1996 II

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answer should **NOT** be awarded. Unless otherwise specified, no marks in the marking scheme are subdivisible.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol pp–1 should be used to denote marks deducted for poor presentation (p.p.). Marks entered in the Page Total Box should be the net total score on that page. Note the following points :

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those parts where candidates could not score any marks.

   (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles ⬚, whereas alternative answers are enclosed by solid rectangles ▭ .

6. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

7. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

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Solution	Marks	Remarks
1. $\sin 5\theta + \sin 3\theta = \cos\theta$		
$2\sin 4\theta \cos\theta = \cos\theta$	1M	For using $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos\theta = 0$ or $\sin 4\theta = \frac{1}{2}$	1A+1A	
$4\theta = n\pi + (-1)^n(\frac{\pi}{6})$ n is an integer		
$\theta = 2n\pi \pm \frac{\pi}{2}$ or $\theta = \frac{n\pi}{4} + (-1)^n(\frac{\pi}{24})$	1A+1A	
or $360n° \pm 90°$ or $45n° + (-1)^n 7.5°$		$2n\pi \pm 90°$ etc (pp − 1)
	5	
2. (a) $(1 + x + ax^2)^6 = [1 + x(1 + ax)]^6$	1M	Accept $[(1 + x) + ax^2]^6$ etc.
$= 1 + 6x(1 + ax) + 15x^2(1 + ax)^2 + 20x^3(1 + ax)^3 + \ldots$	1A	or $(1 + x)^6 + 6(1 + x)^5 ax^2 + \ldots$ Accept ${}_6C_r$ notations
$= 1 + 6x + (6a + 15)x^2 + (30a + 20)x^3 + \ldots$		(pp−1) for omitting dots
$\therefore k_1 = 6a + 15$	1A	
$k_2 = 30a + 20$	1A	
(b) $6 + k_2 = 2k_1$		or $k_2 - k_1 = k_1 - 6$
$6 + (30a + 20) = 2(6a + 15)$	1M	
$a = \frac{2}{9}$	1A	
	6	
3. (a) $\begin{cases} \frac{x^2}{2} + \frac{y^2}{7} = 1 \\ y = mx + c \end{cases}$		
$7x^2 + 2(mx + c)^2 = 14$	1M	For substitution
$(7 + 2m^2)x^2 + 4mcx + (2c^2 - 14) = 0$	1A	
Since the line is a tangent to E,		
$(4mc)^2 - 4(7 + 2m^2)(2c^2 - 14) = 0$	1M	
$16m^2c^2 - 4(14c^2 - 98 + 4m^2c^2 - 28m^2) = 0$		
$14c^2 = 98 + 28m^2$		
$c^2 = 2m^2 + 7$	1	

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Solution	Marks	Remarks
Alternative solution Let (x_0, y_0) be the point of contact of the tangent with E.		
The equation of the tangent is $7(xx_0) + 2(yy_0) = 14$	1A	
Compare with $y = mx + c$,		
$\frac{7x_0}{m} = \frac{2y_0}{-1} = \frac{-14}{c}$	1M	
$\therefore x_0 = \frac{-2m}{c}, y_0 = \frac{7}{c}$		
Substitute into E ,		
$7(\frac{-2m}{c})^2 + 2(\frac{7}{c})^2 = 14$	1M	
$2m^2 + 7 = c^2$	1	
(b) Substitute (0, 5) into $y = mx + c$, $c = 5$	1A	
Put $c = 5$, $25 = 2m^2 + 7$	1M	OR Substituting $y = mx + 5$ into E and set $\Delta = 0$
$m = \pm 3$.		
$\therefore$ The equations of the two tangents are		
$y = 3x + 5$ and $y = -3x + 5$.	1A	
	7	

