香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九五年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1995

附加數學 試卷二
ADDITIONAL MATHEMATICS PAPER II

本評卷參考乃考試局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

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Each year after the examinations, marking schemes will be available for reference at the North Point Teachers' Centre.

只限教師參閱 FOR TEACHERS' USE ONLY

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible too the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits <u>all the marks</u> allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks - awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks - awarded for the accuracy of the answer;

   Marks without 'M' or 'A' - awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous erroneous answers. However, 'A' marks for the corresponding answer should **NOT** be awarded. Unless otherwise specified, no marks in the marking scheme are subdivisible.

3. The symbol pp-1 should be used to denote marks deducted for poor presentation (p.p.). Marks entered in the Page Total Box should be the <u>net</u> total score on that page. Note the following points :

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

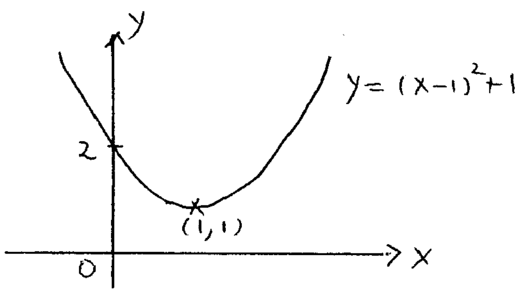
   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those parts where candidates could not score any marks.

4. Unless otherwise specified in the question, numerical answers not given in exact values would not be accepted.

Solution	Marks	Remarks
1. $y = \int 2x\sqrt{x^2+1}\,dx$		
Let $u = x^2 + 1$		
$du = 2x\,dx$		
$y = \int u^{\frac{1}{2}}\,du$	1A	(pp-1) for omitting du
$= \frac{2}{3}u^{\frac{3}{2}} + C$	1A	
$= \frac{2}{3}(x^2+1)^{\frac{3}{2}} + C$ (where C is a constant)	1A	no mark if C is omitted
<u>Alternative solution</u>		
$y = \int 2x\sqrt{x^2+1}\,dx$		(pp-1) for omitting dx
$= \int \sqrt{x^2+1}\,d(x^2+1)$	1A	(Can be omitted)
$= \frac{2}{3}(x^2+1)^{\frac{3}{2}} + C$	2A	1A only if C is omitted
Put $x = 0$, $y = 1$		
$1 = \frac{2}{3} + C$	1M	
$C = \frac{1}{3}$		
$\therefore$ The equation of C is $y = \frac{2}{3}(x^2+1)^{\frac{3}{2}} + \frac{1}{3}$.	1A	
	5	

Solution	Marks	Remarks
2. (a) Substitute (1, 0) into the equation,		
$2 + 4 + k(4 - 1) = 0$	1M	
$k = -2$	1A	
$\therefore$ The equation of the line is $6x + 7y - 6 = 0$.	1A	or $y = \frac{-6}{7}x + \frac{6}{7}$ etc.
(b) $2x - 3y + 4 + k(4x + 2y - 1) = 0$		
$(2 + 4k)x + (2k - 3)y + 4 - k = 0$		
Slope $= -\frac{2+4k}{2k-3}$	1M+1A	
$-\frac{2+4k}{2k-3} = 2$		
$k = \frac{1}{2}$		
$\therefore$ The equation of the line is $8x - 4y + 7 = 0$.	1A	or $y = 2x + \frac{7}{4}$ etc.
Alternative solution		
(a) $\begin{cases} 2x - 3y + 4 = 0 \text{ ----- } (1) \\ 4x + 2y - 1 = 0 \text{ ----- } (2) \end{cases}$		
$(1) \times 2 - (2)$, $-8y + 9 = 0$		
$y = \frac{9}{8}$	1A	
$2x = 3(\frac{9}{8}) - 4$		
$x = \frac{-5}{16}$	1A	
$\therefore$ The family of lines always passes through $(\frac{-5}{16}, \frac{9}{8})$.		
The equation of the line is		
$\frac{y-0}{x-1} = \frac{\frac{9}{8} - 0}{\frac{-5}{16} - 1}$	1M	
$y = \frac{-6}{7}x + \frac{6}{7}$.	1A	or $6x + 7y - 6 = 0$ etc.
(b) The equation of the line is		
$\frac{y - \frac{9}{8}}{x + \frac{5}{16}} = 2$	1M	
$y = 2x + \frac{7}{4}$.	1A	or $8x - 4y + 7 = 0$ etc.
	6	

