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香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九五年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1995

附加數學　試卷一
ADDITIONAL MATHEMATICS PAPER I

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

在今年考試結束後，各科評卷參考將存放於北角教師中心，供教師參閱。
Each year after the examinations, marking schemes will be available for reference at the North Point Teachers' Centre.

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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible too the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous erroneous answers. However, 'A' marks for the corresponding answer should **NOT** be awarded. Unless otherwise specified, no marks in the marking scheme are subdivisible.

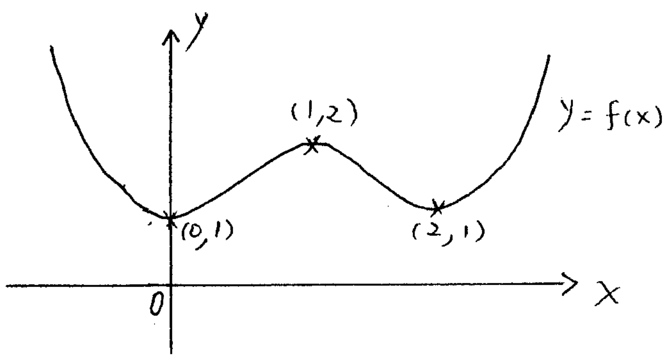
3. The symbol pp–1 should be used to denote marks deducted for poor presentation (p.p.). Marks entered in the Page Total Box should be the net total score on that page. Note the following points :

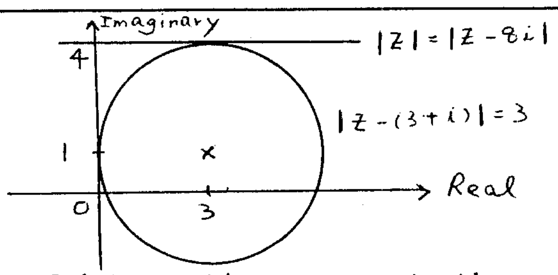
   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those parts where candidates could not score any marks.

4. Unless otherwise specified in the question, numerical answers not given in exact values would not be accepted.

Solution	Marks	Remarks
1. $x^2 + (1-m)x + 2m - 5 = 0$		
Discriminant $= (1-m)^2 - 4(2m-5)$	1M	
$= m^2 - 10m + 21$	1A	
Discriminant < 0		
$m^2 - 10m + 21 < 0$	1M	no mark for ≤ 0
$(m-3)(m-7) < 0$	1A	
$3 < m < 7$	1A	
	5	
$z = -1 + \sqrt{3}i$		
$= 2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$	1A+1A	1A for modulus 1A for argument (Accept degrees) (Accept other equivalent values for argument)
$z^5 + \bar{z}^5 = 2^5(\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}) + 2^5(\cos\frac{-10\pi}{3} + i\sin\frac{-10\pi}{3})$	1M+1M	1M for De moivre's Theorem
or $2^5(\cos\frac{10\pi}{3} - i\sin\frac{10\pi}{3})$		1M for $\bar{z} = \text{cis}(-\theta)$
$(= 64\cos\frac{10\pi}{3})$ (or $= (-16-\sqrt{3}i) + (-16+\sqrt{3}i)$)		(or $\bar{z}^5 = \overline{z^5}$)
$= -32$	1A	
	5	
3. 	1A 1A 1A 2A	For the max. pt. (1, 2) For the min. pt. (0, 1) For the min. pt. (2, 1) Shape Axes not labelled (pp-1)
	5	

Solution	Marks	Remarks
4. $x - \frac{5}{x} > 4$		
Case 1 : $x > 0$		
$x^2 - 5 - 4x > 0$	1A	or $x^2 - 5 > 4x$
$(x - 5)(x + 1) > 0$		
$x > 5$ or $x < -1$		
Since $x > 0$, $\therefore x > 5$	1A	
Case 2 : $x < 0$		
$x^2 - 5 - 4x < 0$	1A	or $x^2 - 5 < 4x$
$(x - 5)(x + 1) < 0$		
$-1 < x < 5$		
Since $x < 0$, $\therefore -1 < x < 0$	1A	
Combining the 2 cases, $x > 5$ or $-1 < x < 0$	2A 6	No mark for using 'and' or ','.
Alternative solution (1)		
$x - \frac{5}{x} > 4$		
$x^3 - 5x - 4x^2 > 0$ $(\because x^2 > 0)$	2A	or $x^3 - 5x > 4x^2$
$x(x - 5)(x + 1) > 0$	1A	
$x > 5$ or $-1 < x < 0$	3A	Withhold 2 marks for using 'and' or ','
Alternative solution (2)		
$x - \frac{5}{x} > 4$		
$\frac{x^2 - 5 - 4x}{x} > 0$	2A	
$\frac{(x - 5)(x + 1)}{x} > 0$	1A	or $x(x - 5)(x + 1) > 0$
$x > 5$ or $-1 < x < 0$	3A	Withhold 2 marks for using 'and' or ','
5. 	1A 1A 1A 1A 1A	For a circle For centred at $(3 + i)$ For radius = 3 For a straight line For the correct line Axes not labelled (pp-1) Two diagrams (pp-1)
The point of intersection represents the complex number $3 + 4i$.	1A 6	

