香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九九年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1999

附加數學　試卷一
ADDITIONAL MATHEMATICS PAPER 1

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。
After the examinations, marking schemes will be available for reference at the Teachers' Centres.

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative solutions. In general, a correct alternative solution merits <u>all the marks</u> allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol (pp–1) should be used to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Note : if the final answers are not expressed in the simplest form, deduct 1 mark for p.p.

   (e) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u–1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points:

   (a) At most deduct 1 mark for wrong/no units for the whole paper.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the <u>net</u> total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles ⬚ ,

   whereas alternative answers are enclosed by solid rectangles ▭ .

8. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

10. Unless the form of answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

Solution	Marks	Remarks
1. (a) $\frac{\mathrm{d}}{\mathrm{d}x}\sin(x^2+1)$		
$\boxed{=\frac{\mathrm{d}}{\mathrm{d}(x^2+1)}\sin(x^2+1)\frac{\mathrm{d}}{\mathrm{d}x}(x^2+1)}$	1M	For chain rule (can be omitted)
$=2x\cos(x^2+1)$	1A	
(b) $\frac{\mathrm{d}}{\mathrm{d}x}\frac{\sin(x^2+1)}{x}$		
$\boxed{=\frac{x\frac{\mathrm{d}}{\mathrm{d}x}\sin(x^2+1)-\sin(x^2+1)\frac{\mathrm{d}}{\mathrm{d}x}(x)}{x^2}}$	1M	For quotient rule (can be omitted)
$=\frac{2x^2\cos(x^2+1)-\sin(x^2+1)}{x^2}$	1A	
Alternative solution		
$\frac{\mathrm{d}}{\mathrm{d}x}\frac{\sin(x^2+1)}{x}$		
$\boxed{=\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}\sin(x^2+1)+\sin(x^2+1)\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x}}$	1M	For product rule (can be omitted)
$=\frac{1}{x}2x\cos(x^2+1)+\sin(x^2+1)(-\frac{1}{x^2})$		
$=2\cos(x^2+1)-\frac{1}{x^2}\sin(x^2+1)$	1A	
	4	

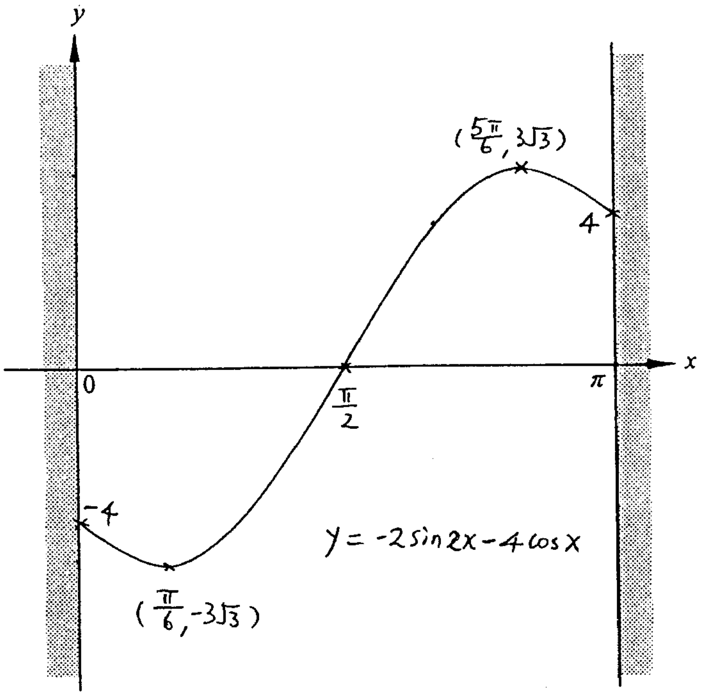
Solution	Marks	Remarks
2. $\frac{x}{x-1} > 2$		
$\frac{x}{x-1} - 2 > 0$	1M	
$\frac{-x+2}{x-1} > 0$	1A	
$1 < x < 2$	2A	
Alternative solution (1)		
Consider the following cases : (i) $x > 1$, (ii) $x < 1$	1M	Awarded even if equality sign is included
Case 1 : $x > 1$		
$x > 2(x-1)$		
$x < 2$		
Since $x > 1$, $\therefore$ $1 < x < 2$.	→ 1A	
Case 2 : $x < 1$		
$x < 2(x-1)$		
$x > 2$		
Since $x < 1$, $\therefore$ there is no solution.		
Combining the 2 cases, $1 < x < 2$.	2A	
Alternative solution (2)		
$\frac{x}{x-1} > 2$		
$x(x-1) > 2(x-1)^2$	1M	
$x^2 - 3x + 2 < 0$		
$(x-1)(x-2) < 0$	1A	(can be omitted)
$1 < x < 2$	2A	
	4	