Solution	Marks	Remarks
3. (a) Let (x, y) be the mid-point of AB.		
$\begin{cases} x = \dfrac{t+2}{2} \end{cases}$	1A	
$y = \dfrac{\frac{1}{2}t^2 + 2}{2}$	1A	
$\therefore \begin{cases} 2x - 2 = t \\ 4y - 4 = t^2 \end{cases}$		
Eliminating t,		
$(2x-2)^2 = 4y - 4$	1M	
$y = (x-1)^2 + 1$	1A	or $(x-1)^2 = y - 1$, $y = x^2 - 2x + 2$ etc.
(b) 	1A	Shape
	1A	Labelling the vertex (1, 1)
		Axes not labelled (pp-1)
	6	

Solution	Marks	Remarks
4. $(x^2+\frac{1}{x})^5-(x^2-\frac{1}{x})^5$		
$=(x^2)^5+5(x^2)^4(\frac{1}{x})+10(x^2)^3(\frac{1}{x})^2+10(x^2)^2(\frac{1}{x})^3+5(x^2)(\frac{1}{x})^4+(\frac{1}{x})^5$	1A	Accept ${}_5Cr$ notations
$-[(x^2)^5-5(x^2)^4(\frac{1}{x})+10(x^2)^3(\frac{1}{x})^2-10(x^2)^2(\frac{1}{x})^3+5(x^2)(\frac{1}{x})^4-(\frac{1}{x})^5]$	1A	
$=10x^7+20x+\frac{2}{x^5}$		
$\therefore a=10,\ b=20,\ c=2$	2A	All correct – 2A 1 or 2 correct – 1A only
Put $x=\sqrt{2}$,		
$(2+\frac{1}{\sqrt{2}})^5-(2-\frac{1}{\sqrt{2}})^5=10(\sqrt{2})^7+20\sqrt{2}+\frac{2}{(\sqrt{2})^5}$	1M	
$=\frac{401\sqrt{2}}{4}$ (or $\frac{401}{2\sqrt{2}}$, $401(2)^{-\frac{3}{2}}$)	1A	141.77 – no mark
	6	
5. (a) Area $=\int_0^{\pi}\sin x\,dx$	1A	(pp–1) for omitting dx
$=[-\cos x]_0^{\pi}$		
$=2$	1A	
(b) (i) The coordinates of B are $(\frac{5\pi}{6},\frac{1}{2})$.	1A	No marks for degrees
(ii) Area $=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(\sin x-\frac{1}{2})\,dx$	1M+1A	1M for limits 1A for $\int(y_2-y_1)\,dx$
$=[-\cos x-\frac{1}{2}x]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$		
$=-\cos\frac{5\pi}{6}-\frac{1}{2}(\frac{5\pi}{6})+\cos\frac{\pi}{6}+\frac{1}{2}(\frac{\pi}{6})$		
$=\sqrt{3}-\frac{\pi}{3}$	1A	
	6	