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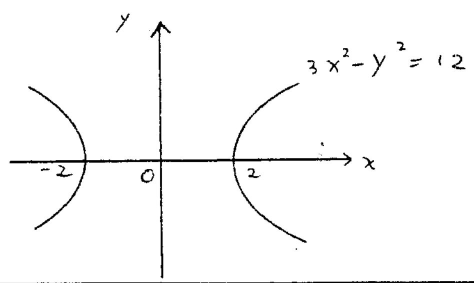
Solution	Marks	Remarks
4. For $n = 1$, $2n^3 + n = 3$ which is divisible by 3.	1	
$\therefore$ the statement is true for $n = 1$.		
Assume $2k^3 + k$ is divisible by 3 for some +ve integer k.	1	
i.e. $2k^3 + k = 3m$, where m is an integer		
Then $2(k+1)^3 + (k+1) = 2(k^3 + 3k^2 + 3k + 1) + (k + 1)$	1	
$= (2k^3 + k) + (6k^2 + 6k + 3)$	2	
$= 3m + 3(2k^2 + 2k + 1)$		
Since both terms are divisibles by 3,		
$\therefore$ $2(k+1)^3 + (k+1)$ is also divisible by 3.		
Alternative solution (1)		
Put $2k^3 = 3m - k$		
$2(k+1)^3 + (k+1) = 2(k^3 + 3k^2 + 3k + 1) + (k + 1)$	1	
$= 3m - k + 6k^2 + 6k + 2 + (k + 1)$	2	
$= 3m + 3(2k^2 + 2k + 1)$		
Alternative solution (2)		
Let $f(k) = 2k^3 + k$		
$f(k+1) - f(k) = 2(k^3 + 3k^2 + 3k + 1) + (k + 1) - (2k^3 + k)$	1	
$= 3(2k^2 + 2k + 1)$	2	
$f(k+1) = f(k) + 3(2k^2 + 2k + 1)$		
Since $f(k)$ is divisible by 3, $\therefore$ $f(k+1)$ is also divisible by 3.		
$\therefore$ The statement is also true for $n = k + 1$ if it is true for $n = k$.		
By the principle of mathematical induction,		
the statement is true for all positive integers n.	1	
	6	
5. (a) Area bounded by C and OA $= \int_0^4 (4x - x^2)dx$	1A	(pp–1) for omitting dx
$= [2x^2 - \frac{x^3}{3}]_0^4$	1A	
$= \frac{32}{3}$	1A	
(b) Area of $\Delta OPA = \frac{1}{2}(4)(3)$		or $= \int_0^1 3x\,dx + \int_1^4 (-x+4)dx$
$= 6$	1A	$= \frac{3}{2} + \frac{9}{2} = 6$
$\therefore$ Area of shaded region $= \frac{32}{3} - 6$	1M	
$= \frac{14}{3}$	1A	
	6	

	Solution	Marks	Remarks
6.	$y=\int \tan^3 x \sec x\,dx$	1A	Withhold this mark if "$y=$" is omitted
	Let $u=\sec x$		
	$du=\sec x \tan x\,dx$	1A	
	$y=\int (u^2-1)du$	1A	Omit 'du' (pp–1)
	$=\frac{u^3}{3}-u+c$		
	$=\frac{1}{3}\sec^3 x-\sec x+c$ where c is a constant	1A	no mark if c is omitted
	Alternative solution		
	$y=\int \tan^3 x \sec x\,dx$	1A	Omit 'dx' (pp–1)
	$=\int \tan^2 x\,d\sec x$	1A	
	$=\int (\sec^2 x-1)\,d\sec x$	1A	
	$=\frac{1}{3}\sec^3 x-\sec x+c$ where c is a constant	1A	
	Since the curve passes through O,		
	$0=\frac{1}{3}-1+c$	1M	
	$c=\frac{2}{3}$		
	$\therefore$ The equation of the curve is $y=\frac{1}{3}\sec^3 x-\sec x+\frac{2}{3}$.	1A	
	or $y=\frac{1}{3\cos^3 x}-\frac{1}{\cos x}+\frac{2}{3}$		
		6	
7.	(a)		
	$\sqrt{(x-4)^2+y^2}=2\lvert x-1\rvert$ or $(x-4)^2+y^2=4(x-1)^2$	1A+1A	1A for LHS, 1A for RHS (pp–1) for omitting absolute sign
	$x^2-8x+16+y^2=4(x^2-2x+1)$		
	$3x^2-y^2=12$ or $\frac{x^2}{4}-\frac{y^2}{12}=1$, $3x^2-y^2-12=0$ etc.	1A	
	The locus is a hyperbola.	1A	
	(b) 	1M+1A	(pp–1) for not labelling the axes.
		6	