Solution	Marks	Remarks
3. $\lvert x-3\rvert=\lvert x^2-4x+3\rvert$		
$\lvert x-3\rvert=\lvert x-1\rvert\,\lvert x-3\rvert$	2A	
$\lvert x-3\rvert=0$ or $\lvert x-1\rvert=1$		
$x=3$ $\quad$ $x-1=\pm1$		
$x=0$ or 2		
$\therefore x=0,2$ or 3.	1A+1A+1A	Can be awarded even if the above '2A' were not given.
Alternative solutions		
(1) $x-3=x^2-4x+3$ or $x-3=-(x^2-4x+3)$	2A	
$x-3=(x-1)(x-3)$ $\quad$ $(x-3)=-(x-1)(x-3)$		
$x=3$ or $x-1=1$ $\quad$ $x=3$ or $x-1=-1$		
$x=2$ $\quad$ $x=0$		
$\therefore\ x=0,2$ or 3.	1A+1A+1A	
(2) $(x-3)^2=(x^2-4x+3)^2$	2A	
$(x-3)^2[(x-1)^2-1]=0$		
$(x-3)^2(x-2)(x)=0$		
$x=0,2$ or 3.	1A+1A+1A	
(3) Consider the following cases : $x\geq3, 1<x<3, x\leq1$.	1A	
Case 1 : $x\geq3$		
$x-3=x^2-4x+3$		
$x^2-5x+6=0$		
$x=2$ (rejected) or 3		
Case 2 : $1<x<3$		
$-(x-3)=-(x^2-4x+3)$		
$x=2$ or 3 (rejected)		
Case 3 : $x\leq1$		
$-(x-3)=x^2-4x+3$	1A	Awarded only if the 3 equations were all correct.
$x^2-3x=0$		
$x=0$ or 3 (rejected)		
Combining the 3 cases, $x=0,2$ or 3.	1A+1A+1A	
	5	
4. (a) Discriminant $=4(k-4)^2-4(2)(k)$	1M	
$=4k^2-40k+64$	1A	
(b) Discriminant <0		
$4k^2-40k+64<0$	2M	1M for $\Delta\leq0$
$k^2-10k+16<0$		
$\boxed{(k-2)(k-8)<0}$		
$2<k<8$.	1A	
	5	

Solution	Marks	Remarks
5. $1+i=\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$	1A+1A	1A for modulus, 1A for argument (Accept degrees, Accept other equivalent values for argument)
$(1+i)^{\frac{1}{3}}=[\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})]^{\frac{1}{3}}$		
$=(\sqrt{2})^{\frac{1}{3}}(\cos\frac{2k\pi+\frac{\pi}{4}}{3}+i\sin\frac{2k\pi+\frac{\pi}{4}}{3})$ $k=-1,0,1$	1M+1A	1M for De Moivre's Theorem, 1A for modulus
$=2^{\frac{1}{6}}[\cos(\frac{2k\pi}{3}+\frac{\pi}{12})+i\sin(\frac{2k\pi}{3}+\frac{\pi}{12})]$ $k=-1,0,1$	1A	1A for argument (accept degrees) (accept k = any 3 consecutive integers)
OR $=2^{\frac{1}{6}}(\cos\frac{-7\pi}{12}+i\sin\frac{-7\pi}{12}),\ 2^{\frac{1}{6}}(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}),$ $2^{\frac{1}{6}}(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})$.		
	5	
6. (a) $3x^2-xy-y^2-a^2=0$		
$6x-y-x\frac{dy}{dx}-2y\frac{dy}{dx}=0$	1A+1A	1A for $\frac{d}{dx}(xy)$, 1A for other terms
$\frac{dy}{dx}=\frac{6x-y}{x+2y}$		
Substitute $x=a, y=a$:		
$\frac{dy}{dx}=\frac{6a-a}{a+2a}$	1M	} (can be awarded in (b))
$=\frac{5}{3}$	1A	
(b) Equation of tangent is		
$\frac{y-a}{x-a}=\frac{5}{3}$	1M	
$5x-3y-2a=0$.	1A	Accept equivalent forms
	6	

