Section A (42 marks)

Answer ALL questions in this section.

1. Let $f(x) = \sin^3 x$.

   Find $f'(x)$ and $f''(x)$.

   (3 marks)

2. Find $\frac{d}{dx}(x^2)$ from first principles.

   (4 marks)

3. Solve the inequality $\frac{2x-3}{x+1} \leq 1$.

   (4 marks)

4. Given $x^2 - 6x + 11 \equiv (x+a)^2 + b$, where x is real.

   (a) Find the values of a and b.

   Hence write down the least value of $x^2 - 6x + 11$.

   (b) Using (a), or otherwise, write down the range of possible values of $\frac{1}{x^2 - 6x + 11}$.

   (5 marks)

5. (a) Express the complex number $\frac{2+4i}{1-i}$ in standard form.

   (b) If $p + qi = \frac{2+4i}{1-i}(q+i)$, where p and q are real constants, find the values of p and q.

   (6 marks)

6. Find the equations of the two tangents to the curve $C : y = \frac{6}{x+1}$ which are parallel to the line $x + 6y + 10 = 0$.

   (7 marks)

7. Given $\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j}$ and C is a point on OA such that $|\overrightarrow{OC}| = \frac{16}{5}$.

   (a) Find the unit vector in the direction of $\overrightarrow{OA}$.

   Hence find $\overrightarrow{OC}$.

   (b) If $\overrightarrow{OB} = \mathbf{i} + 4\mathbf{j}$, show that BC is perpendicular to OA.

   (6 marks)

8. The graph of $y = x^2 - (k-2)x + k + 1$ intersects the x-axis at two distinct points $(\alpha, 0)$ and $(\beta, 0)$, where k is real.

   (a) Find the range of possible values of k.

   (b) Furthermore, if $|\alpha + \beta| < 5$, find the range of possible values of k.

   (7 marks)

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Section B (48 marks)

Answer any THREE questions in this section.
Each question carries 16 marks.

9. C_1 is the curve $y = \dfrac{4x-3}{x^2+1}$.

(a) Find

(i) the x- and y- intercepts of the curve C_1 ;

(ii) the range of values of x for which $\dfrac{4x-3}{x^2+1}$ is decreasing;

(iii) the turning point(s) of C_1 , stating whether each point is a maximum or a minimum point. (Testing for maximum/minimum is not required.)

(9 marks)

(b) In Figure 1(a), sketch the curve C_1 for $-10 \le x \le 10$.

(3 marks)

(c) C_2 is the curve $y = \dfrac{|4x-3|}{x^2+1}$.

Using the result of (b), sketch the curve C_2 for $-10 \le x \le 10$ in Figure 1(b).

Hence write down the greatest and least values of $\dfrac{|4x-3|}{x^2+1}$ for $-10 \le x \le 10$.

(4 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

If you attempt Question 9, fill in the details in the first three boxes above and tie this sheet into your answer book.