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Solution	Marks	Remarks
8. (a) The equation of F is		
$2x-y-4+k(x-2y+4)=0$ where k is real	1A	or $x-2y+4+k(2x-y-4)=0$
Substitute (0, 0) into the equation,		
$-4+4k=0$	1M	
$k=1$	1A	
$\therefore$ The equation of L is $2x-y-4+(x-2y+4)=0$		
i.e. $y-x=0$.	1A	
Alternative solution		
$\begin{cases} x-2y+4=0 \\ 2x-y-4=0 \end{cases}$		
Solving the two equations, $x=4$, $y=4$.		
$\therefore$ The coordinates of the point of intersection of L_1 and L_2 are (4, 4).		
Let m be the slope of a line passing through (4, 4).		
The equation of F is		
$\dfrac{y-4}{x-4}=m$		
$y=mx-4m+4$.	1A	
Substitute (0, 0) into the equation ,	1M	
$m=1$	1A	
$\therefore$ The equation of L is $y=x$.	1A	
	4	
(b) (i) $m_{L_1}=2$, $m_{L_2}=\dfrac{1}{2}$, $m_L=1$	1A	Awarded if at least two are correct.
$\tan\angle CAB=\dfrac{m_{L_1}-m_L}{1+m_L m_{L_1}}$		Accept $\left\lvert\dfrac{m_L-m_{L_1}}{1+m_L m_{L_1}}\right\rvert$
$=\dfrac{2-1}{1+2}$	1M	
$=\dfrac{1}{3}$	1	
$\tan\angle CAD=\dfrac{m_L-m_{L_2}}{1+m_{L_2}m_L}$		
$=\dfrac{1-\frac{1}{2}}{1+\frac{1}{2}}$		
$=\dfrac{1}{3}$	1	

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Solution	Marks	Remarks
(ii) (1) Let $BC = x$		or Let $CD = x$
$\tan\angle CAB = \frac{BC}{AB} = \frac{1}{3}$ $\therefore AB = 3x$	1A	$AD = 3x$
Area of $\Delta CBA = \frac{1}{2}(BC)(AB)$ $= \frac{3x^2}{2}$	1A	Area $= \frac{1}{2}(CD)(AD)$
Since ΔCBA and ΔCDA are congruent,		(can be omitted)
$\frac{3x^2}{2} = \frac{240}{2}$	1M	
$x = 4\sqrt{5}$ or $= \sqrt{80}$	1A	
(2) Let the coordinates of C be (h, h).	1A	
$BC = \left\|\frac{2h-h-4}{\sqrt{5}}\right\|$	1M	For distance formula
Since $BC = 4\sqrt{5}$, $\left\|\frac{2h-h-4}{\sqrt{5}}\right\| = 4\sqrt{5}$	1A	Accept $\frac{h-4}{\sqrt{5}} = -4\sqrt{5}$
$\|h-4\| = 20$		(pp–1) for $\frac{h-4}{\sqrt{5}} = 4\sqrt{5}$
$h = 24$ (rejected) or -16		
$h = -16$		
$\therefore$ The coordinates of C are $(-16, -16)$.	1A	
or $CD = \left\|\frac{h-2h+4}{\sqrt{5}}\right\|$	1M	For distance formula
Since $CD = 4\sqrt{5}$, $\left\|\frac{h-2h+4}{\sqrt{5}}\right\| = 4\sqrt{5}$	1A	Accept $\frac{4-h}{\sqrt{5}} = 4\sqrt{5}$
$\|4-h\| = 20$		(pp–1)for $\frac{h-4}{\sqrt{5}} = 4\sqrt{5}$
$h = 24$ (rejected) or -16		
$h = -16$		
$\therefore$ The coordinates of C are $(-16, -16)$.	1A	

