FORMULAS FOR REFERENCE

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Section A (42 marks)
Answer **ALL** questions in this section.

1. Find $\frac{d}{dx}(\sqrt{x})$ from first principles.

(4 marks)

2. α, β are the roots of the quadratic equation $x^2 - 2x + 7 = 0$.
Find the quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$.

(4 marks)

3. The quadratic equations $x^2 - 6x + 2k = 0$ and $x^2 - 5x + k = 0$ have a common root α. (i.e. α is a root of both equations.)

Show that $\alpha = k$ and hence find the value(s) of k.

(4 marks)

4. (a) Express $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ in polar form.

(b) Using (a), find the value(s) of n such that $(\frac{\sqrt{3}}{2} + \frac{1}{2}i)^n = 1$, where n is a positive integer.

(5 marks)

5.

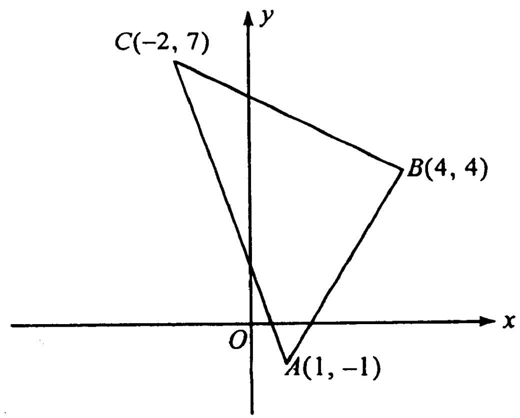

Figure 1

Figure 1 shows the points A, B and C whose position vectors are $\mathbf{i}-\mathbf{j}$, $4\mathbf{i}+4\mathbf{j}$ and $-2\mathbf{i}+7\mathbf{j}$ respectively.

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(b) By considering $\overrightarrow{AB}\cdot\overrightarrow{AC}$, find $\angle BAC$ to the nearest degree.

(6 marks)

6. (a) Solve $x^2-6x-16>0$.

(b) Using (a), or otherwise, solve $(y+1)^2-6|y+1|-16>0$.

(6 marks)

7. Find the complex number(s) z satisfying the following system of equations:

$$\begin{cases}|1+z|=|3-z|\\ z\bar{z}=4\end{cases}.$$

(6 marks)

8. $P(0, 2)$ is a point on the curve $x^2-xy+3y^2=12$.

(a) Find the value of $\dfrac{dy}{dx}$ at P.

(b) Find the equation of the normal to the curve at P.

(7 marks)

Section B (48 marks)

Answer any **THREE** questions in this section.

Each question carries 16 marks.

9.

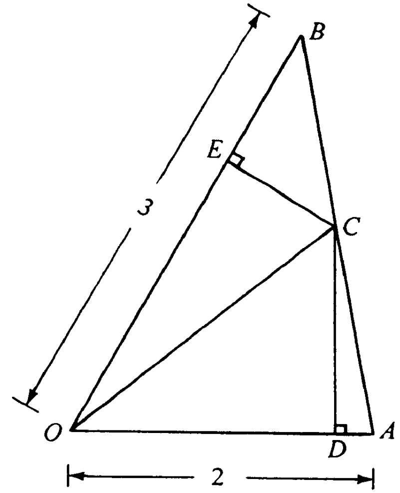

Figure 2

In Figure 2, OAB is a triangle with $OA=2$, $OB=3$ and $\angle AOB=60°$. C is a point on AB such that $AC:CB=t:1-t$, where $0<t<1$. D and E are respectively the feet of perpendicular from C to OA and OB. Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

(a) (i) Find $\mathbf{a}\cdot\mathbf{b}$.

(ii) Express $\overrightarrow{OC}$ in terms of t, $\mathbf{a}$ and $\mathbf{b}$.

(iii) Express $\mathbf{a}\cdot\overrightarrow{OC}$ and $\mathbf{b}\cdot\overrightarrow{OC}$ in terms of t. (7 marks)

(b) (i) Using (a) (iii), show that $\mathbf{a}\cdot\overrightarrow{OD}=4-t$ and $\mathbf{b}\cdot\overrightarrow{OE}=3+6t$.

