Solution	Marks	Remarks
1. (a) $(\sqrt{2(x+\Delta x)}-\sqrt{2x})(\sqrt{2(x+\Delta x)}+\sqrt{2x})$		
$= 2(x+\Delta x) - 2x$		
$= 2\Delta x$	1A	
(b) $\frac{d}{dx}\sqrt{2x} = \lim_{\Delta x\to 0}\frac{1}{\Delta x}(\sqrt{2(x+\Delta x)}-\sqrt{2x})$	1A	
$= \lim_{\Delta x\to 0}\frac{1}{\Delta x}(\sqrt{2(x+\Delta x)}-\sqrt{2x})\cdot\frac{\sqrt{2(x+\Delta x)}+\sqrt{2x}}{\sqrt{2(x+\Delta x)}+\sqrt{2x}}$	1M	
$= \lim_{\Delta x\to 0}\frac{1}{\Delta x}\cdot\frac{2\Delta x}{\sqrt{2(x+\Delta x)}+\sqrt{2x}}$		
$= \lim_{\Delta x\to 0}\frac{2}{\sqrt{2(x+\Delta x)}+\sqrt{2x}}$	1A	
$= \frac{1}{\sqrt{2x}}$	1A	
	5	
2. (a) $\frac{50}{4+3i} = \frac{50}{4+3i}\left(\frac{4-3i}{4-3i}\right)$	1M	
$= 8 - 6i$	1A	
(b) $5z + 3\bar{z} = \frac{50}{4+3i}$		
$5(a+bi) + 3(a-bi) = 8 - 6i$	1A	For $\bar{z} = a - bi$
$\begin{cases} 5a + 3a = 8 \\ 5b - 3b = -6 \end{cases}$	1M	
$\therefore\ a = 1,\quad b = -3$	1A	
$z = 1 - 3i$		
	5	
3. $\alpha + \beta = -p,\quad \alpha\beta = q$	1A	
$-q = (\alpha + 3) + (\beta + 3)$	1M	
$= (\alpha + \beta) + 6$		
$-q = -p + 6$	1A	
$p = (\alpha + 3)(\beta + 3)$	1M	
$= \alpha\beta + 3(\alpha + \beta) + 9$		
$p = q - 3p + 9$	1A	
Solving the equations, $p = 1,\ q = -5$	1A	
	6	

Solution	Marks	Remarks
Alternative solution		
α, β are the roots of $(x+3)^2 + q(x+3) + p = 0$	1M	
$x^2 + (q+6)x + (p+3q+9) = 0$	1A	
Comparing coefficient with $x^2 + px + q = 0$	1M	
$\begin{cases} p = q + 6 \\ q = p + 3q + 9 \end{cases}$	1A+1A	
Solving the equations, $p = 1$, $q = -5$.	1A	
4. $\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3}$		
$= \frac{\sqrt{3}}{2} - \frac{1}{2}i$	1A	
$= \cos\frac{\pi}{6} - i\sin\frac{\pi}{6}$	1A	(can be omitted)
$= \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$	1A	or $\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}$ etc. Accept degree measures
Alternative solution		
$\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3} = \cos(\frac{\pi}{2} - \frac{2\pi}{3}) + i\sin(\frac{\pi}{2} - \frac{2\pi}{3})$	1A	
$= \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$	2A	
$\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3} = \sin(\frac{\pi}{2} + \frac{\pi}{6}) + i\cos(\frac{\pi}{2} + \frac{\pi}{6})$	1A	
$= \cos\frac{\pi}{6} - i\sin\frac{\pi}{6}$	1A	(can be omitted)
$= \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$	1A	
$(\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3})^{\frac{1}{3}} = [\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})]^{\frac{1}{3}}$		
$= \cos\frac{2k\pi - \frac{\pi}{6}}{3} + i\sin\frac{2k\pi - \frac{\pi}{6}}{3}$,	1M+1A	1M for De Moivre's Theorem; 1A if others correct
where $k = -1, 0, 1$,	1A	or $k = 0, 1, 2$ or etc.
OR $(\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3})^{\frac{1}{3}} = \cos(-\frac{\pi}{18}) + i\sin(-\frac{\pi}{18})$,	1A	
$\cos\frac{11\pi}{18} + i\sin\frac{11\pi}{18}$,	1A	
$\cos(-\frac{13\pi}{18}) + i\sin(-\frac{13\pi}{18})$	1A	
(or $\cos\frac{23\pi}{18} + i\sin\frac{23\pi}{18}$)		
	6	

