Section A (42 marks)

Answer ALL questions in this Section.

1. Let $f(x)=\sqrt{3+x^2}$. Find $f'(-1)$.

(3 marks)

2. $P(8,1)$ is a point on the curve $y^2+\sqrt[3]{x}\,y-3=0$. Find the value of $\frac{dy}{dx}$ at P.

(3 marks)

3. (a) Express $\frac{1+i}{1-i}$ in standard form.

(b) Using (a), or otherwise, find the value(s) of n such that $(1+i)^{2n}=(1-i)^{2n}$, where n is a positive integer.

(5 marks)

4.

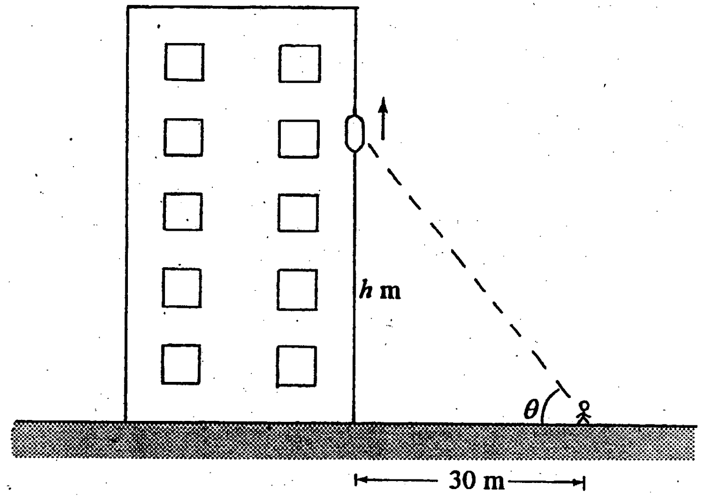

Figure 1

A man stands at a horizontal distance of 30 m from a sight-seeing elevator of a building as shown in Figure 1. The elevator is rising vertically with a uniform speed of 1.5 m s^{-1}. When the elevator is at a height h m above the ground, its angle of elevation from the man is θ. Find the rate of change of θ with respect to time when the elevator is at a height $30\sqrt{3}$ m above the ground. [Note : You may assume that the sizes of the elevator and the man are negligible.]

(5 marks)

5. Solve $\begin{cases} |3x-4|<2 \\ \frac{1}{2x-1}\le 1 \end{cases}$.

(6 marks)

6. In an Argand diagram, P is the point representing the complex number z which satisfies the equation

$$|z-(3-4i)|=3.$$

(a) Sketch the locus of P.

(b) Q is the point on the locus of P such that the modulus of the complex number represented by Q is the smallest. Find the complex number represented by Q in standard form.

(6 marks)

7. Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors such that $\mathbf{a}=2\mathbf{i}+4\mathbf{j}$, $|\mathbf{b}|=\sqrt{5}$ and $\cos\theta=\frac{4}{5}$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

(a) Find $|\mathbf{a}|$.

(b) Find $\mathbf{a}\cdot\mathbf{b}$.

(c) If $\mathbf{b}=m\mathbf{i}+n\mathbf{j}$, find the values of m and n.

(7 marks)

8. Let α and β be the roots of the equation $x^2+(k+2)x+2(k-1)=0$, where k is real.

(a) Show that α and β are real and distinct.

(b) If $|\alpha-\beta|>3$, find the range of possible values of k.

(7 marks)

Section B (48 marks)

Answer any THREE questions in this Section.
Each question carries 16 marks.

9.

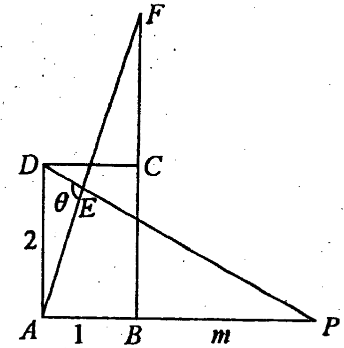

Figure 2

In Figure 2, $ABCD$ is a rectangle with $AB=1$ and $AD=2$. F is a point on BC produced with $BC=CF$. P is a variable point on AB produced such that $BP=m$. AF and DP intersect at a point E. Let $\overrightarrow{AB}=\mathbf{a}$, $\overrightarrow{AD}=\mathbf{b}$ and $\angle AED=\theta$.