(ii) If $\overrightarrow{OD}=k\mathbf{a}$ and $\overrightarrow{OE}=s\mathbf{b}$, express k and s in terms of t.

(6 marks)

(c) Find the value of t such that $\overrightarrow{DE}$ is parallel to $\overrightarrow{AB}$. (3 marks)

10. Let $f(x) = 2\cos 2x + 4\sin x - 3$, where $-\pi \le x \le \pi$.

(a) (i) Find the x- and y-intercepts of the curve $y = f(x)$.

(ii) Find the maximum and minimum points of the curve $y = f(x)$.

(10 marks)

(b) In Figure 3, sketch the curve $y = f(x)$.

Hence write down the greatest and least values of $|2\cos 2x + 4\sin x|$ for $-\pi \le x \le \pi$.

(6 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet to your answer book.

10. (b) **(continued)**

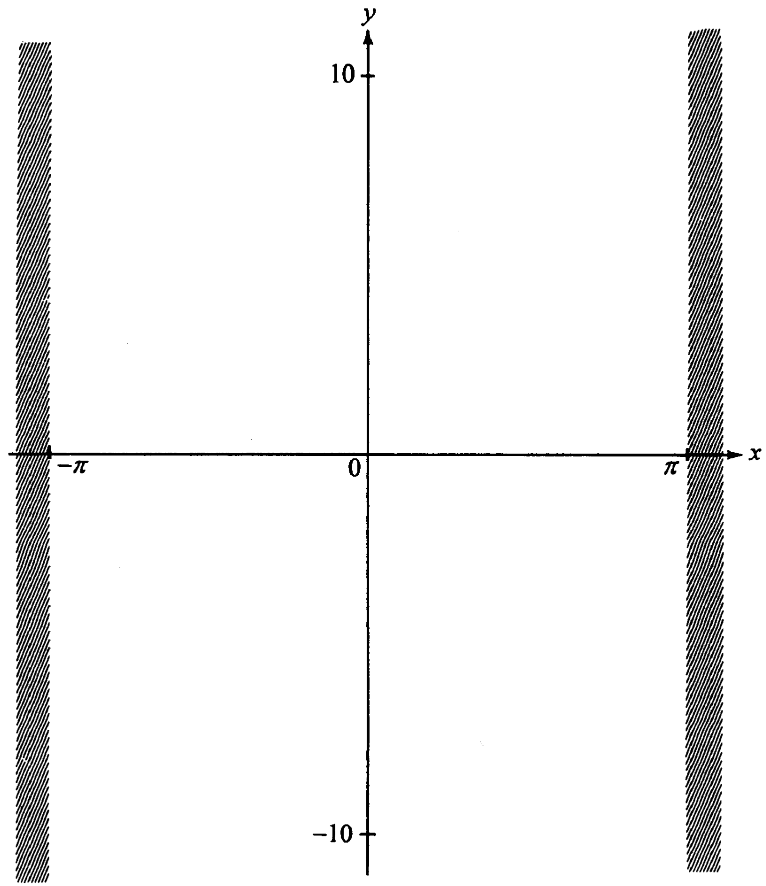

Figure 3

This is a blank page.

11. Let $f(x)=x^2-kx$, where k is a real constant, and $g(x) = -x$.

(a) Show that the least value of $f(x)$ is $-\frac{k^2}{4}$ and find the corresponding value of x.

(3 marks)

(b) Find the coordinates of the two intersecting points of the curves $y=f(x)$ and $y=g(x)$.

(3 marks)

(c) Suppose $k=3$.

(i) In the same diagram, sketch the graphs of $y=f(x)$ and $y=g(x)$ and label their intersecting points.

(ii) Find the range of values of x such that $f(x)\le g(x)$.

Hence find the least value of $f(x)$ within this range of values of x.

(6 marks)

(d) Suppose $k=\frac{3}{2}$.

Find the least value of $f(x)$ within the range of values of x such that $f(x)\le g(x)$.

(4 marks)

12.