Solution	Marks	Remarks
7. (a) $\lvert\vec{a}\rvert=\sqrt{3^2+4^2}=5$	1A	
(b) $\vec{a}\cdot\vec{b}=\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos 60^\circ$		
$=5(4)\cos 60^\circ$	1M	
$=10$	1A	
(c) $(m\vec{a}+\vec{b})\cdot\vec{b}=0$	1M	
$m\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}=0$	1M	For distribution law only
$m(10)+4^2=0$		
$m=-1.6$	1A	Omit vectors sign in most cases (pp–1) Omit dot product sign more than once (pp–1)
	6	
8. (a) (i) $\tan\theta=\dfrac{h+40}{55}$		
$=\dfrac{20t-5t^2+40}{55}$		
$\tan\theta=\dfrac{4t-t^2+8}{11}$ - - - - - (1)	1A	
(ii) At $t=3$, $\tan\theta=\dfrac{4(3)-3^2+8}{11}=1$		
$\theta=\dfrac{\pi}{4}$ (OR $=45^\circ$)	1A	
(b) Differentiate (1) with respect to t:		
$\sec^2\theta\dfrac{d\theta}{dt}=\dfrac{4-2t}{11}$	1M+1A+1A	1M for chain rule, 1A for LHS, 1A for RHS
At $t=3$, $\sec^2\dfrac{\pi}{4}\dfrac{d\theta}{dt}=\dfrac{4-2(3)}{11}$	1M	For substituting t and θ
$2\dfrac{d\theta}{dt}=\dfrac{-2}{11}$		
$\dfrac{d\theta}{dt}=\dfrac{-1}{11}$		
$\therefore$ the rate of change of θ with respect to time at $t=3$ is $\dfrac{-1}{11}\text{s}^{-1}$.	1A	Omit/wrong unit $(u-1)$
(OR θ decreases at a rate of $\dfrac{1}{11}\text{s}^{-1}$ at $t=3$.)		
	7	

Solution	Marks	Remarks
9. (a) (i) $f'(x)=2a\cos 2x-b\sin x$	1A	
(ii) From figure 2 (a), $f'(0)=-4$.		OR $f'(\pi)=-4$
$2a\cos 0-b\sin 0=-4$	1M	
$2a=-4$		
$a=-2$	1	
$f'(\frac{\pi}{6})=0$ $\quad 2(-2)\cos\frac{\pi}{3}-b\sin\frac{\pi}{6}=0$		OR $f'(\frac{5\pi}{6})=0$
$b=-4$	1	
$\therefore f(x)=-2\sin 2x-4\cos x$		
	4	
(b) (i) $f(0)=-4$ $\therefore$ the y-intercept is -4.	1A	(pp–1) for $(0,-4)$
Put $f(x)=0$: $-2\sin 2x-4\cos x=0$	1M	
$-4\sin x\cos x-4\cos x=0$	1A	
$-4\cos x(1+\sin x)=0$		
$\cos x=0$ or $\sin x=-1$ (rejected)		
$x=\frac{\pi}{2}$	1A	No mark for $x=90°$
$\therefore$ the x-intercept is $\frac{\pi}{2}$.		(pp–1) for $(\frac{\pi}{2},0)$
(ii) From Figure 2 (a), $f'(x)=0$ when $x=\frac{\pi}{6}$ or $\frac{5\pi}{6}$.		
As $f'(x)$ changes from –ve to +ve as x increases through $\frac{\pi}{6}$, so $(\frac{\pi}{6},-3\sqrt{3})$ is a minimum point.	1A+1M	Withhold 1M if explanation was omitted
As $f'(x)$ changes from +ve to –ve as x increases through $\frac{5\pi}{6}$, so $(\frac{5\pi}{6},3\sqrt{3})$ is a maximum point.	1A	
Alternative solution		
$f'(x)=0$ at $x=\frac{\pi}{6}$ or $\frac{5\pi}{6}$.		
$f''(x)=4\cos x+8\sin 2x$		
$f''(\frac{\pi}{6})\ \boxed{=6\sqrt{3}}\ >0.$		
$\therefore (\frac{\pi}{6},-3\sqrt{3})$ is a minimum point.	1M+1A	Withhold 1M if checking was omitted
$f''(\frac{5\pi}{6})\ \boxed{=-6\sqrt{3}}\ <0$		
$\therefore (\frac{5\pi}{6},3\sqrt{3})$ is a maximum point.	1A	
	7	

