Section A (42 marks)

Answer ALL questions in this Section.

1. Show that $\dfrac{\sin 3\theta}{\sin\theta} + \dfrac{\cos 3\theta}{\cos\theta} = 4\cos 2\theta$.

(4 marks)

2. Find $\int x\sqrt{x-1}\,dx$.

[Hint : Let $u = x - 1$.]

(4 marks)

3. Given three points $A(0, 2)$, $B(4, 6)$ and $C(3, 0)$. P is a point on AB such that $AP:PB = \lambda:1$, where $\lambda > 0$.

(a) Find the coordinates of P in terms of λ.

(b) If the area of ΔPAC is 6, find the value(s) of λ.

(5 marks)

4. By expressing $6\sin x + 8\cos x$ in the form $r\sin(x+\alpha)$, find the general solution of the equation

$$6\sin x + 8\cos x = 5,$$

and give your answer correct to the nearest degree.

(5 marks)

5. The slope at any point (x, y) of a curve is given by

$$\frac{dy}{dx} = 6x + \frac{1}{x^2},$$

where $x > 0$. If the curve cuts the x-axis at the point (1, 0), find its equation.

(5 marks)

6. L is the line $y = 2x+3$.

(a) A line with slope m makes an angle of 45° with L. Find the value(s) of m.

(b) A family of straight lines is given by the equation

$$2x - 3y + 2 + k(x-y-1) = 0,$$

where k is real. Using (a), find the equation of the line in the family with positive slope which makes an angle of 45° with L.

(6 marks)

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7. Let $T_n = (n^2+1)(n!)$ for any positive integer n.

Prove, by mathematical induction, that

$$T_1 + T_2 + \ldots + T_n = n[(n+1)!]$$

for any positive integer n.

[Note : $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$.]

(6 marks)

8. Expand $(1+x)^n(1-2x)^4$ in ascending powers of x up to the term x^2, where n is a positive integer.

If the coefficient of x^2 is 54, find the coefficient of x.

(7 marks)

Section B (48 marks)

Answer any THREE questions in this Section.
Each question carries 16 marks.

9. $A(x, y)$ is a variable point such that the distance between A and the point (1, 0) is always equal to the distance from A to the line $x+1=0$. Let $\mathcal{P}$ be the locus of A.

(a) Show that the equation of $\mathcal{P}$ is $y^2 = 4x$. (3 marks)

(b) Show that the equation of the tangent to $\mathcal{P}$ at the point $(t^2, 2t)$, where $t \neq 0$, is $x - ty + t^2 = 0$.

Find the equation of the normal to $\mathcal{P}$ at that point. (3 marks)

(c)

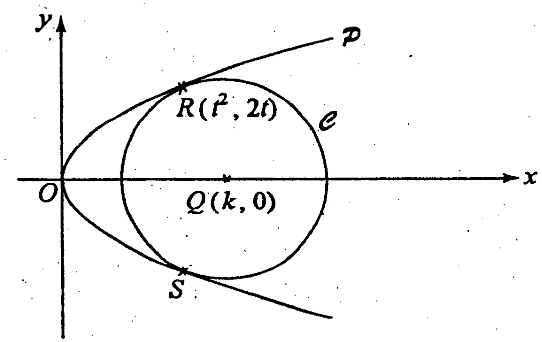

Figure 1

A circle $\mathcal{C}$ centred at point $Q(k, 0)$ touches $\mathcal{P}$ at two distinct points $R(t^2, 2t)$ and S as shown in Figure 1 (i.e. $\mathcal{P}$ and $\mathcal{C}$ have common tangents at these two points), where $t \neq 0$.

(i) Show that $t^2 = k-2$.

(ii) It is known that the tangent to $\mathcal{P}$ at point R cuts the y-axis at the point (0, 2). Find

(1) the equation of the tangent to $\mathcal{P}$ at point S,

(2) the equation of $\mathcal{C}$. (10 marks)

10.