Solution	Marks	Remarks
6. (a) $y^2 + y\sqrt{x} = 3$		
$2y\frac{dy}{dx} + \sqrt{x}\frac{dy}{dx} + \frac{y}{2\sqrt{x}} = 0$	1A+1A	1A for $\frac{d}{dx}(y\sqrt{x})$ 1A for the other 2 terms.
At $P(4, 1)$, $2(1)\frac{dy}{dx} + \sqrt{4}\frac{dy}{dx} + \frac{1}{2\sqrt{4}} = 0$	1M	For substitution
$\frac{dy}{dx} = -\frac{1}{16}$	1A	
Alternative solution		
$y^2 + y\sqrt{x} = 3$		
$x = \frac{(3-y^2)^2}{y^2}$	1A	
$\frac{dx}{dy} = \frac{y^2 2(3-y^2)(-2y) - (3-y^2)^2 2y}{y^4}$ [or $= 2\left(\frac{3-y^2}{y}\right)\frac{y(-2y)-(3-y^2)}{y^2}$]	1A	
At $P(4, 1)$ $\frac{dx}{dy} = \frac{1^2(2)(3-1^2)(-2) - (3-1^2)^2(2)}{1^4}$	1M	
$= -16$		
$\therefore \frac{dy}{dx} = -\frac{1}{16}$	1A	
(b) Slope of normal $= -1 \big/ -\frac{1}{16}$	1M	
$= 16$		
The equation of the normal is $\frac{y-1}{x-4} = 16$	1A	
$y = 16x - 63$	1A	$16x - y - 63 = 0$
	7	

Solution	Marks	Remarks
7. (a) $\vec{OR} = \dfrac{\vec{OP} + k\vec{OQ}}{k+1}$	1M	Omit vector sign (pp-1)
$= \dfrac{(2\vec{i} + 3\vec{j}) + k(-6\vec{i} + 4\vec{j})}{k+1}$		
$= \dfrac{2-6k}{k+1}\vec{i} + \dfrac{3+4k}{k+1}\vec{j}$	1A	
(b) $\vec{OP}\cdot\vec{OR} = \dfrac{2(2-6k)}{k+1} + 3\left(\dfrac{3+4k}{k+1}\right)$	1M	Omit dot sign (pp-1)
$= \dfrac{13}{k+1}$	1A	
$\vec{OQ}\cdot\vec{OR} = -6\dfrac{(2-6k)}{k+1} + 4\left(\dfrac{3+4k}{k+1}\right)$		
$= \dfrac{52k}{k+1}$	1A	
(c) $\cos\angle POR = \cos\angle QOR$		
$\dfrac{\vec{OP}\cdot\vec{OR}}{\lvert\vec{OP}\rvert\lvert\vec{OR}\rvert} = \dfrac{\vec{OQ}\cdot\vec{OR}}{\lvert\vec{OQ}\rvert\lvert\vec{OR}\rvert}$	1M	
$\lvert\vec{OQ}\rvert\dfrac{13}{k+1} = \lvert\vec{OP}\rvert\dfrac{52k}{k+1}$		
$13\sqrt{52} = 52k\sqrt{13}$	1A	For $\lvert\vec{OQ}\rvert = \sqrt{52}$ and $\lvert\vec{OP}\rvert = \sqrt{13}$
$k = \dfrac{1}{2}$	1A	
Alternative solution		
$k = \dfrac{\lvert\vec{OP}\rvert}{\lvert\vec{OQ}\rvert}$	1M	
$= \dfrac{\sqrt{13}}{\sqrt{52}}$	1A	For $\lvert\vec{OP}\rvert = \sqrt{13}$ and $\lvert\vec{OQ}\rvert = \sqrt{52}$
$= \dfrac{1}{2}$	1A	
	8	

