Section A (42 marks)

Answer **ALL** questions in this section.

1. Let $f(x) = x^2 + (1 - m)x + 2m - 5$, where m is a constant. Find the discriminant of the equation $f(x) = 0$.

   Hence find the range of values of m so that $f(x) > 0$ for all real values of x.

   (5 marks)

2. Let $z = -1 + \sqrt{3}i$. Express z in polar form.

   Hence find $z^5 + \bar{z}^5$.

   (5 marks)

3. Using the information in the following table, sketch the graph of $y = f(x)$, where $f(x)$ is a polynomial.

	$x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$1 < x < 2$	$x = 2$	$x > 2$
$f(x)$		1		2		1	
$f'(x)$	< 0	0	> 0	0	< 0	0	> 0

   (5 marks)

4. By considering the two cases $x > 0$ and $x < 0$, or otherwise, solve the inequality

$$x - \frac{5}{x} > 4.$$

   (6 marks)

5. In the same Argand diagram, sketch the locus of the point representing the complex number z in each of the following cases :

   (a) $|z - (3 + i)| = 3$,

   (b) $|z| = |z - 8i|$.

   Hence, or otherwise, find the complex number(s) represented by the point(s) of intersection of the two loci.

   (6 marks)

6. $P(4, 1)$ is a point on the curve $y^2 + y\sqrt{x} = 3$, where $x > 0$.

   (a) Find the value of $\frac{dy}{dx}$ at P.

   (b) Find the equation of the normal to the curve at P.

   (7 marks)

7. Let $\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OQ} = -6\mathbf{i} + 4\mathbf{j}$. Let R be a point on PQ such that $PR : RQ = k : 1$, where $k > 0$.

   (a) Express $\overrightarrow{OR}$ in terms of k, $\mathbf{i}$ and $\mathbf{j}$.

   (b) Express $\overrightarrow{OP} \cdot \overrightarrow{OR}$ and $\overrightarrow{OQ} \cdot \overrightarrow{OR}$ in terms of k.

   (c) Find the value of k such that OR bisects $\angle POQ$.

   (8 marks)

Go on to the next page

Section B (48 marks)

Answer any **THREE** questions in this section.
Each question carries 16 marks.

8.

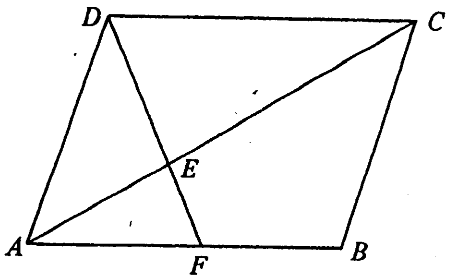

Figure 1

In Figure 1, $ABCD$ is a parallelogram and F is a point on AB. DF meets AC at a point E such that $DE : EF = \lambda : 1$, where λ is a positive number. Let $\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AD} = \mathbf{q}$ and $\overrightarrow{AE} = h\overrightarrow{AC}$, $\overrightarrow{AF} = k\overrightarrow{AB}$, where h, k are positive numbers.

(a) (i) Express $\overrightarrow{AE}$ in terms of h, $\mathbf{p}$ and $\mathbf{q}$.

(ii) Express $\overrightarrow{AE}$ in terms of λ, k, $\mathbf{p}$ and $\mathbf{q}$.

Hence show that $\lambda = \frac{1}{k}$. (5 marks)

(b) It is given that $|\mathbf{p}| = 3$, $|\mathbf{q}| = 2$, $\angle DAB = \frac{\pi}{3}$.

(i) Find $\mathbf{p} \cdot \mathbf{q}$.

(ii) Suppose DF is perpendicular to AC.

(1) By expressing $\overrightarrow{DF}$ in terms of k, $\mathbf{p}$ and $\mathbf{q}$, find the value of k.

(2) Using (a), or otherwise, find the length of AE.

(11 marks)

9.

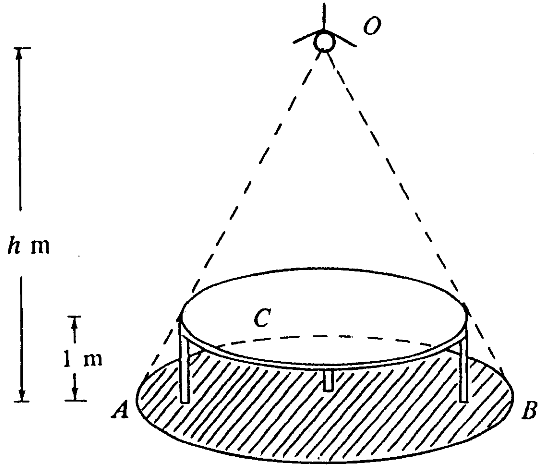

Figure 2

A small lamp O is placed h m above the ground, where $1 < h \leq 5$. Vertically below the lamp is the centre of a round table of radius 2 m and height 1 m. The lamp casts a shadow ABC of the table on the ground (see Figure 2). Let S m^2 be the area of the shadow.