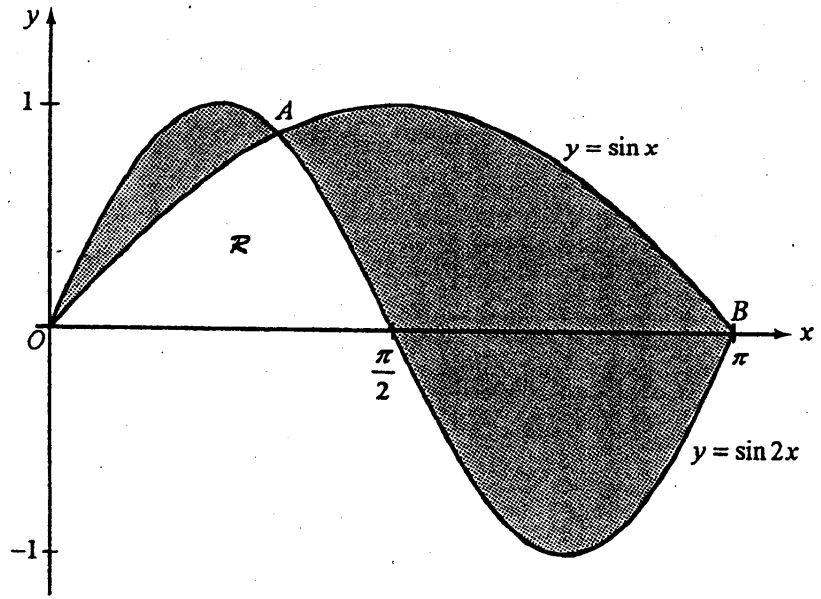

Figure 2(a)

Figure 2(a) shows the curves $y = \sin x$ and $y = \sin 2x$ for $0 \le x \le \pi$. The two curves intersect at the origin O, point A and point $B(\pi, 0)$.

(a) Show that the coordinates of A are $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$. (2 marks)

(b) Find the area of the shaded region in Figure 2(a). (5 marks)

(c) $\mathcal{R}$ is the region bounded by the two curves and the x-axis from $x = 0$ to $\frac{\pi}{2}$. (See Figure 2(a).) If the region $\mathcal{R}$ is revolved about the x-axis, find the volume of the solid of revolution generated.

(5 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet into your answer book.

10. (d) Figure 2(b) shows the curve $y = \sin x$ for $0 \le x \le \pi$.

In Figure 2(b), sketch the curve $y = |\sin 2x|$ for $0 \le x \le \pi$.

In the same figure, shade the region whose area is represented by the expression

$$\int_0^{\pi} \left| |\sin 2x| - \sin x \right| dx .$$ (4 marks)

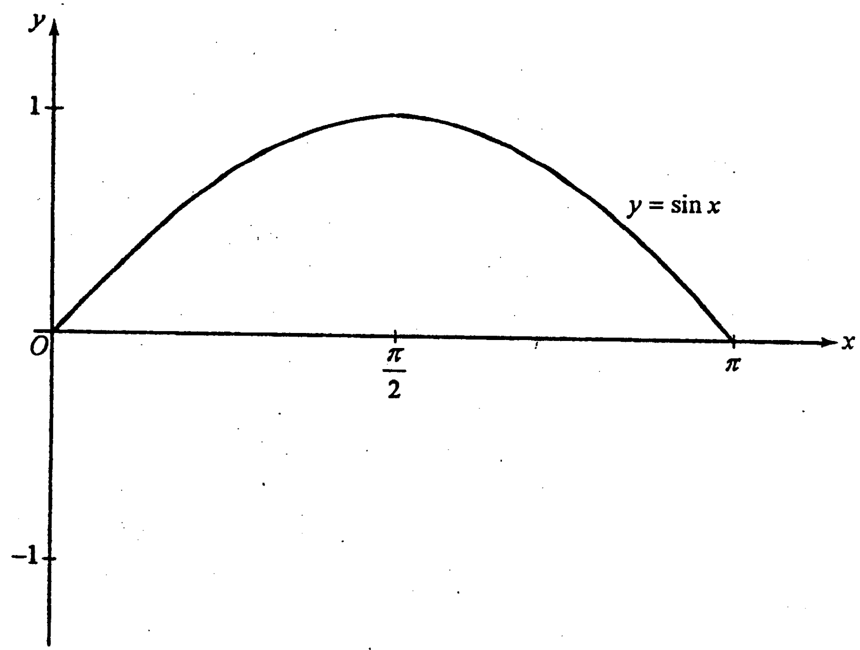

Figure 2(b)

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11. (a) Using the substitution $u = \cot\theta$, find

$$\int \cot^n\theta \csc^2\theta \, d\theta,$$

where n is a non-negative integer.