Solution	Marks	Remarks
(c)		(Awarded even if checking was omitted in (b))
	1A	For shape
	1A	For x-intercepts and turning points
	1A	For end-points
	3	
(d) $6-3\sqrt{3} \le g(x) \le 6+3\sqrt{3}$	1A+1A	1A for LHS, 1A for RHS Award 1A for $6-3\sqrt{3} < g(x) < 6+3\sqrt{3}$
	2	

Solution	Marks	Remarks
10. (a) $\overrightarrow{OC}=\dfrac{7\vec{a}+8\vec{b}}{15}$	1A	
$\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}$	1M	Accept other correct methods
$=\dfrac{16}{21}\vec{b}-\vec{a}$	1A+1A	1A for $\overrightarrow{OD}=\dfrac{16}{21}\vec{b}$
<u>Alternative solution</u>		
$\overrightarrow{AD}=\dfrac{16\overrightarrow{AB}+5\overrightarrow{AO}}{21}$	1M	
$=\dfrac{16(\vec{b}-\vec{a})+5(-\vec{a})}{21}$	1A	1A for $\overrightarrow{AB}=\vec{b}-\vec{a}$
$=\dfrac{16}{21}\vec{b}-\vec{a}$	1A	
	4	
(b) (i) $\overrightarrow{OE}=r\overrightarrow{OC}$		
$=\dfrac{7r}{15}\vec{a}+\dfrac{8r}{15}\vec{b}$	1A	
$=\dfrac{r}{15}(7\vec{a}+8\vec{b})$		
(ii) $\overrightarrow{OE}=\overrightarrow{OA}+\overrightarrow{AE}$		<u>OR</u> $=\overrightarrow{OD}+\overrightarrow{DE}$
$=\vec{a}+k\overrightarrow{AD}$		$=\dfrac{16}{21}\vec{b}+(1-k)\overrightarrow{DA}$
$=\vec{a}+k(\dfrac{16\vec{b}}{21}-\vec{a})$	1M	$=\dfrac{16}{21}\vec{b}+(1-k)(\vec{a}-\dfrac{16}{21}\vec{b})$
$=(1-k)\vec{a}+\dfrac{16k}{21}\vec{b}$	1A	$=(1-k)\,\vec{a}+\dfrac{16k}{21}\vec{b}$
<u>Alternative solution</u>		
$\overrightarrow{OE}=\dfrac{(1-k)\,\overrightarrow{OA}+k\overrightarrow{OD}}{(1-k)+k}$	1M	
$=(1-k)\,\vec{a}+\dfrac{16k}{21}\vec{b}$	1A	
Comparing the two expressions :		
$\begin{cases}\dfrac{7r}{15}=1-k & \text{------(1)}\\ \dfrac{8r}{15}=\dfrac{16}{21}k & \text{------(2)}\end{cases}$	} 1M	For comparing coefficients
$(1)\div(2)\quad \dfrac{7}{8}=\dfrac{21(1-k)}{16k}$		
$14k=21-21k$		
$k=\dfrac{3}{5}$	1	