(a) Show that $S = \frac{4\pi h^2}{(h-1)^2}$. (3 marks)

(b) If the lamp is lowered vertically at a constant rate of $\frac{1}{8}$ m s^{-1}, find the rate of change of S with respect to time when $h = 2$. (5 marks)

(c) Let V m^3 be the volume of the cone $OABC$.

(i) Show that $V = \frac{4\pi h^3}{3(h-1)^2}$.

(ii) Find the minimum value of V as h varies.

Does S attain a minimum when V attains its minimum? Explain your answer. (8 marks)

Go on to the next page

10. Let $f(x) = 12x^2 + 2px - q$

and $g(x) = 12x^2 + 2qx - p$,

where p, q are distinct real numbers. α, β are the roots of the equation $f(x) = 0$ and α, γ are the roots of the equation $g(x) = 0$.

(a) Using the fact that $f(\alpha) = g(\alpha)$, find the value of α.

Hence show that $p + q = 3$.

(3 marks)

(b) Express β and γ in terms of p.

(4 marks)

(c) Suppose $\left|\beta^3 + \gamma^3\right| < \dfrac{7}{24}$.

(i) Find the range of possible values of p.

(ii) Furthermore, if $p > q$, write down the possible integral values of p and q.

(9 marks)

11. (a) Let α, β be the roots of the equation

$$x^2 - x + 1 = 0 \text{ .... } (*),$$

where $-\pi < \arg\beta < \arg\alpha < \pi$.

Express α and β in polar form. (3 marks)

(b)

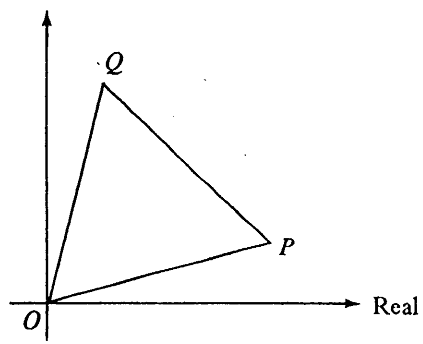

Figure 3

Figure 3 shows an Argand diagram in which OPQ is an equilateral triangle. The complex numbers represented by O, P and Q are 0, z_1 and z_2 respectively.

(i) Find the values of $\left|\dfrac{z_2}{z_1}\right|$ and $\arg\left(\dfrac{z_2}{z_1}\right)$.

Hence show that $\dfrac{z_2}{z_1}$ is a root of the equation (*) in (a).

(ii) Using (i), or otherwise, show that

$$z_1^2 + z_2^2 = z_1 z_2.$$

(iii) It is given that $|z_1| = 2$. Find

(1) $|z_1 + z_2|$,

(2) $|z_1^2 + z_2^2|$.

(13 marks)

Go on to the next page

12.

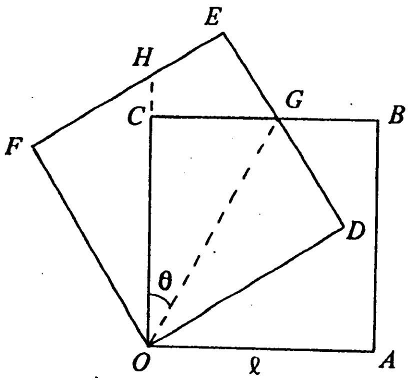

Figure 4

In Figure 4, $OABC$ is the position of a square of side ℓ. The square is rotated anticlockwise about O to a new position $ODEF$. BC cuts DE at G and OC produced cuts EF at H. Let $\angle COG = \theta$, where $\frac{\pi}{8} < \theta < \frac{\pi}{4}$.

(a) Name a triangle which is congruent to ΔOCG.

Hence show that the area of ΔOFH is $\frac{\ell^2}{2\tan 2\theta}$. (3 marks)

(b) Let S be the sum of the areas of ΔOFH and the quadrilateral $ODGC$.

(i) Show that $S = \frac{\ell^2}{2}\left(\frac{2 - \cos 2\theta}{\sin 2\theta}\right)$.

(ii) Find the range of values of θ for which S is

(1) increasing,

(2) decreasing.

Hence find the minimum value of S. (11 marks)

(c) Find the maximum value of the area of the quadrilateral $CGEH$. (2 marks)

END OF PAPER