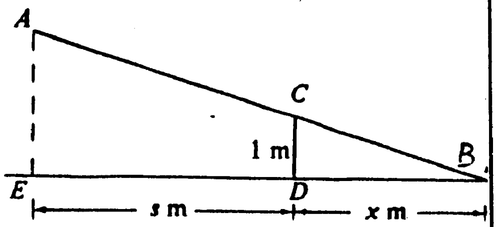
Solution	Marks	Remarks
5. $\lvert -x^2 + 2x + 3 \rvert \geq 5$		
$-x^2 + 2x + 3 \geq 5$ or $-x^2 + 2x + 3 \leq -5$	2A	use 'and' or ' ,,, and' (no mark)
$x^2 - 2x + 2 \leq 0$ or $x^2 - 2x - 8 \geq 0$		
$(x - 1)^2 + 1 \leq 0$ or $(x + 2)(x - 4) \geq 0$	1A	For factorisation
No solution or $x \geq 4$ or $x \leq -2$	1A+1A	
$\therefore x \leq -2$ or $x \geq 4$	1A	cannot omit 'or'
Alternative solution (1)		
$(-x^2 + 2x + 3)^2 \geq 5^2$	1A	
$(-x^2 + 2x + 3 + 5)(-x^2 + 2x + 3 - 5) \geq 0$	1A	
$(-x^2 + 2x + 8)(-x^2 + 2x - 2) \geq 0$		
$(x^2 - 2x + 2)(x^2 - 2x - 8) \geq 0$		
$\because x^2 - 2x + 2 = (x - 1)^2 + 1 > 0$	1M	
$(x^2 - 2x - 8) \geq 0$	1A	
$(x - 4)(x + 2) \geq 0$	1A	
$x \geq 4$ or $x \leq -2$	1A	
Alternative solution (2)		
$\lvert (x + 1)(x - 3) \rvert \geq 5$		
Consider the following cases :	1M	
case 1 : $x \geq 3$		
$(x + 1)(x - 3) \geq 5$		
$(x - 4)(x + 2) \geq 0$		
$x \geq 4$ or $x \leq -2$		
since $x \geq 3$, $\therefore x \geq 4$	1A	
case 2 : $-1 < x < 3$		
$-(x + 1)(x - 3) \geq 5$		
$x^2 - 2x + 2 \leq 0$		
$(x - 1)^2 + 1 \leq 0$		
no solution	1A	
case 3 : $x \leq -1$		
$(x + 1)(x - 3) \geq 5$		
$x \geq 4$ or $x \leq -2$		
since $x \leq -1$, $\therefore x \leq -2$	1A	
Combining the 3 cases,		
$x \leq -2$ or $x \geq 4$	2A	
	6	

	Solution	Marks	Remarks
6.	(a) $\vec{AB} = \vec{OB} - \vec{OA}$	1A	Omit vector sign (pp-1)
	$= -2\vec{i} + 3\vec{j}$	1A	
	(b) $\vec{AB} \cdot \vec{AB} = (-2)^2 + 3^2$	1M	Omit dot sign (pp-1)
	$= 13$	1A	
	or $\vec{AB} \cdot \vec{AB} = (-2i + 3j) \cdot (-2i + 3j)$	1M	
	$= 4 + 9 = 13$	1A	
	Since $AB \perp BC$, $\vec{AB} \cdot \vec{BC} = 0$	1A	
	$\vec{AB} \cdot \vec{AC} = \vec{AB} \cdot (\vec{AB} + \vec{BC})$	1M	For $\vec{AC} = \vec{AB} + \vec{BC}$
	$= \vec{AB} \cdot \vec{AB} + \vec{AB} \cdot \vec{BC}$		
	$= 13$	1A	
		7	
7.	(a) $2x - 2y^2 - 4xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$	1A+1A	1A for $\frac{d}{dx}(-2xy^2)$ 1A for other terms
	$\frac{dy}{dx} = \frac{2x - 2y^2}{4xy - 3y^2}$	1A	
	(b) At $(2, -1)$, $\frac{dy}{dx} = \frac{2(2) - 2(-1)^2}{4(2)(-1) - 3(-1)^2}$	1M	
	$= -\frac{2}{11}$	1A	
	Equation of tangent is		
	$\frac{y + 1}{x - 2} = -\frac{2}{11}$	1M	
	$2x + 11y + 7 = 0$	1A	or $y = -\frac{2}{11}x - \frac{7}{11}$
		7	