Solution	Marks	Remarks
Substitute $k=\frac{3}{5}$ into (1) : $\frac{7}{15}r=1-\frac{3}{5}$		
$r=\frac{6}{7}$	1	
$\therefore k=\frac{3}{5}$ and $r=\frac{6}{7}$.		
	6	
(c) (i) Let $EC=x$.		Let $EC=1$ etc. (pp−1)
Since $EC:ED=1:2$, $ED=2x$.	1M+1A	
From (b), $\overrightarrow{OE}=\frac{6}{7}\overrightarrow{OC}$.		
$\therefore$ $EO:EC=6:1$, i.e. $EO=6x$.		
From (b), $\overrightarrow{AE}=\frac{3}{5}\overrightarrow{AD}$		1A for $EO:EC=6:1$ or $EA:ED=3:2$ etc.
$\therefore$ $EA:ED=3:2$, i.e. $EA=3x$.		
$\therefore$ $EA:EO=3x : 6x$		
$= 1 : 2$.	1A	
(ii) In ΔEAC and ΔEOD,	1A+1	
$\angle AEC=\angle OED$		1A for naming a pair of similar Δs
From (b), $\frac{EA}{EO}=\frac{1}{2}=\frac{EC}{ED}$		1 for completing the proof
$\therefore \Delta EAC \sim \Delta EOD$.		
$\angle EAC=\angle EOD$ (Corr ∠s of similar Δs)	1	
$\therefore$ $OACD$ is a cyclic quadrilateral. (Converse of ∠s in the same segment)		Omit vector sign in most cases (pp−1)
	6	

Solution	Marks	Remarks
11. (a) $z_1 = 2(\cos 60° + i\sin 60°)$	1A	
$= 1+\sqrt{3}i$	1A	
$z_3 = (\sqrt{3}i)\, z_1$		
$= \sqrt{3}i(1+\sqrt{3}i)$		
$= -3+\sqrt{3}i$	1A	
<u>Alternative solution</u>		
$OC = 2\sqrt{3}$		
$\arg(z_3) = 60° + 90° = 150°$		
$z_3 = 2\sqrt{3}(\cos 150° + i\sin 150)$		
$= -3+\sqrt{3}i$	1A	
	3	
(b) $\frac{z_2}{z_1} = \frac{z_1+z_3}{z_1}$	1M	For $z_2 = z_1 + z_3$
$= 1+(\frac{z_3}{z_1})$	1A	
$= 1+\sqrt{3}i \quad (\because\ z_3 = (\sqrt{3}i)z_1)$	1	
<u>Alternative solution</u>		
$z_2 = z_1 + z_3$	1M	
$= (1+\sqrt{3}i)+(-3+\sqrt{3}i)$		
$= -2+2\sqrt{3}i$	1A	
$\frac{z_2}{z_1} = \frac{-2+2\sqrt{3}i}{1+\sqrt{3}i}(\frac{1-\sqrt{3}i}{1-\sqrt{3}i})$		
$= \frac{-2+2\sqrt{3}i+2\sqrt{3}i+6}{4}$		
$= 1+\sqrt{3}i$	1	
$\angle AOB = \arg(\frac{z_2}{z_1})$	1M	
$= \arg(1+\sqrt{3}i)$		
$= 60°$	1A	
<u>Alternative solution (1)</u>		
$\arg(z_2) = 180° + \tan^{-1}(\frac{2\sqrt{3}}{-2})$		
$= 120°$		
$\angle AOB = \arg(z_2) - \arg(z_1)$	1M	
$= 120° - 60°$		
$= 60°$	1A	
<u>Alternative solution (2)</u>		
$\tan \angle AOB = \frac{AB}{OA}$	1M	
$= \frac{2\sqrt{3}}{2}$		
$= \sqrt{3}$		
$\therefore\ \angle AOB = 60°$	1A	
	5	