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Solution	Marks	Remarks
Alternative Solution for (2)		
$AC^2 = BC^2 + AB^2$		or $= CD^2 + DA^2$
$= (4\sqrt{5})^2 + (12\sqrt{5})^2$		
$AC = \sqrt{800}$		
Let the coordinates of C be (h, h).	1A	
Coordinates of A are $(4, 4)$.		
$\sqrt{(h-4)^2+(h-4)^2} = \sqrt{800}$	1M+1A	
$(h-4)^2 = 400$		
$h = 24$ (rejected) or -16		
$\therefore h = -16$		
$\therefore$ The coordinates of C are $(-16, -16)$.	1A	
Alternative Solution for (ii)		
(1)&(2) Let the coordinates of C be (h, h).	1A	
$BC = \left\|\frac{2h-h-4}{\sqrt{5}}\right\|$ or $= -(\frac{2h-h-4}{\sqrt{5}})$	1M	For distance formula (pp–1) for $\frac{2h-h-4}{\sqrt{5}}$
$= \frac{\|h-4\|}{\sqrt{5}}$		
Since $\tan\angle CAB = \frac{BC}{AB} = \frac{1}{3}$		
$AB = 3\frac{\|h-4\|}{\sqrt{5}}$ or $= 3(\frac{4-h}{\sqrt{5}})$	1A	
Area of $\Delta CBA = \frac{1}{2}(BC)(AB)$		
$= \frac{3}{10}(h-4)^2$	1A	
or $CD = \left\|\frac{h-2h+4}{\sqrt{5}}\right\|$ or $= (\frac{h-2h+4}{\sqrt{5}})$	1M	For distance formula (pp–1)for $-(\frac{h-2h+4}{\sqrt{5}})$
$= \frac{\|4-h\|}{\sqrt{5}}$		
Since $\tan\angle CAD = \frac{CD}{AD} = \frac{1}{3}$		
$\therefore AD = 3\frac{\|4-h\|}{\sqrt{5}}$ or $= 3(\frac{4-h}{\sqrt{5}})$	1A	
Area of $\Delta CDA = \frac{1}{2}(CD)(AD)$		
$= \frac{3}{10}(4-h)^2$	1A	

Solution	Marks	Remarks
Since ΔCBA and ΔCDA are congruent,		
$\frac{3}{10}(h-4)^2 = \frac{240}{2}$	1M+1A	
$(h-4)^2 = 400$		
$h = 24$ (rejected) or -16		
$\therefore h = -16$		
$\therefore$ The coordinates of C are $(-16, -16)$	1A	
Put $h = -16$		
$BC = \frac{\lvert -16-4 \rvert}{\sqrt{5}}$		
$= 4\sqrt{5}$.	1A	
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Solution	Marks	Remarks
9. (a) Let $t=\cos x$, $dt=-\sin x\,dx$		
$\int_0^{\pi}\sin^5 x\,dx=\int_1^{-1}-(1-t^2)^2\,dt$	1A+1A	1A for integrand, 1A for limits
$=\int_{-1}^{1}(1-2t^2+t^4)\,dt$		
$=[t-\frac{2t^3}{3}+\frac{t^5}{5}]_{-1}^{1}$	1A	For the primitive function
$=\frac{16}{15}$	1A	
<u>Alternative solution</u>		
$\int_0^{\pi}\sin^5 x\,dx=\int_0^{\pi}-\sin^4 x\,d\cos x$		
$=\int_0^{\pi}-(1-\cos^2 x)^2\,d\cos x$	2A	
$=\int_0^{\pi}-(1-2\cos^2 x+\cos^4 x)\,d\cos x$		
$=-[\cos x-\frac{2\cos^3 x}{3}+\frac{\cos^5 x}{5}]_0^{\pi}$	1A	For the primitive function
$=\frac{16}{15}$	1A	
	4	
(b) $u=\pi-x$, $du=-dx$		
$\int_0^{\pi}x\sin^5 x\,dx=\int_{\pi}^{0}(\pi-u)\sin^5(\pi-u)(-du)$	1A+1A	1A for integrand, 1A for limits
$=\int_0^{\pi}(\pi-u)\sin^5 u\,du$		
$=\pi\int_0^{\pi}\sin^5 u\,du-\int_0^{\pi}u\sin^5 u\,du$		
$\because \int_0^{\pi}x\sin^5 x\,dx=\int_0^{\pi}u\sin^5 u\,du$	1M	(can be omitted)
$\therefore 2\int_0^{\pi}x\sin^5 x\,dx=\pi\int_0^{\pi}\sin^5 u\,du$	1A	
$\int_0^{\pi}x\sin^5 x\,dx=\frac{\pi}{2}(\frac{16}{15})$		
$=\frac{8\pi}{15}$	1A	
	5	