Solution	Marks	Remarks
8. (a) (i) $\overrightarrow{AE} = h\overrightarrow{AC}$		
$= h(\vec{p} + \vec{q})$	1A	Omit vector sign (pp-1)
(ii) $\overrightarrow{AE} = \dfrac{\lambda\overrightarrow{AF} + \overrightarrow{AD}}{1 + \lambda}$		
$= \dfrac{\lambda k\vec{p} + \vec{q}}{1 + \lambda}$	1A	
$h(\vec{p} + \vec{q}) = \dfrac{\lambda k\vec{p} + \vec{q}}{1 + \lambda}$	1M	(can be omitted)
$\therefore \begin{cases} h = \dfrac{\lambda k}{1 + \lambda} \\ h = \dfrac{1}{1 + \lambda} \end{cases}$	1M	
$\therefore \lambda k = 1$		
$\lambda = \dfrac{1}{k}$	1 5	
(b) (i) $\vec{p} \cdot \vec{q} = \lvert\vec{p}\rvert \lvert\vec{q}\rvert \cos\frac{\pi}{3}$	1M	Omit dot sign (pp-1)
$= 3(2)\cos\frac{\pi}{3}$		
$= 3$	1A	
(ii) (1) $\overrightarrow{DF} = k\vec{p} - \vec{q}$	1A	
$\overrightarrow{DF} \cdot \overrightarrow{AC} = 0$	1M	or $\overrightarrow{DF} \cdot \overrightarrow{AE} = 0$
$(k\vec{p} - \vec{q}) \cdot (\vec{p} + \vec{q}) = 0$		
$k\vec{p} \cdot \vec{p} + (k - 1)\vec{p} \cdot \vec{q} - \vec{q} \cdot \vec{q} = 0$	1M	For distribution
$9k + 3(k - 1) - 4 = 0$	1A	For $\vec{p} \cdot \vec{p} = 9$ and $\vec{q} \cdot \vec{q} = 4$ only
$k = \dfrac{7}{12}$	1A	

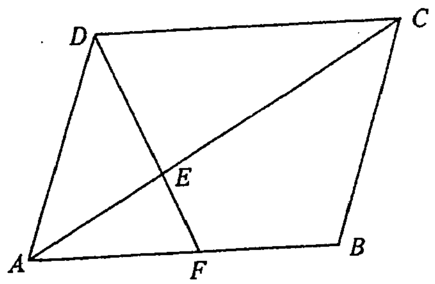


Figure 1

Solution	Marks	Remarks
(2) For $k = \frac{7}{12}$, $\lambda = \frac{12}{7}$, $h = \frac{7}{19}$		
$\vec{AE} = \frac{7}{19}(\vec{p} + \vec{q})$	1M+1A	
$\lvert\vec{AE}\rvert^2 = \vec{AE} \cdot \vec{AE}$	1M	
$= (\frac{7}{19})^2(\vec{p}\cdot\vec{p} + 2\vec{p}\cdot\vec{q} + \vec{q}\cdot\vec{q})$		
$= (\frac{7}{19})^2(9 + 6 + 4)$		
$= \frac{49}{19}$		
$\therefore \lvert\vec{AE}\rvert = \frac{7\sqrt{19}}{19}$ (or $\frac{7}{\sqrt{19}}$)	1A	
Alternative solution (1)		
By Cosine Law,		
$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\frac{2\pi}{3}$	1M	
$= 3^2 + 2^2 - 2(3)(2)\cos\frac{2\pi}{3}$		
$= 19$		
$AC = \sqrt{19}$		
For $k = \frac{7}{12}$, $\lambda = \frac{12}{7}$, $h = \frac{7}{19}$		
$\vec{AE} = \frac{7}{19}\vec{AC}$	1M+1A	
$\therefore \lvert\vec{AE}\rvert = \frac{7}{19}\lvert\vec{AC}\rvert$		
$= \frac{7}{\sqrt{19}}$	1A	
Alternative solution (2)		
$\vec{DF} = \frac{7}{12}\vec{p} - \vec{q}$		
$\lvert\vec{DF}\rvert^2 = \vec{DF}\cdot\vec{DF}$		
$= \frac{49}{144}\vec{p}\cdot\vec{p} - \frac{7}{6}\vec{p}\cdot\vec{q} + \vec{q}\cdot\vec{q}$	1M	
$= \frac{57}{16}$		
$\lvert\vec{EF}\rvert = \frac{7\sqrt{57}}{76}$	1A	
$\lvert\vec{AF}\rvert = \frac{7}{6}$		
$\lvert\vec{AE}\rvert = \sqrt{\lvert AF\rvert^2 - \lvert EF\rvert^2}$	1M	
$= \frac{7\sqrt{19}}{19}$	1A	
	11	