Solution	Marks	Remarks
8. (a) $\overrightarrow{OP} = \dfrac{\vec{a} + r\vec{b}}{1+r}$	1A	Omit vector sign (pp-1)
$\overrightarrow{OQ} = \dfrac{\overrightarrow{OP} + r\overrightarrow{OB}}{1+r}$	1A	
$= \dfrac{\frac{1}{1+r}(\vec{a} + r\vec{b}) + r\vec{b}}{1+r}$		
$= \dfrac{\vec{a} + (r^2 + 2r)\vec{b}}{(1+r)^2}$	1A	
<u>Alternative solutions for $\overrightarrow{OQ}$</u>		
$\overrightarrow{PQ} = \dfrac{r}{r^2 + 2r + 1}\overrightarrow{AB}$	1A	
$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$		
$= \dfrac{\vec{a} + r\vec{b}}{1+r} + \dfrac{r}{(1+r)^2}(\vec{b} - \vec{a})$		
$= \dfrac{\vec{a} + (r^2 + 2r)\vec{b}}{(1+r)^2}$	1A	
$AQ : QB = (r^2 + 2r) : 1$	1A	
$\overrightarrow{OQ} = \dfrac{\vec{a} + (r^2 + 2r)\vec{b}}{(1+r)^2}$	1A	
	3	
(b) $\overrightarrow{OT} = \dfrac{1}{1+r}\vec{b}$	1A	
$\overrightarrow{TQ} = \overrightarrow{OQ} - \overrightarrow{OT}$		
$= \dfrac{\vec{a} + (r^2 + 2r)\vec{b}}{(1+r)^2} - \dfrac{1}{(1+r)}\vec{b}$	1M	
$= \dfrac{\vec{a} + (r^2 + r - 1)\vec{b}}{(1+r)^2}$	1	
	3	
(c) Since $\overrightarrow{OA} \parallel \overrightarrow{TQ}$,		
$r^2 + r - 1 = 0$	2M	or $\dfrac{r^2 + r - 1}{(1+r)^2} = 0$
$r = \dfrac{-1 \pm \sqrt{5}}{2}$		
Since $r > 0$, $\therefore r = \dfrac{-1 + \sqrt{5}}{2}$	1A	
	3	

Solution	Marks	Remarks
Alternative solution		
$AQ : QB = (r^2 + 2r) : 1$		
$OT : TB = 1 : r$		
Since $\overrightarrow{OA} \parallel \overrightarrow{TB}$,		
$\frac{r^2 + 2r}{1} = \frac{1}{r}$	1M+1A	
$r^3 + 2r^2 - 1 = 0$		
$(r + 1)(r^2 + r - 1) = 0$		
$r = -1, \frac{-1 \pm \sqrt{5}}{2}$		
Since $r > 0$, $\therefore r = \frac{-1 + \sqrt{5}}{2}$	1A	
(d) (i) $\vec{a} \cdot \vec{a} = 4$	1A	Omit dot sign (pp-1)
$\vec{a} \cdot \vec{b} = 2(16)(\cos\frac{\pi}{3})$	1M	
$= 16$	1A	
(ii) $\overrightarrow{OA} \cdot \overrightarrow{TQ} = 0$	1M	
$\vec{a} \cdot [\frac{\vec{a} + (r^2 + r - 1)\vec{b}}{(1 + r)^2}] = 0$		
$\frac{1}{(1 + r)^2}[\vec{a} \cdot \vec{a} + (r^2 + r - 1)\vec{a} \cdot \vec{b}] = 0$		
$\frac{1}{(1 + r)^2}[4 + (r^2 + r - 1)16] = 0$	1M	
$16r^2 + 16r - 12 = 0$	1A	
$r = \frac{1}{2}$ or $-\frac{3}{2}$ (rejected)		
$\therefore r = \frac{1}{2}$	1A	
	7	