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Solution	Marks	Remarks
(c) $y = x^2 \sin^5 x$		
$\frac{dy}{dx} = 2x\sin^5 x + 5x^2 \sin^4 x \cos x$	1M+1A	1M for product rule
Integrating with respect to x from 0 to π,		
$[x^2 \sin^5 x]_0^\pi = \int_0^\pi (2x \sin^5 x + 5x^2 \sin^4 x \cos x)\,dx$	1M	(pp−1) for omitting the limits in the L.H.S.
$0 = 2\int_0^\pi x \sin^5 x\,dx + 5\int_0^\pi x^2 \sin^4 x \cos x\,dx$	1A	For LHS = 0
$\int_0^\pi x^2 \sin^4 x \cos x\,dx = -\frac{2}{5}\int_0^\pi x \sin^5 x\,dx$		
$= -\frac{2}{5}(\frac{8\pi}{15})$		
$= -\frac{16\pi}{75}$.	1A	
	5	
(d) I_2 is equal to I_1 because		
$\lvert x \rvert = x$ for $0 < x < \pi$.	2A	
or $\cos\lvert x \rvert = \cos x$ for all values of x.		
	2	

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Solution	Marks	Remarks
10. (a) (i) $x^2+y^2-8ky-6ky+25(k^2-1)=0$		
The centre is $(4k, 3k)$.	1A	
Since $3x-4y=3(4k)-4(3k)=0$,	1	
$\therefore$ the centre always lies on the line $3x-4y=0$.		
(ii) $(x-4k)^2+(y-3k)^2=-25(k^2-1)+25k^2$ $=25$		or $r=\sqrt{(\frac{-8k}{2})^2+(\frac{-6k}{2})^2-25(k^2-1)}$ $=5$
$\therefore$ All circles in F have the same radius 5.	1	
	3	
(b) Slopes of the common tangents $=\frac{3}{4}$.	1A	
Let the equation of the tangents by $y=\frac{3}{4}x+c$.	1M	or $3x-4y+c=0$
Distance between centre $(4k, 3k)$ and the tangents = 5.		or (4, 3) or any point on the line $3x-4y=0$
$\left\lvert\dfrac{\frac{3}{4}(4k)-(3k)+c}{\sqrt{(\frac{3}{4})^2+(-1)^2}}\right\rvert=5$	2M	(pp−1) for omitting absolute sign
$c=\pm\frac{25}{4}$		
$\therefore$ The equations of the two tangents are $y=\frac{3}{4}x+\frac{25}{4}$ and $y=\frac{3}{4}x-\frac{25}{4}$	1A+1A	$3x-4y+25=0$ and $3x-4y-25=0$
Alternative Solution (1)		
Slope of the common tangents $=\frac{3}{4}$.	1A	
Let the equation of the tangent be $y=\frac{3}{4}x+c$.	1M	
Substitute into F, $x^2+(\frac{3}{4}x+c)^2-8kx-6k(\frac{3}{4}x+c)+25(k^2-1)=0$	1M	
$\frac{25}{16}x^2+(\frac{3c}{2}-\frac{25k}{2})x+(c^2-6kc+25k^2-25)=0$		
Discriminant $=(\frac{3c}{2}-\frac{25k}{2})^2-4(\frac{25}{16})(c^2-6kc+25k^2-25)=0$	1M	
$\frac{9c^2}{4}-\frac{625k^2}{4}-\frac{75}{2}kc-\frac{25}{4}c^2+\frac{75}{2}kc-\frac{625}{4}k^2+\frac{625}{4}=0$		
$16c^2=625$		
$c=\pm\frac{25}{4}$		
$\therefore$ The equations for the tangents are $y=\frac{3}{4}x+\frac{25}{4}$ and $y=\frac{3}{4}x-\frac{25}{4}$.	1A+1A	

