

香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九四年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1994

附加數學卷一
ADDITIONAL MATHEMATICS PAPER I

評卷參考
MARKING SCHEME

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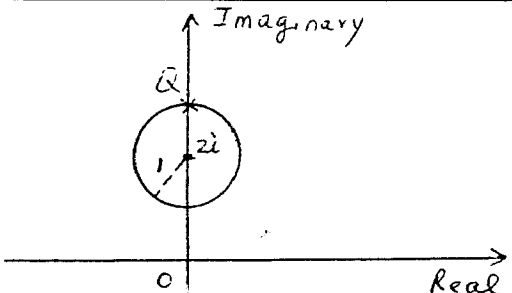
It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

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本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員如向學生披露本評卷參考內容，即違背閱卷員守則。

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Solution	Marks	Remarks
<p>1. $\frac{2(x+1)}{x-2} \geq 1$</p> <p>$\frac{2(x+1)}{x-2} - 1 \geq 0$</p> <p>$\frac{x+4}{x-2} \geq 0$</p> <p>$x > 2$ or $x \leq -4$</p>	<p>1M</p> <p>1A</p> <p>2A</p> <hr style="width: 50%; margin-left: 0;"/> <p>4</p>	<p>1A for $x \geq 2$ or $x \leq -4$</p>
<p><u>Alternative solution (1)</u></p> <p>$\frac{2(x+1)}{x-2} \geq 1$</p> <p>Consider the following 2 cases (i) $x > 2$, (ii) $x < 2$:</p> <p>Case 1 : $x > 2$</p> <p style="margin-left: 40px;">$2(x+1) \geq x-2$ _____</p> <p style="margin-left: 80px;">$x \geq -4$</p> <p style="margin-left: 40px;">Since $x > 2$, $\therefore x > 2$</p> <p>Case 2 : $x < 2$</p> <p style="margin-left: 40px;">$2(x+1) \leq x-2$ _____</p> <p style="margin-left: 80px;">$x \leq -4$</p> <p style="margin-left: 40px;">Since $x < 2$, $\therefore x \leq -4$</p> <p>Combining the 2 cases, $x > 2$ or $x \leq -4$</p>	<p>1M</p> <p>1A</p> <p>2A</p>	<p>Awarded even if equality sign is included.</p> <p>1A for $x \geq 2$ or $x \leq -4$</p>
<p><u>Alternative solution (2)</u></p> <p>$\frac{2(x+1)}{(x-2)} \geq 1$</p> <p>$2(x+1)(x-2) \geq (x-2)^2$ (and $x \neq 2$)</p> <p>$x^2 + 2x - 8 \geq 0$ (and $x \neq 2$)</p> <p>$(x-2)(x+4) \geq 0$ (and $x \neq 2$)</p> <p>$x > 2$ or $x \leq -4$</p>	<p>1M</p> <p>1A</p> <p>2A</p>	<p>1A for $x \geq 2$ or $x \leq -4$</p>
<p>2.</p> <div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> <p>Position of Q</p> </div> </div>	<p>1A</p> <p>1A</p> <p>1A</p> <p>1M+1A</p>	<p>For circle</p> <p>For centred at $z = 2i$</p> <p>For radius = 1</p> <p>1M for being farthest away from O</p> <p>Axes not labelled - (pp-1)</p>
<hr style="width: 50%; margin-left: auto; margin-right: 0;"/> <p>5</p>		

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1994 HKCE Add. Maths. I
Marking Scheme