Solution	Marks	Remarks
9. (a) $\Delta BCD \sim \Delta BAE$		
$\frac{BC}{BA} = \frac{BD}{BE}$	1M	
$\frac{\sqrt{x^2+1}}{8} = \frac{x}{x+s}$	1A	
$s = \frac{8x}{\sqrt{1+x^2}} - x$	1	
	3	
(b) $\frac{ds}{dx} = \frac{8\sqrt{1+x^2} - \frac{8x^2}{\sqrt{1+x^2}}}{1+x^2} - 1$	1M+1A	1M for quotient rule
$= \frac{8}{(1+x^2)^{3/2}} - 1$		
$\frac{ds}{dx} = 0$	1M	
$(1+x^2)^{3/2} = 8$		
$x = \pm\sqrt{3}$		
Since $x > 0$, $\therefore x = \sqrt{3}$	1A	
$\frac{d^2s}{dx^2} = \frac{-24x}{(1+x^2)^{5/2}}$	1A	When $0 < x < \sqrt{3}$, $\frac{ds}{dx} > 0$; When $\sqrt{3} < x\ 3\sqrt{7}$, $\frac{ds}{dx} < 0$ } 1A
At $s = \sqrt{3}$, $\frac{d^2s}{dx^2}\ (= -\frac{3\sqrt{3}}{4}) < 0$ $\therefore s$ is a maximum	1M	$\therefore s$ is a maximum 1M
$s_{max} = \frac{8\sqrt{3}}{\sqrt{1+3}} - \sqrt{3} = 3\sqrt{3}$	1A	Awarded if checking is omitted
	7	
(c) (i) P = Area of ΔABE − area of ΔCBD		
$= \frac{1}{2}(s+x)(8)\sin\angle CBD - \frac{x}{2}$	1M	
$= \frac{1}{2}(s+x)(8)\cdot\frac{1}{\sqrt{1+x^2}} - \frac{x}{2}$	1A	
$= \frac{1}{2}\left(\frac{8x}{\sqrt{1+x^2}} - x + x\right)\cdot\frac{8}{\sqrt{1+x^2}} - \frac{x}{2}$		
$= \frac{32x}{1+x^2} - \frac{x}{2}$	1	



Solution	Marks	Remarks
Alternative solution		
Area of ΔABE : Area of ΔCBD		
$= (s + x)^2 : x^2$	1M	
$= \dfrac{64x^2}{1 + x^2} : x^2$		
Area of $\Delta ABE = \dfrac{32x}{1 + x^2}$	1A	
Area of $\Delta CBD = \dfrac{x}{2}$		
$\therefore P = \dfrac{32x}{1 + x^2} - \dfrac{x}{2}$	1	
$P = \dfrac{1}{2}(1 + AE)s$	1M	
$= \dfrac{1}{2}(1 + \dfrac{s + x}{x})s$		
$= \dfrac{1}{2}(1 + \dfrac{8}{\sqrt{1 + x^2}})(\dfrac{8x}{\sqrt{1 + x^2}} - x)$	1A	
$= \dfrac{x}{2}(1 + \dfrac{8}{\sqrt{1 + x^2}})(\dfrac{8}{\sqrt{1 + x^2}} - 1)$		
$= \dfrac{32x}{1 + x^2} - \dfrac{x}{2}$	1	
(ii) $\dfrac{dp}{dx} = \dfrac{32(1 + x^2) - 32x(2x)}{(1 + x^2)^2} - \dfrac{1}{2}$	1M	
$= \dfrac{32(1 - x^2)}{(1 + x^2)^2} - \dfrac{1}{2}$		
From (b), s attains its maximum at $x = \sqrt{3}$		
At $x = \sqrt{3}$, $\dfrac{dp}{dx} = -\dfrac{9}{2}$	1A	
Since $\dfrac{dp}{dx} \neq 0$ at $x = \sqrt{3}$, P does not attain a maximum when s attains its maximum	1	
	6	

Solution	Marks	Remarks
10. (a) Let α, β be the roots of the equation		
$\frac{1}{k+1}[2x^2 + (k+7)x + 4] = 0$		
$\begin{cases} \alpha + \beta = -\frac{k+7}{2} \\ \alpha\beta = 2 \end{cases}$	1A	
$PQ = \lvert \alpha - \beta \rvert = 1$	1M	
$(\alpha - \beta)^2 = 1$		
$(\alpha + \beta)^2 - 4\alpha\beta = 1$	1M	
$(-\frac{k+7}{2})^2 - 8 = 1$	1A	
Alternative solution		
$\frac{1}{k+1}[2x^2 + (k+7)x + 4] = 0$		
$x = \frac{-(k+7) \pm \sqrt{(k+7)^2 - 32}}{4}$	1A	
$PQ = \frac{-(k+7) + \sqrt{(k+7)^2 - 32}}{4} - \frac{-(k+7) - \sqrt{(k+7)^2 - 32}}{4}$		2M
$1 = \frac{\sqrt{(k+7)^2 - 32}}{2}$	1A	
$k^2 + 14k + 13 = 0$	1A	
$k = -1$ or -13		(can be omitted)
$\because k \neq -1, \quad \therefore k = -13$	1A 6	
(b) Discriminant $= \frac{(k+7)^2 - 32}{(k+1)^2} < 0$	1M+1A	Accept $\frac{(k+7)^2}{4} - 8 < 0$, $(k+7)^2 - 32 < 0$
$-7 - 4\sqrt{2} < k < -7 + 4\sqrt{2}$	2A 4	