Solution	Marks	Remarks
(c) (i) $z_3 = 2\sqrt{3}(\cos 150° + i\sin 150°)$		
$\arg(z_3) = 150°$	1A	can be awarded in (ii)
Let u be the complex number represented by E.		
$u = wz_3$ $= (\cos\theta + i\sin\theta)2\sqrt{3}(\cos 150° + i\sin 150°)$ $= 2\sqrt{3}[\cos(150° + \theta) + i\sin(150° + \theta)]$		
$\arg(u) = 150° + \theta$	1M	
$\bar{z}_3 = 2\sqrt{3}[\cos(-150°) + i\sin(-150°)]$		
$\arg(\bar{z}_3) = -150°$	1A	(OR $= 210°$)
If E represents the complex number $\bar{z}_3$,		
$150° + \theta = -150° + 360°$	1M	OR $150° + \theta = 210°$
$\theta = 60°$	1A	
Alternative solution $wz_3 = \bar{z}_3$ $w(-3 + \sqrt{3}i) = -3 - \sqrt{3}i$	1M	
$w = \dfrac{-3 - \sqrt{3}i}{-3 + \sqrt{3}i}\left(\dfrac{-3 - \sqrt{3}i}{-3 - \sqrt{3}i}\right)$ $\cos\theta + i\sin\theta = \dfrac{6 + 6\sqrt{3}i}{12}$		
$\cos\theta = \dfrac{1}{2}$ and $\sin\theta = \dfrac{\sqrt{3}}{2}$ (OR $\tan\theta = \sqrt{3}$)	1A	
$\theta = 60°$	1A	
(ii) If E, O and A lie on a straight line,		
$150° + \theta = 60° + 360\,k°$ or $150° + \theta = -120° + 360\,k°$	1M	Awarded if either one was correct
OR $150° + \theta = 60° + 360°$ or $150° + \theta = -120° + 360°$ or $= 60° + 180°$		
$\theta = 270°$ or $90°$.	1A+1A 8	

Solution	Marks	Remarks
12. (a) $S =$ Area of $ABCD$ – Area of ΔABE – Area of ΔCEF – Area of ΔADF		
$= 2(2k) - \frac{1}{2}(2)(2x) - \frac{1}{2}(x)(2k-2x) - \frac{1}{2}(2k)(2-x)$	1M+1A	
$= 4k - 2x - (kx - x^2) - (2k - kx)$		
$= x^2 - 2x + 2k$	1	
	3	
(b) (i) As E lies on BC, so $0 \le 2x \le 2k$ $0 \le x \le \frac{3}{2}$.	1A	
As F lies on CD, so $0 \le x \le 2$. Combining the two inequalities, $0 \le x \le \frac{3}{2}$.	}1	
(ii) $S = x^2 - 2x + 2k$ $= x^2 - 2x + 3$		
$S = (x-1)^2 + 2$	1M+1A	1M for method of completing squares
As $x = 1$ lies in the range of possible value of $x\left(0 \le x \le \frac{3}{2}\right)$,		
∴ the least value of $S = 2$, which occurs when $x = 1$.	1A+1A	$S = 2\text{ cm}^2, x = 1\text{ cm}$ $(u-1)$
Alternative solution $S = x^2 - 2x + 3$ $\frac{dS}{dx} = 2x - 2$		
$\frac{dS}{dx} = 0$ when $x = 1$.	1A	
$\frac{d^2S}{dx^2} = 2 > 0$ ∴ S is a minimum at $x = 1$.	1M	For checking
As $x = 1$ lies in the range of possible values of x,		
∴ the least value of $S = 1^2 - 2(1) + 3$ $= 2$ which occurs when $x = 1$.	1A+1A	
(iii) Since $S = x^2 - 2x + 3$ is a parabola and there is only a minimum in the range $0 \le x \le \frac{3}{2}$, so greatest value of S occurs at the end points.	1M	(can be omitted)
At $x = 0, S = 3$. At $x = \frac{3}{2}, S = (\frac{3}{2})^2 - 2(\frac{3}{2}) + 3 = \frac{9}{4}$.	} 1M	For evaluating the end-values
∴ the greatest value of S is 3.	1A	
	9	