P.2

Solution	Marks	Remarks
<p>3. (a) $\vec{PQ} = \vec{OQ} - \vec{OP}$ $= 2\vec{i} - \vec{j}$ $\vec{PQ} = \sqrt{2^2 + (-1)^2} = \sqrt{5}$</p> <p>(b) Let $\angle QPR = \theta$ $\vec{PQ} \cdot \vec{PR} = (2\vec{i} - \vec{j}) \cdot (-3\vec{i} - 2\vec{j}) = -4$ $\cos \theta = \frac{\vec{PQ} \cdot \vec{PR}}{ \vec{PQ} \vec{PR} }$ $= \frac{-4}{\sqrt{65}}$</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1M</p> <p><u>1A</u></p> <p><u>6</u></p>	<p>Omit vector sign (pp-1)</p> <p>Omit dot sign (pp-1)</p>
<p><u>Alternative solution</u></p> <p>(b) $\vec{PR} = \sqrt{13}$ $\vec{RQ} = 5\vec{i} + \vec{j}$ $\vec{RQ} = \sqrt{26}$</p> $\cos \angle QPR = \frac{ \vec{PQ} ^2 + \vec{PR} ^2 - \vec{QR} ^2}{2 \vec{PQ} \vec{PR} }$ $= \frac{5 + 13 - 26}{2\sqrt{5}\sqrt{13}}$ $= \frac{-4}{\sqrt{65}}$	<p>1A</p> <p>1M</p> <p>1A</p>	
<p>4. $y = \tan\left(\frac{1}{x}\right)$</p> $\frac{dy}{dx} = -\frac{1}{x^2} \sec^2\left(\frac{1}{x}\right)$ $x^2 \frac{dy}{dx} + (y^2 + 1) = -\sec^2\left(\frac{1}{x}\right) + \tan^2\left(\frac{1}{x}\right) + 1$ $= 0$ <p>Differentiating $x^2 \frac{dy}{dx} + (y^2 + 1) = 0$ with respect to x</p> $2x \frac{dy}{dx} + x^2 \frac{d^2y}{dx^2} + 2y \frac{dy}{dx} = 0$ $x^2 \frac{d^2y}{dx^2} + 2(x + y) \frac{dy}{dx} = 0$ $\frac{d^2y}{dx^2} + \frac{2(x + y)}{x^2} \frac{dy}{dx} = 0$	<p>1M+1A</p> <p>1M</p> <p>1</p> <p>1A</p> <p><u>1</u></p> <p><u>6</u></p>	<p>1M for $\frac{d}{dx}(\tan x) = \sec^2 x$</p> <p>or $= -(1 + y^2) + y^2 + 1$</p>

Solution	Marks	Remarks
5. $z^2 - \sqrt{2}z + 1 = 0$ $z = \frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}i$ $= \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \quad \text{or} \quad \cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4})$	1A	Accept $\frac{\sqrt{2}}{2} \pm \frac{\sqrt{-2}}{2}$
$w^4 - \sqrt{2}w^2 + 1 = 0$ $w^2 = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \quad \text{or} \quad w^2 = \cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4})$	1A+1A	Do not accept degrees Do not accept $\cos \frac{\pi}{4} - i \sin \frac{\pi}{4}$ (Note : Mark the rest of the Q if z is correct but not in the specified format.)
$w = \cos(\frac{2k\pi + \frac{\pi}{4}}{2}) + i \sin(\frac{2k\pi + \frac{\pi}{4}}{2})$ $= \cos(k\pi + \frac{\pi}{8}) + i \sin(k\pi + \frac{\pi}{8})$	1M	
$\text{or } w = \cos(\frac{2k\pi - \frac{\pi}{4}}{2}) + i \sin(\frac{2k\pi - \frac{\pi}{4}}{2}),$ $= \cos(k\pi - \frac{\pi}{8}) + i \sin(k\pi - \frac{\pi}{8})$	1M+1A +1A	1M for De Moivre's Theorem
where $k = 0, 1$ (or any 2 consecutive integers)	<u>7</u>	
$\text{or } w = \cos \theta + i \sin \theta,$ where $\theta = \frac{\pi}{8}$ (or $-15 \frac{\pi}{8}$ etc.), $-\frac{\pi}{8}$ (or $\frac{15\pi}{8}$), $\frac{7\pi}{8}$ (or $-\frac{9\pi}{8}$), $-\frac{7\pi}{8}$ (or $\frac{9\pi}{8}$)	1A	Accept other equivalent values. Accept degrees