9. (b) **(continued)**

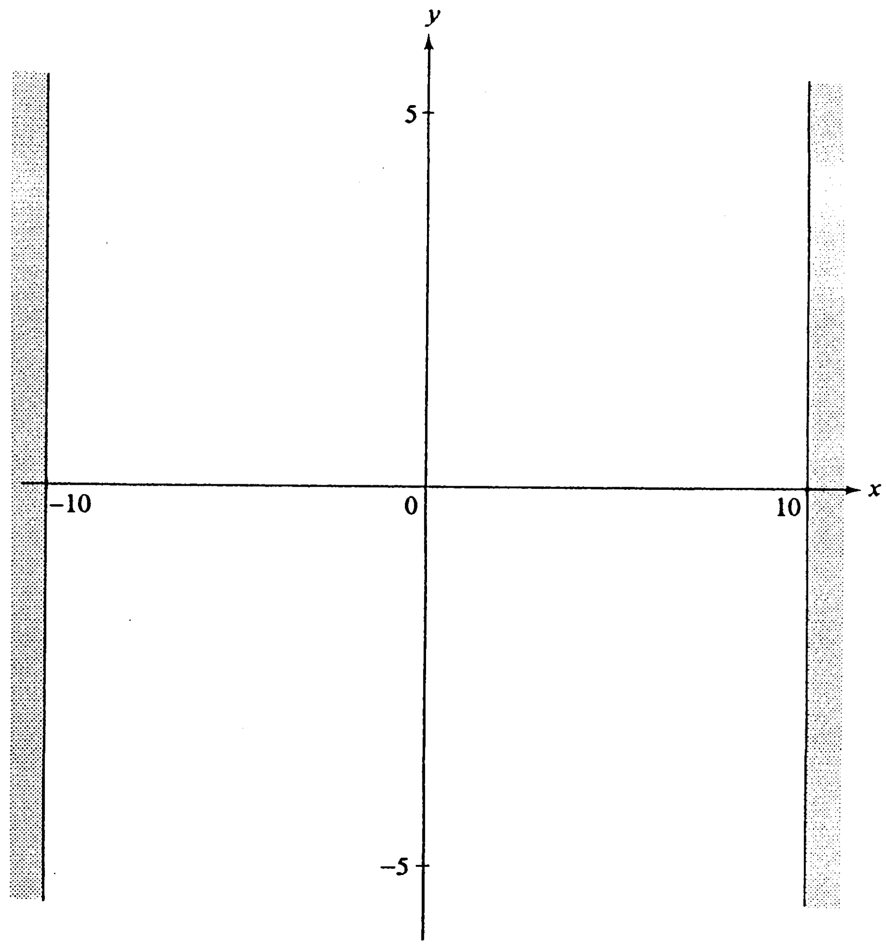

Figure 1(a)

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9. (c) **(continued)**

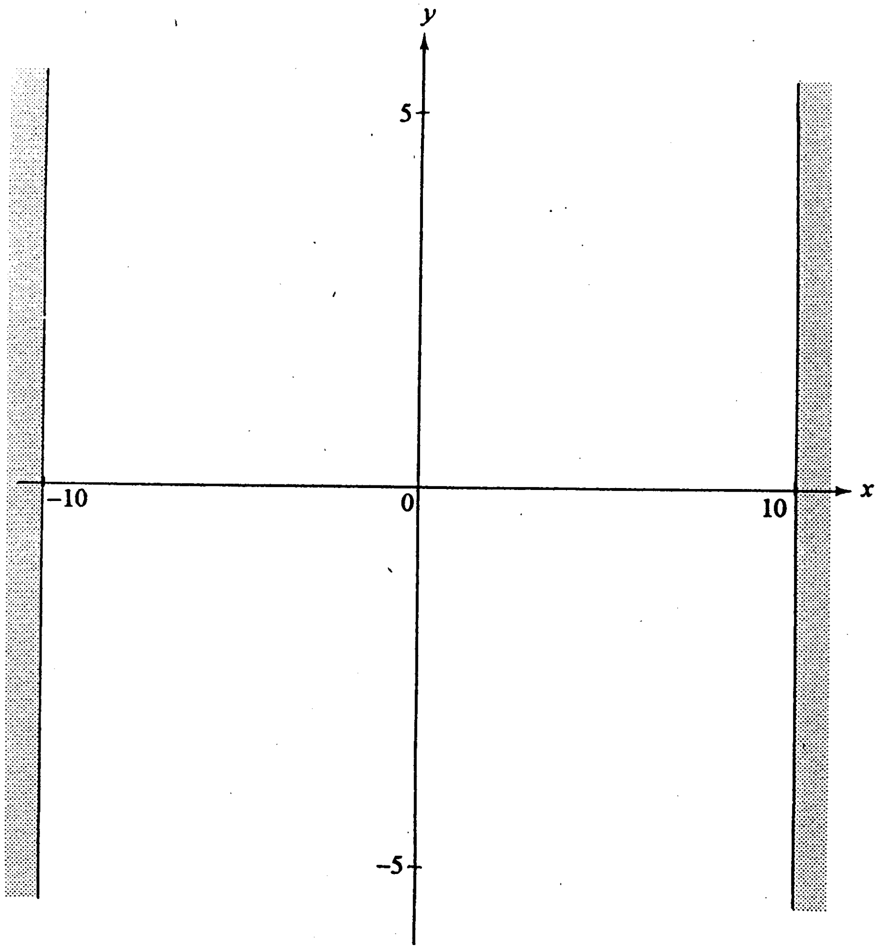

Figure 1(b)

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10.

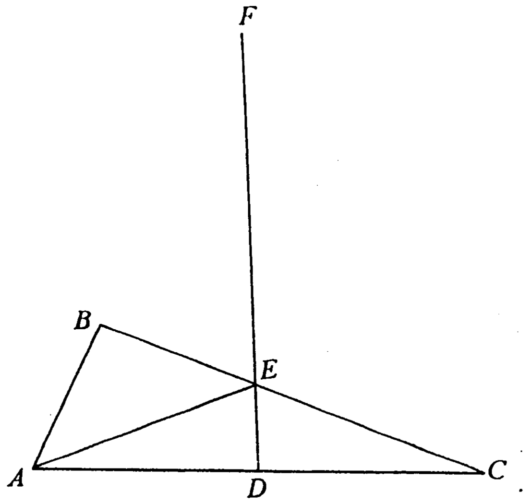

Figure 2

In Figure 2, D is the mid-point of AC and E is a point on BC such that $BE : EC = 1 : t$, where $t > 0$. DE is produced to a point F such that $DE : EF = 1 : 7$. Let $\overrightarrow{AD} = \mathbf{a}$ and $\overrightarrow{AB} = \mathbf{b}$.