Solution	Marks	Remarks
9. (a) Let r m be the radius of the shadow		
$\frac{2}{r} = \frac{h-1}{h}$	1M	For considering similar Δs
$r = \frac{2h}{h-1}$	1A	
$S = \pi r^2 = \frac{4\pi h^2}{(h-1)^2}$	1	
Alternative solution		
Area of Table $= \pi(2)^2$		
$\frac{\text{Area of Table}}{S} = \left(\frac{h-1}{h}\right)^2$	1M+1A	
$\therefore S = \frac{4\pi h^2}{(h-1)^2}$	1	
	3	
(b) $\frac{dS}{dh} = \frac{(h-1)^2 8\pi h - 4\pi h^2 . 2(h-1)}{(h-1)^4}$	1M+1A	1M For quotient rule
$= \frac{-8\pi h}{(h-1)^3}$		
$\frac{dS}{dt} = \frac{dS}{dh}\frac{dh}{dt}$	1M	For chain rule
$= \frac{-8\pi h}{(h-1)^3}\left(-\frac{1}{8}\right) = \frac{\pi h}{(h-1)^3}$	1A	For $\frac{dh}{dt} = -\frac{1}{8}$
At $h = 2$, $\frac{dS}{dt} = \frac{\pi(2)}{(2-1)^3} = 2\pi$ (s^{-1})	1A	
	5	
(c) (i) $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi\left(\frac{2h}{h-1}\right)^2 h$ $= \frac{4\pi h^3}{3(h-1)^2}$	1	
(ii) $\frac{dV}{dh} = \frac{4\pi}{3}\frac{(h-1)^2 3h^2 - h^3 . 2(h-1)}{(h-1)^4}$	1A	
$= \frac{4\pi}{3}\frac{h^3 - 3h^2}{(h-1)^3}$		
$\frac{dV}{dh} = 0$	1M	
$\frac{4\pi}{3}\frac{h^3 - 3h^2}{(h-1)^3} = 0$		
$h^2(h-3) = 0$		
$h = 3$ ($\because h > 1$)	1A	
When $h > 3$, $\frac{dV}{dh} > 0$		

Solution	Marks	Remarks
When $(1<)\ h<3$, $\frac{dV}{dh}<0$ $\therefore V$ is minimum at $h=3$.	1M	For checking
Alternative Solution for checking $\frac{d^2V}{dh^2}=\frac{4\pi}{3}\frac{(h-1)^3(3h^2-6h)-(h^3-3h^2)3(h-1)^2}{(h-1)^6}$ $=\frac{8\pi h}{(h-1)^4}$ At $h=3$, $\frac{d^2V}{dh^2}\left(=\frac{3\pi}{2}\right)>0$ $\therefore V$ is minimum at $h=3$	1M	For checking
$\therefore$ Minimum value of V $=\frac{4\pi(3^3)}{3(3-1)^2}$ $=9\pi$	1A	Awarded even if checking is omitted.
At $h=3$, $\frac{dS}{dh}=\frac{-8\pi(3)}{(3-1)^3}\ (=-3\pi)\neq 0$ Since $\frac{dS}{dh}\neq 0$ at $h=3$, S does not attain a minimum when V attains its minimum.	1M+1A	
Alternative solution From (b), $\frac{dS}{dh}=\frac{-8\pi h}{(h-1)^3}<0$ $\therefore S$ is a (strictly) decreasing function. $\therefore S$ does not attain a minimum at $h=3$	1M+1A	
As the lamp is lowered, S (strictly) increases. $\therefore S$ does not attain a minimum at $h=3$.	1M+1A	
	8	