Solution	Marks	Remarks
<p>6. (a) $x^2 + y \cos x - y^2 = 0$</p> $2x + \cos x \frac{dy}{dx} - y \sin x - 2y \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{y \sin x - 2x}{\cos x - 2y}$ <p>(b) At P, $\frac{dy}{dx} = \frac{-\frac{\pi}{2} \sin \frac{\pi}{2} - \pi}{\cos \frac{\pi}{2} - (-\pi)}$</p> $= -\frac{3}{2}$ <p>Equation of tangent is</p> $\frac{y + \frac{\pi}{2}}{x - \frac{\pi}{2}} = -\frac{3}{2}$ $6x + 4y - \pi = 0$	<p>1A+1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p><u>1A</u></p> <p><u>7</u></p>	<p>1A for $\frac{d}{dx} (y \cos x)$</p> <p>1A for other terms</p> <p></p> <p></p> <p></p> <p></p> <p></p> <p>or $y = -\frac{3}{2}x + \frac{\pi}{4}$</p>

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Solution	Marks	Remarks
<p>7. $(x - 3)^2 - x - 3 - 12 = 0$</p> <p><u>Solution (1) :</u></p> $(x - 3)^2 = x - 3 ^2$ $ x - 3 ^2 - x - 3 - 12 = 0$ $(x - 3 + 3)(x - 3 - 4) = 0$ $ x - 3 = 4 \quad \text{or} \quad x - 3 = -3$ $\therefore x - 3 = 4$ $\therefore x = 7 \quad \text{or} \quad -1$	<p>2M</p> <p>1A</p> <p>1A+1A+2A <u>7</u></p>	<p>2A for rejecting $x - 3 = -3$</p>
<p><u>Solution (2) :</u></p> <p>Consider 2 cases : (1) $x \geq 3$ (2) $x < 3$</p> <p>Case (1) : $x \geq 3$</p> $(x - 3)^2 - (x - 3) - 12 = 0$ $[(x - 3) + 3][(x - 3) - 4] = 0$ $x^2 - 7x = 0$ $x = 0 \quad \text{or} \quad 7$ <p>Rejecting $x = 0$, $\therefore x = 7$</p> <p>Case (2) : $x < 3$</p> $(x - 3)^2 + (x - 3) - 12 = 0$ $[(x - 3) - 3][(x - 3) + 4]$ $x^2 - 5x - 6 = 0$ $x = 6 \quad \text{or} \quad -1$ <p>Rejecting $x = 6$, $\therefore x = -1$</p> <p>Combining the 2 cases, $x = -1$ or 7</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>Accept omitting equality sign</p>
<p><u>Solution (3) :</u></p> $(x - 3)^2 - x - 3 - 12 = 0$ <p>Let $x - 3 = u$</p> $u^2 - 12 = u $ $u^4 - 24u^2 + 144 = u^2$ $u^4 - 25u^2 + 144 = 0$ $(u^2 - 9)(u^2 - 16) = 0$ $u = \pm 3 \quad \text{or} \quad u = \pm 4$ $x = 0 \quad \text{or} \quad 6 \quad \text{or} \quad x = -1 \quad \text{or} \quad 7$ <p>Rejecting $x = 0$ and 6, $\therefore x = -1$ or 7</p>	<p>2M</p> <p>1A</p> <p>1A+1A</p> <p>1A+1A</p>	