(a) (i) Express $\overrightarrow{AF}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Express $\overrightarrow{DP}$ in terms of m, $\mathbf{a}$ and $\mathbf{b}$. (3 marks)

(b) Suppose $\theta=90°$.

(i) Show that $m=7$.

(ii) Let $AE:EF=1:r$ and $DE:EP=1:k$.

(1) Express $\overrightarrow{AE}$ in terms of r, $\mathbf{a}$ and $\mathbf{b}$.

(2) Express $\overrightarrow{AE}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.

Hence find the values of r and k. (10 marks)

(c) As m tends to infinity, θ approaches a certain value θ_1. Find θ_1 correct to the nearest degree. (3 marks)

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10. The function $f(x)=\dfrac{x^2+kx+9}{x^2+1}$, where k is a constant, attains a stationary value at $x=3$.

(a) Find $f'(x)$ in terms of k and x.

Hence show that $k=-6$.

(4 marks)

(b) (i) Find the x- and y- intercepts of the curve $y=f(x)$.

(ii) Find the maximum and minimum points of the curve $y=f(x)$.

(7 marks)

(c) Sketch the graph of $y=f(x)$ for $-6\leq x\leq 6$ in Figure 3.

Hence sketch the graph of $y=-f(x)-1$ for $-6\leq x\leq 6$ in the same figure.

(5 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet into your answer book.

10. (c) **(continued)**

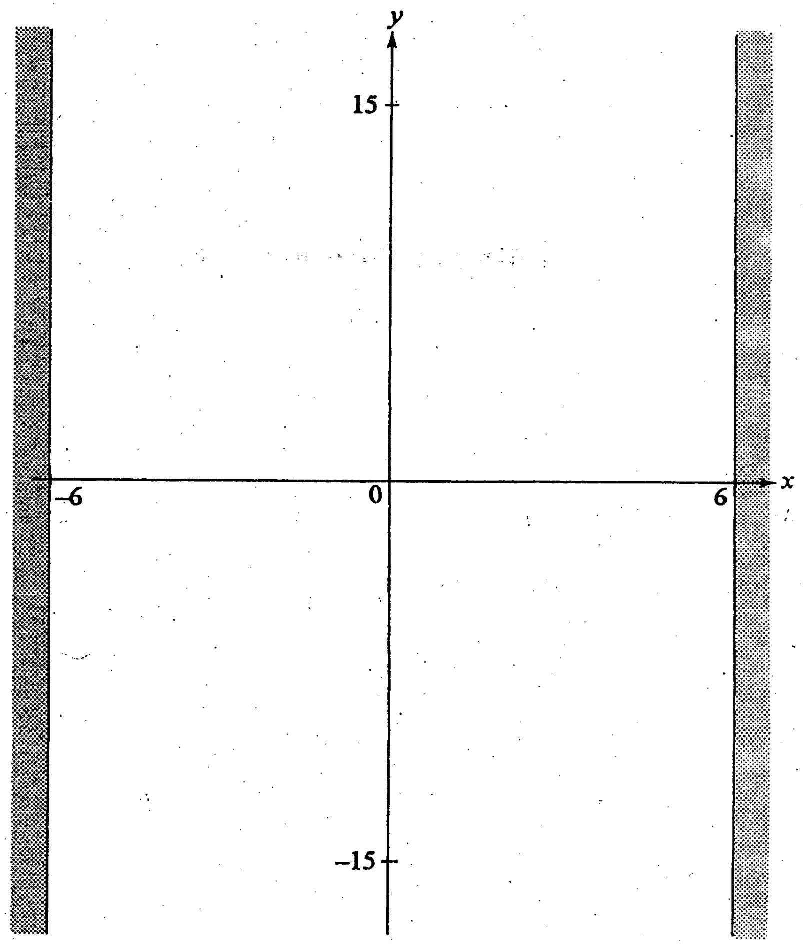

Figure 3

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11. (a) Solve the equation $\cos 5\theta = 0$ for $0° \le \theta \le 180°$.

(2 marks)

(b) Using De Moivre's Theorem, show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta .$$

(4 marks)

(c) Let $f(x) = 16x^4 - 20x^2 + 5$.

(i) By putting $x = \cos\theta$ and using the results of (a) and (b), find the four values of θ for which $\cos\theta$ is a root of $f(x) = 0$.