Solution	Marks	Remarks
6. For $n = 1$,		
$8^n - 1 = 8 - 1 = 7$ which is divisible by 7.	1	
$\therefore$ The statement is true for $n = 1$.		
Assume $8^k - 1$ is divisible by 7. (for some +ve integer k.)	1	
(or $8^k - 1 = 7N$, where N is positive integer)		
Then $8^{k+1} - 1 = 8(8^k) - 1$	1	
$= 8(7N + 1) - 1$ $= 56N + 7$ $= 7(8N + 1)$	2	
$\therefore (8^{k+1} - 1)$ is also divisible by 7. ($\therefore$ the statement is also true for $n = k + 1$ if it is true for $n = k$.) (By the principle of mathematical induction.) The statement is true for all positive integers n.	1	
	6	
7. (a) $\angle PQU = \frac{1}{2}(180° - 42°) = 69°$	1A	
$PU = 10\sin 69°$		
$= 9.34$ (cm)	1A	
$\angle PQR = 108°$	1A	
$PR = 2(10)\sin 54°$		or $\sqrt{10^2 + 10^2 - 2(10)^2\cos 108°}$
$= 16.2$ (cm)	1A	
(b) The angle between the faces is $\angle PUR$.	1A	Can be omitted
In ΔPUR , $PU = UR$		
$\sin(\frac{1}{2}\angle PUR) = \dfrac{\frac{1}{2}PR}{PU}$	1M	
$= \dfrac{\frac{1}{2}(20\sin 54°)}{10\sin 69°}$		
$\angle PUR = 120°$ (2.10)	1A	
Alternative solution		
$\cos\angle PUR = \dfrac{PU^2 + UR^2 - PR^2}{2(PU)(UR)}$	1M	
$= \dfrac{2(10\sin 69°)^2 - (20\sin 54°)^2}{2(10\sin 69°)^2}$		
$\angle PUR = 120°$ (2.10)	1A	
	7	

Solution	Marks	Remarks
8. (a) $\frac{d}{dx}[x^{n-1}(1-x^2)^{\frac{3}{2}}]$		
$= (n-1)x^{n-2}(1-x^2)^{\frac{3}{2}} + \frac{3}{2}(1-x^2)^{\frac{1}{2}}(-2x)x^{n-1}$	1A+1A	1A for each term
$= (n-1)x^{n-2}(1-x^2)\sqrt{1-x^2} - 3x^n\sqrt{1-x^2}$	1M	For $(1-x^2)^{\frac{3}{2}} = (1-x^2)\sqrt{1-x^2}$
$= (n-1)x^{n-2}\sqrt{1-x^2} - (n-1+3)x^n\sqrt{1-x^2}$		
$= (n-1)x^{n-2}\sqrt{1-x^2} - (n+2)x^n\sqrt{1-x^2}$	1	
	4	
(b) Integrating with respect to x,		
$[x^{n-1}(1-x^2)^{\frac{3}{2}}]_0^1 = \int_0^1 [(n-1)x^{n-2}\sqrt{1-x^2}$		
$- (n+2)x^n\sqrt{1-x^2}]\,dx$	1A	
$0 = \int_0^1 (n-1)x^{n-2}\sqrt{1-x^2}dx - \int_0^1 (n+2)x^n\sqrt{1-x^2}dx$	1A	For L.H.S. = 0 (can be omitted)
$\int_0^1 x^n\sqrt{1-x^2}dx = \frac{n-1}{n+2}\int_0^1 x^{n-2}\sqrt{1-x^2}dx$	1	
	3	
(c) $dx = \cos\theta d\theta$		
$\int_0^1 \sqrt{1-x^2}dx = \int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$	1A	
$= \int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2\theta)d\theta$	1M	For $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$
$= \frac{1}{2}[\theta + \frac{1}{2}\sin 2\theta]_0^{\frac{\pi}{2}}$		
$= \frac{\pi}{4}$	1A	
	3	
(d) (i) $\int_0^1 x^4\sqrt{1-x^2}dx$		
$= \frac{4-1}{4+2}\int_0^1 x^2\sqrt{1-x^2}dx$	1A	For using (b)
$= \frac{3}{6}\cdot\frac{2-1}{2+2}\int_0^1 \sqrt{1-x^2}dx$		
$= \frac{3}{6}\cdot\frac{1}{4}\cdot\frac{\pi}{4}$	1M	For using (c) (accept using the susbstitution $x = \sin\theta$)
$= \frac{\pi}{32}$	1A	

Solution	Marks	Remarks
(ii) Put $x = \sin\theta$	1A	
$\int_0^{\frac{\pi}{2}} \sin^6\theta\cos^2\theta\, d\theta = \int_0^1 x^6\sqrt{1-x^2}\,dx$	1A	
$= \frac{6-1}{6+2}\int_0^1 x^4\sqrt{1-x^2}\,dx$		
$= \frac{5}{8}\cdot\frac{\pi}{32}$		
$= \frac{5\pi}{256}$	1A	
	6	