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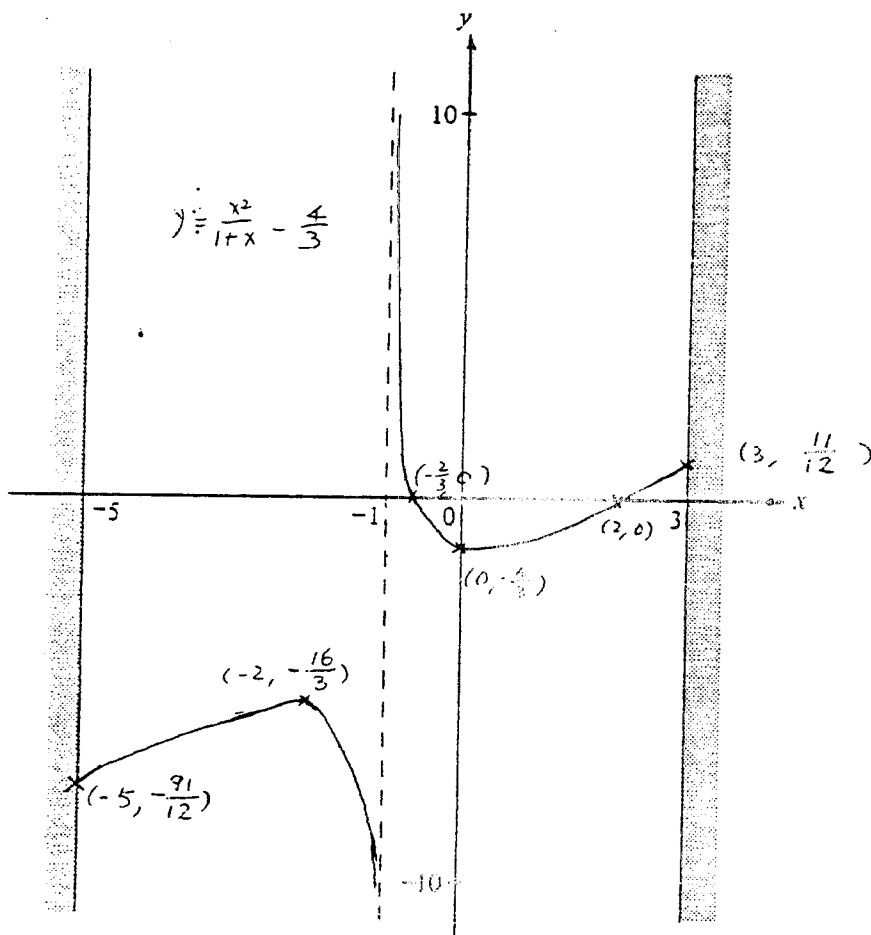
Solution	Marks	Remarks
8. (a) $x = 0$	<u>1A</u> 1	
(b) $x^2 + kx + (2k - 3) = 0$ has no real root. $\Delta = k^2 - 4(2k - 3) < 0$ $k^2 - 8k + 12 < 0$ $(k - 2)(k - 6) < 0$ $2 < k < 6$	1M 1M 1A <u>1</u> 4	Can be omitted
(c) (i) $f'(x) = 3x^2 + 2kx + (2k - 3)$ $\begin{cases} \alpha + \beta = \frac{-2k}{3} \\ \alpha\beta = \frac{2k - 3}{3} \end{cases}$ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ $= \left(\frac{-2k}{3}\right)^2 - 4\left(\frac{2k - 3}{3}\right)$ $= \frac{4}{9}(k^2 - 6k + 9)$ $= \frac{4}{9}(k - 3)^2$ Since $\alpha \neq \beta$, $\therefore k \neq 3$	1A 1M 1A 1A 1A 1	Can be omitted $\Delta = 4k^2 - 12(2k - 3)$ $= 4(k - 3)^2 \dots\dots\dots 1A$ Since $\Delta > 0$, $\therefore k \neq 3 \dots 1$
(ii) $\left \frac{2}{3}(k - 3)\right \leq \frac{2}{3}$ $ k - 3 \leq 1$ $2 \leq k \leq 4$ Combining with $2 < k < 6$, $k \neq 3$ and k is an integer, $k = 4$	1M 1A 1A <u>2A</u> <u>11</u>	or $-1 \leq k - 3 \leq 1$
<u>Alternative solution</u> (c) (ii) $\frac{4}{9}(k^2 - 6k + 9) \leq \frac{4}{9}$ $k^2 - 6k + 8 \leq 0$ $(k - 2)(k - 4) \leq 0$ $2 \leq k \leq 4$ $\therefore k = 4$	1M 1A 1A 2A	