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Solution	Marks	Remarks
Alternative Soluition (2)		
Slope of the common tangents $= \frac{3}{4}$.	1A	
Let the equation of the tangent be $y = \frac{3}{4}x + c$.	1M	or using other circles in F
Substitute into the circle $x^2 + y^2 - 25 = 0$,		
$x^2 + (\frac{3}{4}x + c)^2 - 25 = 0$	1M	
$25x^2 + 24cx + 16(c^2 - 25) = 0$		
Discriminant $= (24c)^2 - 4(25)(16)(c^2 - 25) = 0$	1M	
$16c^2 = 625$		
$c = \pm\frac{25}{4}$		
$\therefore$ The equations for the tangents are		
$y = \frac{3}{4}x + \frac{25}{4}$ and $y = \frac{3}{4}x - \frac{25}{4}$.	1A+1A	
Alternative Solution (3)		
Slope of the common tangents $= \frac{3}{4}$.	1A	
Equation of the line through O and perpendicular to the common tangents is $y = -\frac{4}{3}x$.	1M	
Putting $k = 0$, equation of circle in F with O as centre is $x^2 + y^2 = 25$.	1M	
$\begin{cases} x^2 + y^2 = 25 \\ y = -\frac{4}{3}x \end{cases}$	1M	
$x^2 + (\frac{-4}{3}x)^2 = 25$		
$x = \pm 3$		
When $x = 3$, $y = -4$		
When $x = -3$, $y = 4$		
$\therefore$ Equations of the common tangents are		
$\frac{y+4}{x-3} = \frac{3}{4}$ and $\frac{y-4}{x+3} = \frac{3}{4}$		
$3x - 4y - 25 = 0$ and $3x - 4y + 25 = 0$.	1A+1A	
	6	

Solution	Marks	Remarks
(c) Since the coordinates of the centre are $(4k, 3k)$, distance from centre to x-axis		
$=3\|k\|$ $\left(\text{or } = \begin{cases} 3k & \text{if } k \geq 0 \\ -3k & k < 0 \end{cases}\right)$	2A	Withhold 1 mark if absolute sign was omitted.
Let M be the mid-point of AB and C be the centre.		
Since $AB = 8$, $AM = 4$	1A	(can be omitted)
$AC = 5$		
$\therefore CM = \sqrt{5^2 - 4^2} = 3$	1A	
$\therefore 3\|k\| = 3$	1M	
$k = \pm 1$		
The equations of the two circles are		
$x^2 + y^2 - 8x - 6y = 0$ and $x^2 + y^2 + 8x + 6y = 0$.	1A+1A	$(x-4)^2 + (y-3)^2 = 25$, $(x+4)^2 + (y+3)^2 = 25$
<u>Alternative Solution (1)</u>		
Since $AB = 8$, $AM = 4$.	1A	(can be omitted)
Coordinates of A are $(4k-4, 0)$.	1A	<u>or</u> $B(4k+4, 0)$
Substitute $A(4k-4, 0)$ into F,		
$(4k-4)^2 - 8k(4k-4) + 25k^2 - 25 = 0$	1M	
$9k^2 - 9 = 0$		
$k = \pm 1$		
$\therefore$ The equations of the two circles are $x^2 + y^2 - 8x - 6y = 0$ and $x^2 + y^2 + 8x + 6y = 0$.	1A+1A	
<u>Alternative solution (2)</u>		
Since $AB = 8$, $AM = 4$.	1A	(can be omitted)
Coordinates of A are $(4k-4, 0)$.	1A	<u>or</u> $B(4k+4, 0)$
Distance between A (or B) and the centre is 5.		
$\therefore \sqrt{(4k-4-4k)^2 + (0-3k)^2} = 5$	1M	
$9k^2 + 16 = 25$		
$k = \pm 1$		
$\therefore$ The equation of the circles are		
$x^2 + y^2 - 8x - 6y = 0$ and $x^2 + y^2 + 8x + 6y = 0$.	1A+1A	