Solution	Marks	Remarks
10. (a) $f(\alpha) = g(\alpha)$		
$12\alpha^2 + 2p\alpha - q = 12\alpha^2 + 2q\alpha - p$		
$2(p - q)\alpha = q - p$		
$\alpha = -\frac{1}{2}$ $(\because p \neq q)$	1A	
$f(\alpha) = 0$		or $g(\alpha) = 0$, $f(\alpha) + g(\alpha) = 0$ etc.
$f(-\frac{1}{2}) = 0$		
$12(-\frac{1}{2})^2 + 2p(-\frac{1}{2}) - q = 0$	1M	For substituting $\alpha = -\frac{1}{2}$
$3 - p - q = 0$		
$p + q = 3$	1	
	3	
(b) (i) $\alpha\beta = -\frac{q}{12}$	1M	
$\therefore \alpha = -\frac{1}{2}$ $\therefore \beta = \frac{q}{6} = \frac{3-p}{6}$	1A	
Alternative solution		
(1) $\alpha + \beta = -\frac{2p}{12}$	1M	
$-\frac{1}{2} + \beta = -\frac{2p}{12}$		
$\beta = \frac{1}{2} - \frac{p}{6}$	1A	
(2) $f(x) = 12x^2 + 2px + p - 3 = 0$		
$(2x + 1)(6x + p - 3) = 0$	1M	
$x = -\frac{1}{2}, \frac{3-p}{6}$		
$\therefore \beta = \frac{1}{2} - \frac{p}{6}$	1A	

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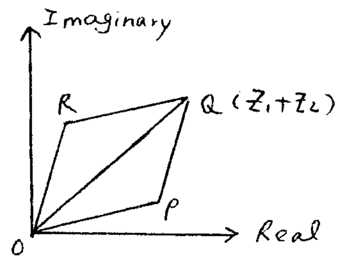
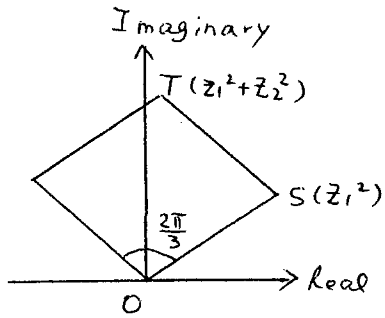
Solution	Marks	Remarks
(ii) $\alpha\gamma = -\frac{p}{12}$	1M	
$\because \alpha = -\frac{1}{2}$ $\therefore \gamma = \frac{p}{6}$	1A	
<u>Alternative solution</u>		
(1) $\alpha + \gamma = -\frac{2q}{12}$	1M	
$-\frac{1}{2} + \gamma = -\frac{2q}{12}$		
$\gamma = \frac{1}{2} - \frac{q}{6}$		
$= \frac{p}{6}$	1A	
(2) $g(x) = 12x^2 + 2qx + q - 3 = 0$		
$(2x + 1)(6x + q - 3) = 0$	1M	
$x = -\frac{1}{2}, \frac{3-q}{6}$		
$\therefore \gamma = \frac{3-q}{6} = \frac{p}{6}$	1A	
	4	
(c) (i) $\lvert\beta^3 + \gamma^3\rvert < \frac{7}{24}$		
$\lvert(\frac{1}{2} - \frac{p}{6})^3 + (\frac{p}{6})^3\rvert < \frac{7}{24}$	1M	For substitution
$\lvert\frac{1}{8} - \frac{p}{8} + \frac{p^2}{24} - \frac{p^3}{216} + \frac{p^3}{216}\rvert < \frac{7}{24}$	1A	$\lvert(3-p)^3 + p^3\rvert < 63$
$\lvert\frac{p^2}{24} - \frac{p}{8} + \frac{1}{8}\rvert < \frac{7}{24}$	1A	$\lvert 27 - 27p + 9p^2 - p^3 + p^3\rvert < 63$
		$\lvert 9p^2 - 27p + 27\rvert < 63$
$\lvert p^2 - 3p + 3\rvert < 7$		
$p^2 - 3p + 3 < 7$ and $p^2 - 3p + 3 > -7$	1M	For handling absolute values
$p^2 - 3p - 4 < 0$ $\quad$ $p^2 - 3p + 10 > 0$		
$(p + 1)(p - 4) < 0$ $\quad$ $(p - \frac{3}{2})^2 + \frac{31}{4} > 0$		
$-1 < p < 4$ $\quad$ All real numbers	1A+1A	
$\therefore -1 < p < 4$	1A	
<u>OR</u> $\because p^2 - 3p + 3 = (p - \frac{3}{2})^2 + \frac{3}{4} > 0$	1A	
$\therefore \lvert p^2 - 3p + 3\rvert = p^2 - 3p + 3$	1M	
$p^2 - 3p + 3 < 7$		
$(p + 1)(p - 4) < 0$	1A	
$-1 < p < 4$	1A	