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Solution	Marks	Remarks
<p>9. (a) Put $x = 0$, $y = -\frac{4}{3}$ \therefore y-intercept is $-\frac{4}{3}$</p> <p>Put $y = 0$, $\frac{x^2}{1+x} - \frac{4}{3} = 0$</p> $3x^2 - 4x - 4 = 0$ $x = 2, -\frac{2}{3}$ <p>\therefore x-intercepts are 2 and $-\frac{2}{3}$.</p>	<p>1A</p> <p><u>1A</u></p> <p><u>2</u></p>	
<p>(b) $\frac{dy}{dx} = \frac{2x(1+x) - x^2}{(1+x)^2}$</p> $= \frac{2x + x^2}{(1+x)^2}$ $\frac{d^2y}{dx^2} = \frac{(2 + 2x)(1+x)^2 - 2(1+x)(2x + x^2)}{(1+x)^4}$ $= \frac{2}{(1+x)^3}$	<p>1M</p> <p>1A</p> <p><u>1</u></p> <p><u>3</u></p>	<p>For quotient rule</p>
<p>(c) $\frac{2x + x^2}{(1+x)^2} = 0$</p> <p>$x = 0$ or -2</p> <p>When $x = 0$, $y = -\frac{4}{3}$</p> $\frac{d^2y}{dx^2} = 2 \quad (\text{or } > 0)$ <p>$\therefore (0, -\frac{4}{3})$ is a minimum point</p> <p>When $x = -2$, $y = -\frac{16}{3}$</p> $\frac{d^2y}{dx^2} = -2 \quad (\text{or } < 0)$ <p>$\therefore (-2, -\frac{16}{3})$ is a maximum point</p>	<p>1M</p> <p>1A+1A</p> <p>1M</p> <p>1A</p> <p><u>1A</u></p> <p><u>6</u></p>	<p>No mark if checking is omitted</p> <p>No mark if checking is omitted</p>

Solution	Marks	Remarks
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(d)



(i) For $-5 \leq x < -1$

1A Shape
 $\frac{1}{2}$ A For $(-5, -\frac{91}{12})$.
Accept $-\frac{91}{12} \approx -7.6$

$\frac{1}{2}$ A For $(-2, -\frac{16}{3})$ as a maximum point

(ii) For $-1 < x \leq 3$

1A Shape
 $\frac{1}{2}$ A For $(-\frac{2}{3}, 0)$

$\frac{1}{2}$ A For $(2, 0)$

$\frac{1}{2}$ A For $(3, \frac{11}{12})$.
Accept $\frac{11}{12} \approx 0.92$

$\frac{1}{2}$ A For $(0, -\frac{4}{3})$ as a minimum point

Note : Round up to the nearest mark

5

Solution	Marks	Remarks
10. (a) $\vec{OC} = \frac{1}{3}\vec{a} + \frac{2}{3}\vec{b}$ $\vec{DA} = \vec{OA} - \vec{OD}$ $= \vec{a} - \frac{1}{2}\vec{b}$	1A 1M $\frac{1A}{3}$	Omit vector sign (pp-1) Can be omitted
(b) $\vec{BE} = \vec{OE} - \vec{OB}$ $= (k+1)\vec{OC} - \vec{OB}$ $= \frac{k+1}{3}\vec{a} + \frac{2k-1}{3}\vec{b}$	1A $\frac{1}{2}$	For $\vec{OE} = (k+1)\vec{OC}$ $\frac{\vec{OC}}{\vec{CE}} = \frac{1}{k}$ (pp-1)
<p style="margin: 0;"><u>Alternative solution</u></p> $\vec{BE} = \vec{BC} + \vec{CE}$ $= \frac{1}{3}\vec{BA} + k\vec{OC}$ $= \frac{1}{3}(\vec{a} - \vec{b}) + k(\frac{\vec{a}}{3} + \frac{2\vec{b}}{3})$ $= \frac{k+1}{3}\vec{a} + \frac{2k-1}{3}\vec{b}$		
(c) $\vec{a} - \frac{1}{2}\vec{b} = \lambda(\frac{k+1}{3}\vec{a} + \frac{2k-1}{3}\vec{b})$ $\begin{cases} 1 = \lambda(\frac{k+1}{3}) \\ -\frac{1}{2} = \lambda(\frac{2k-1}{3}) \end{cases}$ $k = \frac{1}{5}$	1M 1M 1A	No mark if λ is omitted No mark if λ is omitted
<p style="margin: 0;"><u>Alternative solution</u></p> $\frac{\frac{k+1}{3}}{1} = \frac{\frac{2k-1}{3}}{-\frac{1}{2}}$ $k = \frac{1}{5}$		
(d) (i) $\vec{a} \cdot \vec{b} = \vec{a} \vec{b} \cos \frac{\pi}{3}$ $= (1)(2) \cos \frac{\pi}{3} = 1$	$\frac{3}{3}$ 1M 1A	Omit dot sign (pp-1)

