\sin\theta)$
Since $-\pi < \theta \le \pi$, $-2\pi < 2\theta \le 2\pi$. $\cos 2\theta \le 1$	1M	$\lvert z^2+1\rvert^2 = 4\cos^2\theta\lvert\cos\theta + i\sin\theta\rvert$
$\therefore$ the greatest value of $\lvert z^2+1\rvert = \sqrt{2(1+1)}$ $= 2$	1A	$= 2(2\cos^2\theta)$
	5	$= 2(\cos 2\theta + 1)$
(b) (i) $w = 3z$ (OR $w = 3(\cos\theta + i\sin\theta)$)	1A	$\lvert z^2+1\rvert = 2\cos\theta$.
$\lvert w^2+9\rvert = \lvert (3z)^2+9\rvert$ $= 9\lvert z^2+1\rvert$		
From (a), $\lvert z^2+1\rvert \le 2$.		
$\therefore$ greatest value of $\lvert w^2+9\rvert = 9(2)$	1M	For using (a)
$= 18$	1	
(ii) $w^4 - 81 = 100i(w^2-9)$		
$(w^2+9)(w^2-9) - 100i(w^2-9) = 0$	1M	For factorization
$(w^2-9)(w^2+9-100i) = 0$		
$w^2 - 9 = 0$ ---(1) or $w^2+9-100i = 0$ ---(2)		
Consider (1): $w = \pm 3$ which satisfies the condition $\lvert w\rvert = 3$	1A	
Consider (2): $w^2+9 = 100i$. From (i), $\lvert w^2+9\rvert \le 18$ but $\lvert 100i\rvert = 100$. So equation (2) has no solutions	1M	
$\therefore$ the equation has only two roots.	1	
<u>Alternative solution for explaining why (2) has no solution</u>		
$w^2 = -9 + 100i$. Let $w = a + bi$. $(a^2 - b^2) + 2abi = -9 + 100i$. $\begin{cases} a^2 - b^2 = -9 & \text{---(3)} \\ 2ab = 100 & \text{---(4)} \end{cases}$ Since $\lvert w\rvert = 3$, $a^2 + b^2 = 9$ ---(5). From (3) and (5), $a = 0$.	1M	For attempting to solve (3), (4), (5)
Substitute $a = 0$ into (4): LHS $= 0 \ne$ RHS. So the equation has no solution.	1	
	7	

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