(3 marks)

(b) By writing $\cot^{n+2}\theta$ as $\cot^n\theta \cot^2\theta$, show that

$$\int \cot^{n+2}\theta \, d\theta = -\frac{\cot^{n+1}\theta}{n+1} - \int \cot^n\theta \, d\theta,$$

where n is a non-negative integer.

(4 marks)

(c) Using (b), or otherwise, show that

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^2\theta \, d\theta = 1 - \frac{\sqrt{3}}{3} - \frac{\pi}{12}.$$

(3 marks)

(d) Using the substitution $x = \sec\theta$, evaluate

$$\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{(x^2-1)^5}}.$$

(6 marks)

12.

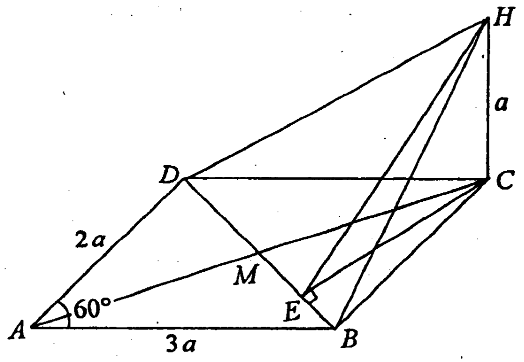

Figure 3(a)

In Figure 3(a), $ABCD$ is a parallelogram on a horizontal plane with $AB = 3a$, $AD = 2a$ and $\angle BAD = 60°$. H is a point vertically above C and $HC = a$.

(a) (i) Find AC in terms of a.

(ii) If M is the mid-point of AC, find the angle of elevation of H from M to the nearest degree.

(4 marks)

(b) E is a point on BD such that CE is perpendicular to BD.

(i) Find BD and CE in terms of a.

(ii) Using Pythagoras' theorem and its converse, show that HE is perpendicular to BD.

Hence find the angle between the planes HBD and $ABCD$ to the nearest degree.

(9 marks)

(c)

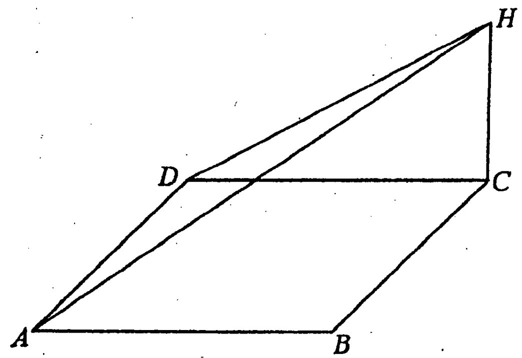

Figure 3(b)

Figure 3(b) shows the planes HAD and $ABCD$. X is a point lying on both planes such that the angle between the two planes is $\angle HXC$. Find AX in terms of a.

(3 marks)

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13. Given a family of circles $F: x^2+y^2-6x-2y+k(2x-4y+3)=0$, where k is real. All circles in F pass through two fixed points A and B.

(a) Find, in terms of k, the centre of a circle in F and show that the radius of the circle is $\sqrt{5(k^2-k+2)}$.

(4 marks)

(b) By considering the radius of the smallest circle in F, or otherwise, find the length of AB.

(4 marks)

(c) Given a straight line $L: 4x+2y-9=0$.

(i) Show that the distance from the centre of a circle in F to the line L is a constant.

State the geometrical relationship between the locus of the centres of the circles in F and the line L.

(ii)

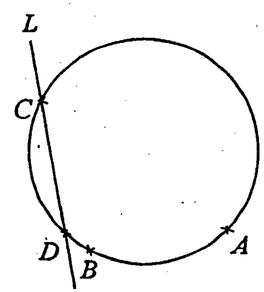

Figure 4

A circle in F cuts the line L at two points C and D such that the chords CD and AB are equal. (See Figure 4.) Find the equations of the two possible circles satisfying this condition.

(8 marks)

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