Solution	Marks	Remarks
(ii) $\vec{BE} \cdot \vec{OE} = 0$ $(\frac{k+1}{3}\vec{a} + \frac{2k-1}{3}\vec{b}) \cdot \frac{k+1}{3}(\vec{a} + 2\vec{b}) = 0$ $\frac{k+1}{9} [(k+1)\vec{a} \cdot \vec{a} + (2k+2+2k-1)$ $\vec{a} \cdot \vec{b} + 2(2k-1)\vec{b} \cdot \vec{b}] = 0$ $k+1 + 4k+1 + 8(2k-1) = 0 \quad (k \neq -1)$ $k = \frac{2}{7}$	1M 1M 1A	 For distribution
For $k = \frac{2}{7}$, $\vec{BE} = \frac{3}{7}\vec{a} - \frac{1}{7}\vec{b}$	1M	Trying to find \vec{BE}
$ \vec{BE} ^2 = \vec{BE} \cdot \vec{BE}$ $= (\frac{3}{7}\vec{a} - \frac{1}{7}\vec{b}) \cdot (\frac{3}{7}\vec{a} - \frac{1}{7}\vec{b})$ $= \frac{9}{49}\vec{a} \cdot \vec{a} - \frac{6}{49}\vec{a} \cdot \vec{b} + \frac{1}{49}\vec{b} \cdot \vec{b}$ $= \frac{9}{49} - \frac{6}{49} + \frac{4}{49} = \frac{1}{7}$	1M	Trying to find $ \vec{BE} $
\therefore Distance of B from OC = $\frac{\sqrt{7}}{7}$.	1A	
	<hr/> 8	

Solution	Marks	Remarks
11. (a) (i) $\frac{z^2}{z} = \frac{r^2(\cos 2\theta + i\sin 2\theta)}{r(\cos\theta - i\sin\theta)}$ $= \frac{r^2(\cos 2\theta + i\sin 2\theta)}{r(\cos(-\theta) + i\sin(-\theta))}$ $= r(\cos 3\theta + i\sin 3\theta)$	1A+1A 1A 1	1A for denominator 1A for numerator For denominator
(ii) $z^2 = i\bar{z}$ $r(\cos 3\theta + i\sin 3\theta) = i$ $= \cos \frac{\pi}{2} + i\sin \frac{\pi}{2}$	1A 1A	Or other equivalent polar form (can be omitted)
$r = 1$ $3\theta = 2n\pi + \frac{\pi}{2}$ $\theta = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ (or $\theta = \frac{2n\pi}{3} + \frac{\pi}{6}$, where $k = -1, 0, 1$)	1A 1A+1A	Any one correct - 1A All correct - 2A
<u>Alternative solution</u> (ii) $r(\cos 3\theta + i\sin 3\theta) = i$ $\begin{cases} r\cos 3\theta = 0 \\ r\sin 3\theta = 1 \end{cases}$ $r^2(\sin^2 3\theta + \cos^2 3\theta) = 1$ $r = 1$ $3\theta = 2n\pi + \frac{\pi}{2}$ $\theta = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$	1A 1A 1A 1A+1A	or $\begin{cases} \cos 3\theta = 0 \\ \sin 3\theta = 1 \end{cases}$ (can be omitted)
	9	