Hence show that

$$f(x) = 16(x^2 - \cos^2 18°)(x^2 - \cos^2 54°) \quad \ldots (*).$$

(ii) Using (*), form a quadratic equation with integral coefficients whose roots are $\sin^2 18°$ and $\sin^2 54°$. [Hint : You may treat $f(x)$ as a quadratic function of x^2.]

(10 marks)

12.

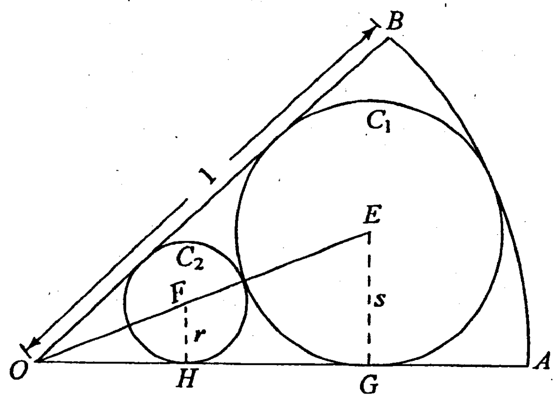

Figure 4

In Figure 4, OAB is a sector of unit radius and $\angle AOB = 2\theta$, where $0<\theta<\frac{\pi}{2}$. C_1 is an inscribed circle of radius s in the sector. C_2 is another circle of radius r touching OA, OB and C_1. Let E and F be the centres of C_1 and C_2 respectively. OA touches C_1 and C_2 at G and H respectively.

(a) Show that $s=\dfrac{\sin\theta}{1+\sin\theta}$.

Hence find $\dfrac{ds}{d\theta}$.

(4 marks)

(b) By considering ΔOFH and ΔOEG, express r in terms of s.

Hence show that $\dfrac{dr}{d\theta}=\dfrac{\cos\theta(1-3\sin\theta)}{(1+\sin\theta)^3}$.

(5 marks)

(c) By considering the ranges of values of θ for which r is

(i) increasing, and

(ii) decreasing,

find the maximum area of circle C_2. [Note : You may give your answers correct to three significant figures.]

(5 marks)

(d) Does the area of circle C_1 attain a minimum when the area of circle C_2 attains its maximum ? Explain your answer.

(2 marks)

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13. **In this question, numerical answers may be given correct to three significant figures.**

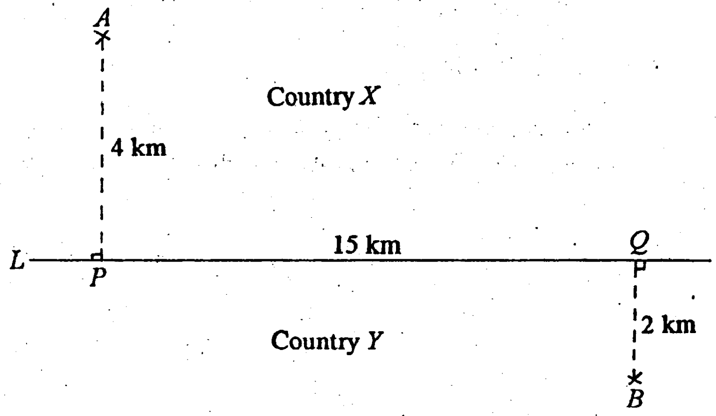

Figure 5

In Figure 5, line L represents the border of two countries X and Y. Amy lives at place A in Country X while Billy lives at place B in Country Y. P and Q are respectively the feet of perpendicular from A and B to the border and $AP = 4$ km, $PQ = 15$ km, $QB = 2$ km. Amy and Billy want to meet each other as early as possible at a certain point **on the border**. They start walking from home to that point at the same time. If one arrives earlier, he/she has to wait for the other.

(a) Let R be a point on the border such that $AR = RB$.

(i) Find the distance of R from Q.

(ii) Suppose Amy and Billy walk at equal speeds of 4 km h^{-1}. Explain briefly why they should walk to R in order to meet each other within the shortest time. Find this shortest time. (6 marks)

(b) (i) Suppose Billy runs at a speed of 8 km h^{-1} instead and Amy still walks at a speed of 4 km h^{-1}. To which point on the border should they go in order to meet each other within the shortest time ?

(ii) Suppose Billy rides on a bicycle at a speed of 16 km h^{-1} instead and Amy still walks at a speed of 4 km h^{-1}. To which point on the border should they go in order to meet each other within the shortest time ? (10 marks)

END OF PAPER