Solution	Marks	Remarks
Alternative solution		
From (b), $\beta + \gamma = \frac{1}{2}$		
$\lvert \beta^3 + \gamma^3 \rvert < \frac{7}{24}$		
$\lvert (\beta + \gamma)(\beta^2 - \beta\gamma + \gamma^2) \rvert < \frac{7}{24}$	1A	
$\lvert \frac{1}{2} [(\frac{1}{2})^2 - 3(\frac{1}{2} - \frac{p}{6})\frac{p}{6}] \rvert < \frac{7}{24}$	1M	For substitution
$\lvert \frac{1}{8} - \frac{p}{8} + \frac{p^2}{24} \rvert < \frac{7}{24}$	1A	
$\lvert p^2 - 3p + 3 \rvert < 7$		
$p^2 - 3p + 3 < 7$ and $p^2 - 3p + 3 > -7$	1M	For handling absolute values
$p^2 - 3p - 4 < 0$ $p^2 - 3p + 10 > 0$		
$(p + 1)(p - 4) < 0$ $(p - \frac{3}{2})^2 + \frac{31}{4} > 0$		
$-1 < p < 4$ All real numbers	1A+1A	
$\therefore -1 < p < 4$	1A	
(ii) From (i), $-1 < p < 4$		
Combining with $p > q$, $p + q = 3$ and p, q are integers,		
$p = 2$, $q = 1$	1A	Withhold 1 mark for giving each extra answer
or $p = 3$, $q = 0$	1A	
	9	

Solution	Marks	Remarks
11. (a) $x^2 - x + 1 = 0$		
$x = \frac{1 \pm \sqrt{-3}}{2}$	1A	
$= \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$		
$\alpha = \frac{1}{2} + \frac{\sqrt{3}}{2} i = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$	1A	(pp-1) for degrees
$\beta = \frac{1}{2} - \frac{\sqrt{3}}{2} i = \cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})$	1A 3	
(b) (i) $\left\lvert\frac{z_2}{z_1}\right\rvert = \frac{\lvert z_2\rvert}{\lvert z_1\rvert}$		
$= 1$	1A	
$\arg(\frac{z_2}{z_1}) = \arg z_2 - \arg z_1$ (or $= \angle POQ$)	1M	(can be omitted)
$= \frac{\pi}{3}$	1A	Accept degrees
Alternative solution		
Let $z_1 = r(\cos\theta + i\sin\theta)$ (where $r > 0$, $0 < \theta < \frac{\pi}{2}$)		
$z_2 = r[\cos(\theta + \frac{\pi}{3}) + i\sin(\theta + \frac{\pi}{3})]$		
$\frac{z_2}{z_1} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$	1M	
$\therefore \left\lvert\frac{z_2}{z_1}\right\rvert = 1$	1A	
$\arg(\frac{z_2}{z_1}) = \frac{\pi}{3}$	1A	
$\therefore \frac{z_2}{z_1} = \alpha$ (or $= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$)	1	
$\therefore \frac{z_2}{z_1}$ is a root of (*)		
(ii) As $\frac{z_2}{z_1}$ satisfies (*),		
$(\frac{z_2}{z_1})^2 - \frac{z_2}{z_1} + 1 = 0$	1M	For substitution
$z_2^2 - z_1 z_2 + z_1^2 = 0$		
$z_1^2 + z_2^2 = z_1 z_2$	1	

Solution	Marks	Remarks
Alternative solution (1)		
$z_2 = \alpha z_1$		
L.H.S. $= z_1^2 + z_2^2$		
$= z_1^2 + \alpha^2 z_1^2$	1M	For substitution
$= (1 + \alpha^2) z_1^2$		
$= \alpha z_1^2 \quad (\because \alpha^2 - \alpha + 1 = 0)$		
$= z_1 z_2 \quad (\because z_2 = \alpha z_1)$	1	
$=$ R.H.S.		
Alternative solutin (2)		
$z_1^2 + z_2^2 = z_1 z_2$		
$\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$		
$\frac{z_2}{z_1} + \frac{z_1}{z_2} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} + [\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})]$	1M	OR $= \alpha + \overline{\alpha}$
$= 2\cos\frac{\pi}{3}$		
$= 1$		
$\therefore z_1^2 + z_2^2 = z_1 z_2$	1	
Alternative solution (3)		
Let $z_1 = r(\cos\theta + i\sin\theta)$ (where $r > 0$, $0 < \theta < \frac{\pi}{2}$)		
$z_2 = r[\cos(\theta + \frac{\pi}{3}) + i\sin(\theta + \frac{\pi}{3})]$		
L.H.S. $= z_1^2 + z_2^2$		
$= r^2(\cos 2\theta + i\sin 2\theta) + r^2[\cos(2\theta + \frac{2\pi}{3}) + i\sin(2\theta + \frac{2\pi}{3})]$	1M	For substitution
$= r^2[2\cos(2\theta + \frac{\pi}{3})\cos\frac{\pi}{3} + i2\sin(2\theta + \frac{\pi}{3})\cos\frac{\pi}{3}]$		
$= r^2[\cos(2\theta + \frac{\pi}{3}) + i\sin(2\theta + \frac{\pi}{3})]$		
$= r(\cos\theta + i\sin\theta)\, r[\cos(\theta + \frac{\pi}{3}) + i\sin(\theta + \frac{\pi}{3})]$		
$= z_1 z_2 =$ R.H.S.	1	