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Solution	Marks	Remarks
Alternative Solution (3)		
Put $y=0$, $x^2-8kx+25(k^2-1)=0$		
$x=\dfrac{8k\pm\sqrt{64k^2-100(k^2-1)}}{2}$		
$=4k\pm\sqrt{25-9k^2}$	1A	
$AB=(4k+\sqrt{25-9k^2})-(4k-\sqrt{25-9k^2})$	1A	
$=2\sqrt{25-9k^2}$		
$\therefore 2\sqrt{25-9k^2}=8$	1M	
$25-9k^2=16$		
$k=\pm1$		
$\therefore$ The equations of the two circles are $x^2+y^2-8x-6y=0$ and $x^2+y^2+8x+6y=0$.	1A+1A	
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Solution	Marks	Remarks
11. (a) The coordinates of B are (3, 4).	1A	
	1	
(b) (i) $y=4-(x-3)^2$ $(x-3)^2=4-y$ $x-3=-\sqrt{4-y}$ $\because x<3$ $x=3-\sqrt{4-y}$	1	
(ii) The equation of BC is $x=3+\sqrt{4-y}$.	1A	
	2	
(c) (i) Volume $=\pi\int_0^h[(3+\sqrt{4-y})^2-(3-\sqrt{4-y})^2]\,dy$	1M+1A+1A	1M for $V=\pi\int_a^b x^2dy$, 1A for integrand, 1A for limits
$=12\pi\int_0^h\sqrt{4-y}\,dy$		
$=12\pi[-\frac{2}{3}(4-y)^{\frac{3}{2}}]_0^h$	1A	For the primitive function or $=-12\pi[\frac{2}{3}u^{\frac{3}{2}}]_4^{4-h}$ $(u=4-y)$
$=8\pi[8-(4-h)^{\frac{3}{2}}]$	1	
Alternative Solution		
Volume formed by revolving lower part of curve BC		
$V_1=\pi\int_0^h(3+\sqrt{4-y})^2]\,dy$	1M+1A	1M for $V=\pi\int_a^b x^2dy$
$=\pi\int_0^h(13-y+6\sqrt{4-y})\,dy$		
$=\pi[13y-\frac{y^2}{2}-4(4-y)^{\frac{3}{2}}]_0^h$		
$=\pi[13h-\frac{1}{2}h^2-4(4-h)^{\frac{3}{2}}+32]$	1A	
Volume formed by revolving lower part of curve AB		
$V_2=\pi\int_0^h(3-\sqrt{4-y})^2]\,dy$		
$=\pi\int_0^h(13-y-6\sqrt{4-y})\,dy$		
$=\pi[13y-\frac{1}{2}y^2+4(4-y)^{\frac{3}{2}}]_0^h$		
$=\pi[13h-\frac{1}{2}h^2+4(4-h)^{\frac{3}{2}}-32]$	1A	
Volume of lower layer of jelly ring		
$=V_1-V_2$		
$=64\pi-8\pi(4-h)^{\frac{3}{2}}$		
$=8\pi[8-(4-h)^{\frac{3}{2}}]$	1	
(ii) Volume of jelly ring		
$=8\pi[8-(4-4)^{\frac{3}{2}}]$	1M	For putting $h=4$
$=64\pi$	1A	

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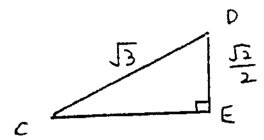
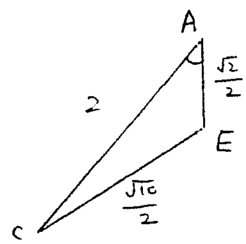
Solution	Marks	Remarks
If the two layers have equal volumes,		
$8\pi[8-(4-h)^{\frac{3}{2}}]=\frac{1}{2}(64\pi)$	1M	
$(4-h)^{\frac{3}{2}}=4$		
h = 1.48 (correct to 3 sig. figures)	1A	
	9	
Alternative solution for (ii)		
Volume of upper layer		
$=\pi\int_h^4[(3+\sqrt{4-y})^2-(3-\sqrt{4-y})^2]\,dy$	1M	
$=12\pi\int_h^4\sqrt{4-y}\,dy$		
$=12\pi[-\frac{2}{3}(4-y)^{\frac{3}{2}}]_h^4$		
$=8\pi(4-h)^{\frac{3}{2}}$	1A	
If the two layers have equal volumes,		
$=8\pi(4-h)^{\frac{3}{2}}=8\pi[8-(4-h)^{\frac{3}{2}}]$	1M	
$(4-h)^{\frac{3}{2}}=4$		
h = 1.48 (correct to 3 sig. figures)	1A	
(d) Volume of milk		
$=\pi\int_0^4(3-\sqrt{4-y})^2\,dy$	1A+1M	1A for integrand, 1M for limits
$=\pi\int_0^4(13-y-6\sqrt{4-y})\,dy$		
$=\pi[13y-\frac{1}{2}y^2+4(4-y)^{\frac{3}{2}}]_0^4$	1A	For the primitive function
$=\pi[52-8-32]=12\pi$	1A	
Alternative Solution (1)		
Substitute $h=4$ in v_2 in the alternative solution (c) (i),		
Volume of milk $=\pi[13(4)-\frac{1}{2}(4)^2+4(4-4)^{\frac{3}{2}}-32]$	2M+1A	
$=\pi(52-8-32)$		
$=12\pi$	1A	