Solution	Marks	Remarks
9. (a) $\cos^2 A - \cos^2 B$		
$= (\cos A + \cos B)(\cos A - \cos B)$		
$= (2\cos\frac{A+B}{2}\cos\frac{A-B}{2})(-2\sin\frac{A+B}{2}\sin\frac{A-B}{2})$	1A	
$= (2\sin\frac{A+B}{2}\cos\frac{A+B}{2})(-2\sin\frac{A-B}{2}\cos\frac{A-B}{2})$		
$= -\sin(A+B)\sin(A-B)$	1A	
$= \sin(A+B)\sin(B-A)$	1	
Alternative solution 1		
$\sin(A+B)\sin(B-A)$		
$= (\sin A\cos B + \sin B\cos A)(\sin B\cos A - \sin A\cos B)$	1A	
$= \sin^2 B\cos^2 A - \sin^2 A\cos^2 B$		
$= (1 - \cos^2 B)\cos^2 A - (1 - \cos^2 A)\cos^2 B$	1A	
$= \cos^2 A - \cos^2 A\cos^2 B - \cos^2 B + \cos^2 A\cos^2 B$		
$= \cos^2 A - \cos^2 B$	1	
Alternative solution 2		
$\sin(A+B)\sin(B-A)$		
$= -\frac{1}{2}(\cos 2B - \cos 2A)$	1A	
$= -\frac{1}{2}[2\cos^2 B - 1 - (2\cos^2 A - 1)]$	1A	
$= \cos^2 A - \cos^2 B$	1	
Alternative solution 3		
$\cos^2 A - \cos^2 B$		
$= \frac{1}{2}(1 + \cos 2A) - \frac{1}{2}(1 + \cos 2B)$	1A	
$= \frac{1}{2}(\cos 2A - \cos 2B)$		
$= -\sin(A + B)\sin(A - B)$	1A	
$= \sin(A + B)\sin(B - A)$	1	
	3	
(b) (i) $\cos^2 A - \cos^2 B + \sin^2 C$		
$= \sin(A + B)\sin(B - A) + \sin^2 C$	1A	For using (a)
$= \sin(\pi - C)\sin(B - A) + \sin^2 C$	1A	For using $A + B + C = \pi$
$= \sin C[\sin(B - A) + \sin(A + B)]$		
$= 2\sin C\sin B\cos A$	1	

Solution	Marks	Remarks
(ii) $\cos^2 A - \cos^2 B - \cos^2 C = -1$		
$\cos^2 A - \cos^2 B - \cos^2 C + 1 = 0$		
$\cos^2 A - \cos^2 B + \sin^2 C = 0$	1A	
$2\cos A \sin C \sin B = 0$	1A	
$(\because \sin C \neq 0,\ \sin B \neq 0\quad \therefore \cos A = 0)$, $\angle A = \frac{\pi}{2}$	1A	
$\therefore \Delta ABC$ is a right angled triangle.	6	
(c) $\cos^2 x - \sin^2 y$		
$= \cos^2 x - \cos^2(\frac{\pi}{2} - y)$	1A	
$= \sin(x + \frac{\pi}{2} - y)\sin(\frac{\pi}{2} - y - x)$	1A	For using (a)
$= \cos(x + y)\cos(x - y)$	1	
Alternative solution (1)		
$\cos(x + y)\cos(x - y)$		
$= (\cos x\cos y - \sin x\sin y)(\cos x\cos y + \sin x\sin y)$	1A	
$= \cos^2 x\cos^2 y - \sin^2 x\sin^2 y$		
$= \cos^2 x(1 - \sin^2 y) - (1 - \cos^2 x)\sin^2 y$	1A	
$= \cos^2 x - \sin^2 y$	1	
Alternative solution (2)		
$\cos^2 x - \sin^2 y = \frac{1}{2}(1 + \cos 2x) - \frac{1}{2}(1 - \cos 2y)$	1A+1A	
$= \frac{1}{2}(\cos 2x + \cos 2y)$		
$= \cos(x + y)\cos(x - y)$	1	
$\cos^2 2\theta - \sin^2 3\theta + \cos\theta\sin 5\theta = 0$		
$\cos 5\theta\cos(-\theta) + \cos\theta\sin 5\theta = 0$	1A	
$\cos\theta(\cos 5\theta + \sin 5\theta) = 0$		
$\cos\theta = 0$ or $\sin 5\theta + \cos 5\theta = 0$		
$\tan 5\theta = -1$	1M	For any correct method of solving $\sin 5\theta + \cos 5\theta = 0$
$5\theta = n\pi - \frac{\pi}{4}$		
$\theta = 2n\pi \pm \frac{\pi}{2}$ (or $360n° \pm 90°$) $\quad \theta = \frac{n\pi}{5} - \frac{\pi}{20}$ (or $36n° - 9°$) (n is an integer)	1A+1A	$2n\pi \pm 90°$ etc (pp-1)