Solution	Marks	Remarks
(b) (i) $w - i = \cos\alpha + i\sin\alpha$ $ w - i = (\sqrt{\cos^2\alpha + \sin^2\alpha}) = 1$	1A 1A	
(ii) Since $(w - i)^2 = i\bar{w} - 1$ $= i(\overline{w - i}),$ $\therefore w - i$ satisfies the equation $z^2 = i\bar{z}$ Using the result of (a), $w - i = \cos\theta + i\sin\theta$, where $\theta = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$	1A 1M	
$w - i = -i, \frac{\sqrt{3}}{2} + \frac{1}{2}i, -\frac{\sqrt{3}}{2} + \frac{1}{2}i$ $w = 0, \frac{\sqrt{3}}{2} + \frac{3}{2}i, -\frac{\sqrt{3}}{2} + \frac{3}{2}i$	1A+1A+1A <u>7</u>	(pp-1) for $w = \cos\alpha + i(1 + \sin\alpha)$ where $\alpha = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

Solution	Marks	Remarks
12. (a) $x = 4\sin\theta$	1A	
$\frac{dx}{dt} = 4\cos\theta \frac{d\theta}{dt}$	1M	For differentiating wrt t
Put $\frac{dx}{dt} = \frac{1}{2}$		
$\frac{d\theta}{dt} = \frac{1}{8\cos\theta}$	<u>1A</u>	
	<u>3</u>	
(b) $y = 4\cos\theta$	1A	
$\frac{dy}{dt} = -4\sin\theta \frac{d\theta}{dt}$		
$= -\frac{\tan\theta}{2}$	1A	
$z = \sqrt{25 - 16\sin^2\theta}$	1M	or $\sqrt{25 - x^2}$
$\frac{dz}{dt} = \frac{-32\sin\theta\cos\theta}{2\sqrt{25 - 16\sin^2\theta}} \frac{d\theta}{dt}$		
$= \frac{-2\sin\theta}{\sqrt{25 - 16\sin^2\theta}}$	1A	
Rate of change $= \frac{dy}{dt} + \frac{dz}{dt}$	1M	
At $\theta = \frac{\pi}{6}$, Rate $= -\frac{\tan\frac{\pi}{6}}{2} - \frac{2\sin\frac{\pi}{6}}{\sqrt{25 - 16\sin^2\frac{\pi}{6}}}$		
$= -0.507 \text{ (m s}^{-1}\text{)}$	<u><u>1A</u></u>	
	<u>6</u>	
(c) Let A be the area of $\triangle OPR$		
$A = \frac{1}{2}xy$		
$= 8\sin\theta\cos\theta$	1A	
$= 4\sin 2\theta$		
A is maximum when $\sin 2\theta = 1$	2M	
$2\theta = \frac{\pi}{2}$		
$\therefore \theta = \frac{\pi}{4}$	1A	

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Solution	Marks	Remarks
<p><u>Alternative solution (1)</u></p> $A = 8\sin\theta\cos\theta$ $\frac{dA}{d\theta} = 8(\cos^2\theta - \sin^2\theta)$ $\frac{dA}{d\theta} = 0, \cos^2\theta - \sin^2\theta = 0$ $\tan^2\theta = 1$ $\theta = \frac{\pi}{4}$ $\frac{d^2A}{d\theta^2} = -32\sin\theta\cos\theta$ $\frac{d^2A}{d\theta^2} < 0 \text{ at } \theta = \frac{\pi}{4}$ <p>$\therefore A$ is maximum when $\theta = \frac{\pi}{4}$</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p>	<p>For checking</p>
<p><u>Alternative solution (2)</u></p> $A = 4\sin 2\theta$ $\frac{dA}{d\theta} = 8\cos 2\theta$ $\frac{dA}{d\theta} = 0, \cos 2\theta = 0$ $\theta = \frac{\pi}{4}$ $\frac{d^2A}{d\theta^2} = -16\sin 2\theta$ $\frac{d^2A}{d\theta^2} < 0 \text{ at } \theta = \frac{\pi}{4}$ <p>$\therefore A$ is a maximum when $\theta = \frac{\pi}{4}$</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p>	<p>For checking</p>
<p>Similarly, area of $\triangle ORQ$ is maximum when</p> $\angle OQR = \frac{\pi}{4}$ <p>By Sine Law,</p> $\frac{\sin \frac{\pi}{4}}{4} = \frac{\sin \theta}{5}$ $\theta = 1.08$	<p>1M</p> <p>1M</p> <p><u>1A</u></p> <p><u>7</u></p>	