(a) (i) Express $\overrightarrow{AE}$ in terms of t, $\mathbf{a}$ and $\mathbf{b}$.

(ii) Express $\overrightarrow{AE}$ in terms of $\mathbf{a}$ and $\overrightarrow{AF}$.

Hence, or otherwise, show that $\overrightarrow{AF} = \dfrac{9-7t}{1+t}\mathbf{a} + \dfrac{8t}{1+t}\mathbf{b}$.

(5 marks)

(b) Suppose that A, B and F are collinear.

(i) Find the value of t.

(ii) It is given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$ and $\cos\angle BAC = \dfrac{1}{3}$.

(1) Find $\mathbf{a} \cdot \mathbf{b}$.

(2) Find $\overrightarrow{AB} \cdot \overrightarrow{BC}$ and $\overrightarrow{AD} \cdot \overrightarrow{DE}$.

(3) Does the circle passing through points B, C and D also pass through point F? Explain your answer.

(11 marks)

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11.

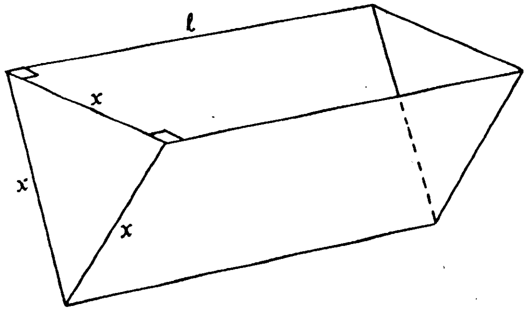

Figure 3(a)

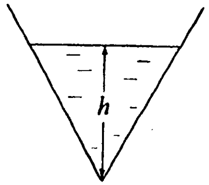

Figure 3(b)

Figure 3(a) shows a vessel with a capacity of 24 cubic units. The length of the vessel is ℓ and its vertical cross-section is an equilateral triangle of side x. The vessel is made of thin metal plates and has no lid. Let S be the total area of metal plates used to make the vessel.

(a) Show that $S = \frac{\sqrt{3}}{2}x^2 + \frac{64\sqrt{3}}{x}$. (4 marks)

(b) Find the values of x and ℓ such that the area of metal plates used to make the vessel is minimum.

(5 marks)

(c) At time $t = 0$, the vessel described in part (b) is completely filled with water. Suppose the water evaporates at a rate proportional to the area of water surface at that instant such that

$$\frac{dV}{dt} = -\frac{1}{10}A,$$ where V and A are respectively the volume of water and the area of water surface at time t.

(i) Let h be the depth of water in the vessel at time t. (See Figure 3(b).) Show that $A = 4h$ and $V = 2h^2$.

Hence, or otherwise, find $\frac{dh}{dt}$.

(ii) Find the time required for the water in the vessel to evaporate completely.

(7 marks)

12. (a) P is a point in an Argand diagram representing a non-zero complex number z such that

$$\arg\left(\frac{z}{1-i}\right) = \frac{\pi}{2}.$$

Find $\arg(1 - i)$ and $\arg z$.

Hence, or otherwise, sketch the locus of P in an Argand diagram.

(6 marks)

(b) R is a point in an Argand diagram representing a non-zero complex number w such that

$$w^3 = (\bar{w})^3 \text{ and } \frac{\pi}{2} < \arg w < \pi.$$

Find $\arg w$.

[Hint : You may let $w = r(\cos\theta + i\sin\theta)$.]

(5 marks)

(c) It is given that in (a), $|z| = 2\sqrt{2}$ and in (b), $|w| = 2$. Furthermore, $OPQR$ is a parallelogram in an Argand diagram, where O represents the complex number 0.

Find the complex number represented by the point Q, giving your answer in standard form.

(5 marks)

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13. Let α, β be the real roots of the quadratic equation

$$x^2 - \lambda x + 1 = 0, \text{ where } \lambda \geq 2.$$

Let $S_n = \alpha^n + \beta^n$, where n is a positive integer.

(a) Express S_2 and S_3 in terms of λ.

(5 marks)

(b) Find the value of $\alpha^5 - \lambda\alpha^4 + \alpha^3$.

Hence show that $S_5 - \lambda S_4 + S_3 = 0$ ...... (*).

(4 marks)

(c) It is known that S_3, S_4 and S_5 are non-zero.
Suppose $S_3 : S_4 : S_5 = 10 : 7\lambda : 25$.

Using (*) in (b), find the value of λ.

Hence (i) find the value of S_3,

(ii) evaluate $\left(\frac{\sqrt{5}+1}{2}\right)^5 + \left(\frac{\sqrt{5}-1}{2}\right)^5$ without using a binomial expansion.

(7 marks)

END OF PAPER

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