Solution	Marks	Remarks
Alternative solution of solving $\sin 5\theta + \cos 5\theta = 0$		
$\sin 5\theta + \cos 5\theta = 0$		
$\cos 5\theta = \cos(\frac{\pi}{2} + 5\theta)$	1M	
$5\theta = 2n\pi \pm (\frac{\pi}{2} + 5\theta)$		
$10\theta = 2n\pi - \frac{\pi}{2}$		
$\theta = \frac{n\pi}{5} - \frac{\pi}{20}$	1A	
	7	

Solution	Marks	Remarks
10. (a) $x^2 + y^2 - 16x - 36 = 0$		
$(x - 8)^2 + y^2 = 100$		
$\therefore$ The centre is $(8, 0)$.	1A	
The radius is 10.	1A	
Since C_1 touches C_2, distance between centres = sum of radius		
$8 - (-7) = 10 + r$ (where r = radius of C_2)	1M	
$r = 5$	1A	
	4	
(b) Let R be the radius of the circle centred at P.		
$\sqrt{(h-8)^2 + k^2} = R + 10$ $\sqrt{(h+7)^2 + k^2} = R + 5$	1M	(Can be omitted)
$\sqrt{(h-8)^2 + k^2} - 10 = \sqrt{(h+7)^2 + k^2} - 5$	1M	(Can be omitted)
$\sqrt{(h-8)^2 + k^2} = \sqrt{(h+7)^2 + k^2} + 5$		
$h^2 - 16h + 64 + k^2 = h^2 + 14h + 49 + k^2 + 10\sqrt{(h+7)^2 + k^2} + 25$	1M	For squaring both sides
$10\sqrt{(h+7)^2 + k^2} = -30h - 10$	1A	OR $10\sqrt{(h-8)^2 + k^2} = -30h + 40$
$\sqrt{(h+7)^2 + k^2} = -3h - 1$		
$9h^2 + 6h + 1 = h^2 + 14h + 49 + k^2$		
$8h^2 - k^2 - 8h - 48 = 0$	1	
	5	
(c) (i) By symmetry, the equation of the locus of P is $y = 20$.	2A	or $k = 20$
Alternative solution		
$\sqrt{(h+7)^2 + (k-40)^2} = \sqrt{(h+7)^2 + k^2}$	1M	
$(k - 40)^2 = k^2$		
$k = 20$		
$\therefore$ The equation of the locus is $y = 20$.	1A	or $k = 20$
(ii) $\begin{cases} 8h^2 - k^2 - 8h - 48 = 0 \\ k = 20 \end{cases}$	1M	
$8h^2 - 8h - 448 = 0$		
$h^2 - h - 56 = 0$		
$(h + 7)(h - 8) = 0$		
$h = -7$ or 8 (rejected $\because h < 0$)	1A+1A	
$\therefore$ The centre of the circle is $(-7, 20)$.		
Radius $= 20 - 5 = 15$	1A	
$\therefore$ The equation of the circle is $(x + 7)^2 + (y - 20)^2 = 15^2$.	1A	
(or $x^2 + y^2 + 14x - 40y + 224 = 0$)		
	7	