Solution	Marks	Remarks
(iii) (1) $(z_1 + z_2)^2 = z_1^2 + z_2^2 + 2z_1z_2$	1A	
$(z_1 + z_2)^2 = 3z_1z_2$		
$\lvert z_1 + z_2\rvert^2 = 3\lvert z_1z_2\rvert$	1A	
$= 3\lvert z_1\rvert^2$ $(\because \lvert z_1\rvert = \lvert z_2\rvert)$	1A	
$= 12$		
$\therefore \lvert z_1 + z_2\rvert = 2\sqrt{3}$	1A	
Alternative solution (1)		
$\lvert z_2\rvert = \lvert z_1\rvert = 2$	1A	(can be omitted)
$\lvert z_1 + z_2\rvert$ is the diagonal of the parallelogram *OPRQ*.	1M	(can be omitted)
$\lvert z_1 + z_2\rvert = 2\lvert z_1\rvert\cos\frac{\pi}{6}$	1A	
$= 2\sqrt{3}$	1A	
or $\lvert z_1 + z_2\rvert^2 = \lvert z_1\rvert^2 + \lvert z_2\rvert^2 - 2\lvert z_1\rvert\lvert z_2\rvert\cos\frac{2\pi}{3}$	1A	
$= 2^2 + 2^2 - 2(2)(2)\cos\frac{2\pi}{3} = 12$		
$\therefore \lvert z_1 + z_2\rvert = 2\sqrt{3}$	1A	