Solution	Marks	Remarks
Alternative Solution (2)		
Volume formed by revolving curve BC		
$=\pi\int_0^4(3+\sqrt{4-y})^2\,dy$	1M	or putting $h=4$ in V_1 in the alt. soln. in (c) (i)
$=\pi\int_0^4(13-y+6\sqrt{4-y})\,dy$		
$=\pi[13y-\frac{y^2}{2}-4(4-y)^{\frac{3}{2}}]_0^4$		
$=76\pi$	1A	
Volume of milk		
$=76\pi-$ Volume of jelly ring	1M	
$=76\pi-64\pi$		
$=12\pi$	1A	
	4	

Solution	Marks	Remarks
12. (a) By Sine Law,		
$\dfrac{AB}{\sin 30^\circ}=\dfrac{2}{\sin 45^\circ}$		
$AB=\sqrt{2}$	1A	
$BD=\sqrt{2}\cos 45^\circ=1$	1A	
$DC=2\cos 30^\circ=\sqrt{3}$	1A	
	3	
(b) (i) θ is $\angle DCE$.	1A	(can be omitted)
$DE=\sin 45^\circ=\dfrac{\sqrt{2}}{2}$	1A	
In ΔCDE, $\sin\theta=\dfrac{DE}{CD}$		
$=\dfrac{\sqrt{2}/2}{\sqrt{3}}$	1M	
$\sin\theta=\dfrac{\sqrt{6}}{6}$	1	
(ii) $CE=\sqrt{(CD)^2-(DE)^2}$ $=\sqrt{(\sqrt{3})^2-(\frac{\sqrt{2}}{2})^2}$ $=\dfrac{\sqrt{10}}{2}$ (or $\sqrt{\frac{5}{2}}$) OR $CE=CD\cos\theta$ $=\sqrt{3}(\frac{\sqrt{30}}{6})$ $=\dfrac{\sqrt{10}}{2}$	1M 1A	
$AE=\dfrac{1}{2}AB=\dfrac{\sqrt{2}}{2}$	1A	
In ΔEAC, by Cosine Law		
$\cos\angle EAC=\dfrac{(AE)^2+(AC)^2-(CE)^2}{2(AE)(AC)}$	1M	
$=\dfrac{(\frac{\sqrt{2}}{2})^2+2^2-(\frac{\sqrt{10}}{2})^2}{2(\frac{\sqrt{2}}{2})(2)}$		
$=\dfrac{1}{\sqrt{2}}$		
$\therefore\ \angle EAC=45^\circ$	1	

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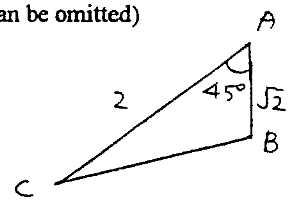
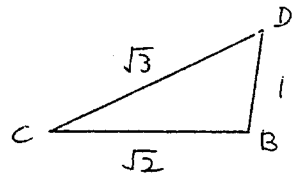
Solution	Marks	Remarks
(iii) $\angle BDC$ represents the angle between the 2 planes.	1A	(can be omitted) 
$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 45°$ $= (\sqrt{2})^2 + 2^2 - 2(\sqrt{2})(2)\cos 45°$ $= 2$		
$\therefore BC = \sqrt{2}$	1A	
In ΔBDC, by Cosine Law,		
$\cos\angle BDC = \dfrac{(BD)^2 + (CD)^2 - (BC)^2}{2(BD)(CD)}$	1M	
$= \dfrac{1^2 + (\sqrt{3})^2 - (\sqrt{2})^2}{2(1)(\sqrt{3})} = \dfrac{1}{\sqrt{3}}$		
$\therefore\ \angle BDC = 55°$ (correct to the nearest degree)	1A	(Do not accept radian)
or Since $BC^2 + BD^2 = CD^2$, $\therefore \angle CBD = 90°$		
$\sin\angle BDC = \dfrac{BC}{CD}$ $= \dfrac{\sqrt{2}}{\sqrt{3}}$	1M	
$\angle BDC = 55°$ (correct to the nearest degree)	1A	
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