Solution	Marks	Remarks
11. (a) (i) In ΔPAQ, by Sine Law,		
$\dfrac{QA}{\sin\alpha} = \dfrac{PA}{\sin\phi}$		
$\therefore \dfrac{QA}{PA} = \dfrac{\sin\alpha}{\sin\phi}$	1A	
(ii) In ΔPQB,		
$\dfrac{QB}{PB}\ (= \dfrac{\sin\beta}{\sin(\pi-\phi)}) = \dfrac{\sin\beta}{\sin\phi}$	1A	
If $\dfrac{QA}{PA} = \dfrac{QB}{PB}$, $\dfrac{\sin\alpha}{\sin\phi} = \dfrac{\sin\beta}{\sin\phi}$		
$\sin\alpha = \sin\beta$		
$\alpha = \beta$	1	
	3	
(b) (i) $9x^2 + 25y^2 = 225$		
$18x + 50y\dfrac{dy}{dx} = 0$	1A	
$\dfrac{dy}{dx} = \dfrac{-9x}{25y}$		
Slope of tangent at $P = \left.\dfrac{dy}{dx}\right\|_{(5\cos\theta,\ 3\sin\theta)}$		
$= -\dfrac{3\cos\theta}{5\sin\theta}$		
Slope of normal at $P = \dfrac{5\sin\theta}{3\cos\theta}$	1A	
Alternative solution		
The equation of the tanget at P is		
$9x(5\cos\theta) + 25y(3\sin\theta) = 225$	1A	or $\dfrac{x}{5}\cos\theta + \dfrac{y}{3}\sin\theta = 1$
$45x\cos\theta + 75y\sin\theta = 225$		
Slope $= -\dfrac{3\cos\theta}{5\sin\theta}$		
$\therefore$ Slope of normal at $p = \dfrac{5\sin\theta}{3\cos\theta}$	1A	
Equation of PQ is		
$\dfrac{y-3\sin\theta}{x-5\cos\theta} = \dfrac{5\sin\theta}{3\cos\theta}$	1M	
$3y\cos\theta - 9\sin\theta\cos\theta = 5x\sin\theta - 25\sin\theta\cos\theta$		
$5x\sin\theta - 3y\cos\theta - 16\sin\theta\cos\theta = 0$	1	
Put $y = 0$, $x = \dfrac{16\cos\theta}{5}$		
$\therefore$ The coordinates of Q are $(\dfrac{16\cos\theta}{5}, 0)$.	1A	

Solution	Marks	Remarks
(ii) $PA = \sqrt{(5\cos\theta + 4)^2 + (3\sin\theta)^2}$		
$= \sqrt{25\cos^2\theta + 40\cos\theta + 16 + 9\sin^2\theta}$		
$= \sqrt{25\cos^2\theta + 40\cos\theta + 16 + 9(1 - \cos^2\theta)}$	1M	(can be omitted)
$= \sqrt{16\cos^2\theta + 40\cos\theta + 25}$		
$= 5 + 4\cos\theta$	1	
$PB = \sqrt{(5\cos\theta - 4)^2 + (3\sin\theta)^2}$		
$= \sqrt{25\cos^2\theta - 40\cos\theta + 16 + 9\sin^2\theta}$		
$= \sqrt{16\cos^2\theta - 40\cos\theta + 25}$		
$= 5 - 4\cos\theta$	1A	Accept $4\cos\theta - 5$
$QA = 4 + \dfrac{16\cos\theta}{5}$	1A	
$QB = 4 - \dfrac{16\cos\theta}{5}$	1A	
$\dfrac{QA}{PA} = \dfrac{\frac{16\cos\theta}{5} + 4}{5 + 4\cos\theta} = \dfrac{4}{5}$	1M+1A	1M for considering the ratio of length
$\dfrac{QB}{PB} = \dfrac{4 - \frac{16\cos\theta}{5}}{5 - 4\cos\theta} = \dfrac{4}{5}$		1A for showing that the two ratios are equal
(Since $\dfrac{QA}{PA} = \dfrac{QB}{PB}$, $\angle APQ = \angle QPB$,)		
$\therefore PQ$ bisects $\angle APB$.	1	
	13	