Solution	Marks	Remarks
Alternative solution (2)		
$\frac{z_2}{z_1} = \alpha = \frac{1}{2} + \frac{\sqrt{3}}{2}i$		
$\lvert z_1 + z_2 \rvert = \lvert (1 + \alpha) z_1 \rvert$	1A	
$= \lvert (\frac{3}{2} + \frac{\sqrt{3}}{2}i) z_1 \rvert$	1A	
$= \lvert \frac{3}{2} + \frac{\sqrt{3}}{2}i \rvert \lvert z_1 \rvert$	1M	
$= 2\sqrt{3}$	1A	
(2) $z_1^2 + z_2^2 = z_1 z_2$		
$\lvert z_1^2 + z_2^2 \rvert = \lvert z_1 z_2 \rvert$	1A	
$= \lvert z_1 \rvert^2$	1A	
$\therefore \lvert z_1^2 + z_2^2 \rvert = 4$	1A	
Alternative solution (1)		
(2) Suppose $z_1 = 2(\cos\theta + i\sin\theta)$		
$z_2 = 2[\cos(\theta + \frac{\pi}{3}) + i\sin(\theta + \frac{\pi}{3})]$		
then $z_1^2 = 4(\cos 2\theta + i\sin 2\theta)$	1A	
$z_2^2 = 4[\cos(2\theta + \frac{2\pi}{3}) + i\sin(2\theta + \frac{2\pi}{3})]$		
Let points S, T denote the complex numbers z_1^2 and $z_1^2 + z_2^2$ respectively.		
ΔOST is equilateral.	1M	
$\therefore \lvert z_1^2 + z_2^2 \rvert = \lvert z_1^2 \rvert$		
$= 4$	1A	
Alternative solution (2)		
$\lvert z_1^2 + z_2^2 \rvert = \lvert (1 + \alpha^2) z_1^2 \rvert$	1A	
$= \lvert (1 + \frac{-2 + 2\sqrt{3}i}{4}) z_1^2 \rvert$		
$= \lvert (\frac{1}{2} + \frac{\sqrt{3}}{2}i) z_1^2 \rvert$	1A	
$= \lvert \frac{1}{2} + \frac{\sqrt{3}}{2}i \rvert \lvert z_1 \rvert^2$		
$= 4$	1A	
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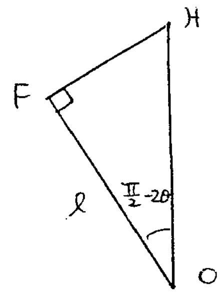
Solution	Marks	Remarks
12. (a) ΔODG	1A	
$\angle HOF = \frac{\pi}{2} - 2\theta$		
Area of $\Delta OFH = \frac{1}{2}\ell(\ell\tan(\frac{\pi}{2} - 2\theta))$	1M	
$= \frac{\ell^2}{2\tan 2\theta}$	1	
(b) (i) Area of $\Delta OCG = \frac{1}{2}\ell(\ell\tan\theta)$		
$= \frac{\ell^2}{2}\tan\theta$	1A	
$S = \frac{\ell^2}{2\tan 2\theta} + 2(\frac{\ell^2}{2}\tan\theta)$	1M	
$= \frac{\ell^2}{2}(2\tan\theta + \frac{\cos 2\theta}{\sin 2\theta})$		
$= \frac{\ell^2}{2}(\frac{4\sin^2\theta + \cos 2\theta}{\sin 2\theta})$		
$= \frac{\ell^2}{2}\ \frac{2(1 - \cos 2\theta) + \cos 2\theta}{\sin 2\theta}$	1M	For expressing in terms of $\cos 2\theta$ and $\sin 2\theta$
$= \frac{\ell^2}{2}(\frac{2 - \cos 2\theta}{\sin 2\theta})$	1	
Alternative solution		
Area of $\Delta OCG = \frac{\ell^2}{2}\tan\theta$	1A	
$S = \frac{\ell^2}{2\tan 2\theta} + 2(\frac{\ell^2}{2}\tan\theta)$	1M	
$= \frac{\ell^2}{2\tan 2\theta} + \ell^2(\frac{1 - \cos 2\theta}{\sin 2\theta})$	1M	For expressing in terms of $\cos 2\theta$ and $\sin 2\theta$
$= \frac{\ell^2}{2}(\frac{\cos 2\theta + 2(1 - \cos 2\theta)}{\sin 2\theta})$		
$= \frac{\ell^2}{2}(\frac{2 - \cos 2\theta}{\sin 2\theta})$	1	
(ii) (1) $\frac{dS}{d\theta} = \frac{\ell^2}{2}\ \frac{\sin 2\theta(2\sin 2\theta) - (2 - \cos 2\theta)(2\cos 2\theta)}{\sin^2 2\theta}$	1M+1A	For quotient rule
$= \frac{\ell^2}{2}\ \frac{2\sin^2 2\theta - 4\cos 2\theta + 2\cos^2 2\theta}{\sin^2 2\theta}$		
$= \ell^2(\frac{1 - 2\cos 2\theta}{\sin^2 2\theta})$		
S is increasing when $\frac{dS}{d\theta} > 0$	Accept	≥ 0
$\ell^2(\frac{1 - 2\cos 2\theta}{\sin^2 2\theta}) > 0$	1M	
$1 - 2\cos 2\theta > 0$		

Solution	Marks	Remarks
$\cos 2\theta < \frac{1}{2}$		
$2\theta > \frac{\pi}{3}$		
$(\frac{\pi}{4} >)\ \theta > \frac{\pi}{6}$	1A	OR $\theta \geq \frac{\pi}{6}$, (pp-1) for degrees
(2) S is decreasing when $\frac{dS}{d\theta} < 0$		Accept ≤ 0
$\cos 2\theta > \frac{1}{2}$		(can be omitted)
$2\theta < \frac{\pi}{3}$		
$(\frac{\pi}{8} <)\ \theta < \frac{\pi}{6}$	1A	(pp-1) for degrees
(Since $\frac{dS}{d\theta}$ changes from -ve to +ve at $\theta = \frac{\pi}{6}$,)		
S is minimum at $\theta = \frac{\pi}{6}$	1M	
$S_{\min} = \frac{\ell^2}{2}\left(\frac{2 - \cos\frac{\pi}{3}}{\sin\frac{\pi}{3}}\right)$		
$= \frac{\sqrt{3}\,\ell^2}{2}$	1A	
	11	
(c) Maximum area $= \ell^2 - S_{\min}$	1M	
$= \ell^2 - \frac{\sqrt{3}\,\ell^2}{2}$	1A	
$= (1 - \frac{\sqrt{3}}{2})\ell^2$		
	2	