Solution	Marks	Remarks
(c) (i) Put $k=\frac{3}{8}$, $S=x^2-2x+\frac{3}{4}$.		
The range of possible values of x is $0\le x\le\frac{3}{8}$.	1A	(can be awarded in (ii))
As $x=1$ does not lie in the above interval, the least value of S will not happen when $x=1$. $\therefore$ the student is incorrect.	} 1M+1	
Alternative solution Put $x=1$:		
$S=1^2-2(1)+\frac{3}{4}=-\frac{1}{4}$	1M	
As $S<0$ at $x=1$, so the least value of S will not happen when $x=1$.	1	
(ii) As S is monotonic decreasing on $0\le x\le\frac{3}{8}$, least value of S occurs when $x=\frac{3}{8}$. $\therefore$ least value of $S=(\frac{3}{8})^2-2(\frac{3}{8})+\frac{3}{4}$		
$=\frac{9}{64}$	1A	
Alternative solution (i) Put $k=\frac{3}{8}$, $S=x^2-2x+\frac{3}{4}$.		
The range of possible values of x is $0\le x\le\frac{3}{8}$.	1A	
$\frac{dS}{dx}=2x-2$		
As $\frac{dS}{dx}<0$ for $0\le x\le\frac{3}{8}$, the least value of S occurs when $x=\frac{3}{8}$. (S is monotonic decreasing on $0\le x\le\frac{3}{8}$.) $\therefore$ the student is incorrect.	} 1M+1	
(ii) Least value of $S=(\frac{3}{8})^2-2(\frac{3}{8})+\frac{3}{4}$		
$=\frac{9}{64}$	1A	
	4	

Solution	Marks	Remarks
13. (a) $V=\pi x^2 h$		
$h=\dfrac{V}{\pi x^2}$	1A	
$C=(2\pi xh)+k(\pi x^2)2$	1A	
$=2\pi x(\dfrac{V}{\pi x^2})+2\pi x^2 k$		
$=\dfrac{2V}{x}+2\pi k x^2$	1	
	3	
(b) $\dfrac{dC}{dx}=-\dfrac{2V}{x^2}+4\pi kx$	1A	
$\dfrac{dC}{dx}=0 \quad -\dfrac{2V}{x^2}+4\pi kx=0$		
$x^3=\dfrac{V}{2\pi k}$	1A	
$\dfrac{d^2C}{dx^2}=\dfrac{4V}{x^3}+4\pi k$	1A	
Put $x^3=(\dfrac{V}{2\pi k})$: $\dfrac{d^2C}{dx^2}\ \boxed{=12\pi k} > 0.$	1M	For checking
$\therefore$ C is a minimum.		<u>OR</u> $\dfrac{d^2C}{dx^2}>0$ for all x.
<u>Alternative solution</u>		
$\dfrac{dC}{dx}=\dfrac{4\pi k}{x^2}(x^3-\dfrac{V}{2\pi k})$	1A	
When $x>(\dfrac{V}{2\pi k})^{\frac{1}{3}}$, $\dfrac{dC}{dx}>0$	1M	For checking
When $\boxed{0<}\ x<(\dfrac{V}{2\pi k})^{\frac{1}{3}}$, $\dfrac{dC}{dx}<0$		
$\therefore$ C is a minimum at $x=(\dfrac{V}{2\pi k})^{\frac{1}{3}}$.		
$\dfrac{x}{h}=\dfrac{x}{V/\pi x^2}$	1M	
$=\dfrac{\pi x^3}{V}$		
$=\dfrac{\pi}{V}(\dfrac{V}{2\pi k})$		
$=\dfrac{1}{2k}$	1	
	6	

Solution	Marks	Remarks
(c) (i) From (b), $x^3=(\frac{V}{2\pi k})$		
$=(\frac{256\pi}{2\pi(2)})$	1M	For substitution
$=64$		
$x=4$	1A	
Since $\frac{x}{h}=\frac{1}{2k}$, $\frac{4}{h}=\frac{1}{2(2)}$		
$h=16$	1A	
(ii) Since $x^3=\frac{V}{2\pi k}$, so x decreases when k increases.	1A	
As $h=\frac{V}{\pi x^2}$, so h increases when x decreases.	1A	
∴ the base radius of the can decreases and the height of the can increases.		
	5	
(d) The costs of the curved and plane surfaces remain unchanged.		
From (b), the ratio $\frac{x}{h}=\frac{1}{2k}$ is independent of the volume of the can.	1	
∴ the ratio $\frac{\text{base radius}}{\text{height}}$ of the bigger can should remain identical to that of the smaller can OR need not be twice that of the smaller can	1	'Incorrect' without explanation – no mark
in order to minimise the cost. So the worker is incorrect.		
	2	