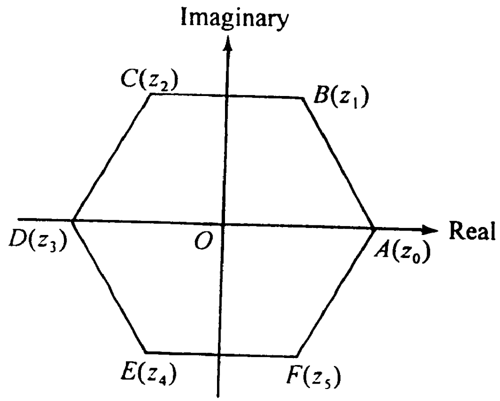

Figure 4(a)

In Figure 4 (a), $ABCDEF$ is a regular hexagon in an Argand diagram. The points A, B, C, D, E and F represent the complex numbers z_0, z_1, z_2, z_3, z_4 and z_5 respectively, where z_0, z_1, z_2, z_3, z_4 and z_5 are the roots of the equation $z^6 = 64$.

(a) Find z_0, z_1, z_2, z_3, z_4 and z_5 in standard form.

(4 marks)

(b) z is a complex number represented by a point on or inside the hexagon $ABCDEF$. For each of the two cases below, copy Figure 4(a) into your answer book and shade the region which satisfies the specified condition :

(i) $\mathrm{Re}(z) \geq 1$. (Note : $\mathrm{Re}(z)$ denotes the real part of z.)

(ii) $-\dfrac{\pi}{3} \leq \arg z \leq \dfrac{\pi}{3}$.

(5 marks)

(c)

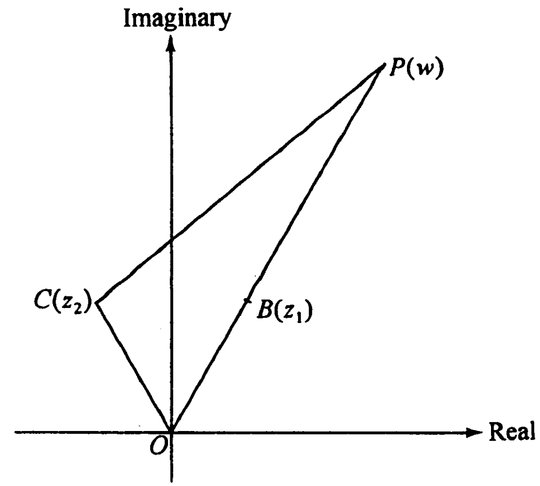

Figure 4(b)

In Figure 4 (b), P is a point on OB produced such that $OP = 3OB$. Let the complex number represented by P be w.

(i) Find w in standard form.

(ii) Find $\arg(w - z_2)$ correct to 3 significant figures.

Hence find $\angle OPC$ correct to 3 significant figures.

(7 marks)

13.

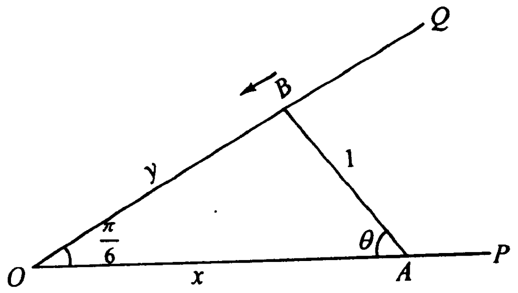

Figure 5

In Figure 5, POQ is a rail and $\angle POQ = \frac{\pi}{6}$. AB is a rod of length 1 m which is free to slide on the rail with end A on OP and end B on OQ. Initially, end A is at the point on OP such that $\angle OAB = \frac{4\pi}{9}$. End B is pushed towards O at a constant speed. After t seconds, $OA = x$ m, $OB = y$ m and $\angle OAB = \theta$, where $0 \le \theta \le \frac{4\pi}{9}$.

(a) Express x and y in terms of θ. (3 marks)

(b) Let S m^2 be the area of ΔOAB.

Show that $\frac{dS}{d\theta} = \sin(\frac{5\pi}{6} - 2\theta)$.

Hence find the value of θ such that S is a maximum. (6 marks)

(c) Using (a), show that $\frac{dx}{dt} = \frac{-\cos(\frac{5\pi}{6} - \theta)}{\cos\theta}\frac{dy}{dt}$. (4 marks)

(d) A student makes the following prediction regarding the motion of end A of the rod :

> As end B moves from its initial position to point O, end A will first move away from O and then it will change its direction and move towards O.

Is the student's prediction correct ? Explain your answer. (3 marks)

END OF PAPER