Solution	Marks	Remarks
12. (a) $y - k = \cos\theta$		
$dy = -\sin\theta\, d\theta$		
$\int_{k-1}^{k+1} \sqrt{1-(y-k)^2}\,dy = \int_{\pi}^{0} \sqrt{1-\cos^2\theta}\,(-\sin\theta\, d\theta)$	1A+1A	1A for integrand,1A for limits
$= \int_0^{\pi} \sin^2\theta\, d\theta$		
$= \int_0^{\pi} \frac{1}{2}(1-\cos 2\theta)\, d\theta$	1M	For $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$
$= \frac{1}{2}[\theta - \frac{1}{2}\sin 2\theta]_0^{\pi}$		
$= \frac{\pi}{2}$	1	
(i) $\int_0^2 [2+\sqrt{1-(y-1)^2}]^2 dy$		
$= \int_0^2 [4+1-(y-1)^2+4\sqrt{1-(y-1)^2}]\,dy$		
$= [5y - \frac{1}{3}(y-1)^3]_0^2 + 4\int_0^2 \sqrt{1-(y-1)^2}]\,dy$	1A	For integrating the 1st three terms
$= (10 - \frac{1}{3} - \frac{1}{3}) + 4(\frac{\pi}{2})$	1M	For using the earlier result to evaluate the last integral
$= \frac{28}{3} + 2\pi$	1	
(ii) $\int_2^4 [2-\sqrt{1-(y-3)^2}]^2 dy$		
$= \int_2^4 4+1-(y-3)^2-4\sqrt{1-(y-3)^2}]\,dy$		
$= [5y - \frac{1}{3}(y-3)^3]_2^4 - 4\int_2^4 \sqrt{1-(y-3)^2}]\,dy$		
$= (20 - \frac{1}{3} - 10 - \frac{1}{3}) - 4(\frac{\pi}{2})$		
$= \frac{28}{3} - 2\pi$	1	
	8	
(b) (i) Equation of ABC		
$(x-2)^2 + (y-3)^2 = 1$		
$x - 2 = -\sqrt{1-(y-3)^2}$ $(\because x \le 2)$	1	
$x = 2 - \sqrt{1-(y-3)^2}$		
Equation of CDE		
$(x-2)^2 + (y-1)^2 = 1$		
$x - 2 = \sqrt{1-(y-1)^2}$ $(\because x \ge 2)$	1	
$x = 2 + \sqrt{1-(y-1)^2}$		

Solution	Marks	Remarks
(ii) Capacity		
$= \pi \int_0^2 [2 + \sqrt{1 - (y-1)^2}]^2 dy$		
$+ \pi \int_2^4 [2 - \sqrt{1 - (y-3)^2}]^2 dy$	1A	
$= \pi(\frac{28}{3} + 2\pi) + \pi(\frac{28}{3} - 2\pi)$	1M	For using results of (a)
$= \frac{56\pi}{3}$	1A	
	5	
(c) Volume		
= Volume of solid *CDE* − Volume of solid *ABC*	1M	(Can be omitted)
$= \pi(\frac{28}{3} + 2\pi) - \pi(\frac{28}{3} - 2\pi)$	1M	For using result of (a)
$= 4\pi^2$	1A	
<u>Alternative solution</u>		
Equation of circle : $(x-2)^2 + (y-1)^2 = 1$		
$x = 2 \pm \sqrt{1 - (y-1)^2}$		
Volume $= \pi \int_0^2 [2 + \sqrt{1 - (y-1)^2}]^2 dy$		
$- \pi \int_0^2 [2 - \sqrt{1 - (y-1)^2}]^2 dy$	1M	
$= 8\pi \int_0^2 \sqrt{1 - (y-1)^2} dy$		
$= 8\pi(\frac{\pi}{2})$	1M	For using result of (a)
$= 4\pi^2$	1A	
	3	