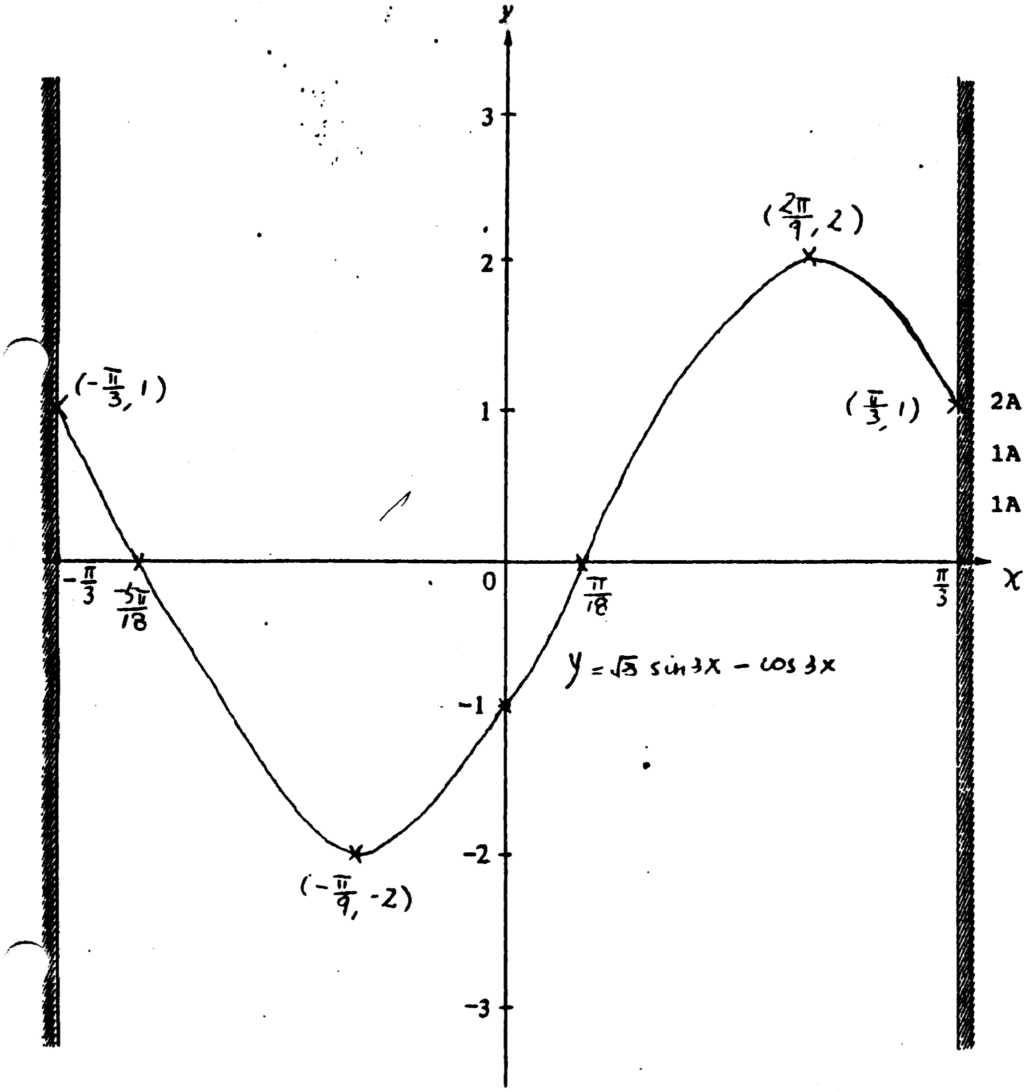
Solution	Marks	Remarks
(c) Put $k = 0$, C becomes $y = 2x^2 + 7x + 4$ .... (1)	2M	or any other values
$k = 1$, C becomes $y = x^2 + 4x + 2$ .... (2)		
Solving (1) and (2), the points of intersection are		
$(-1, -1)$ and $(-2, -2)$	1A+1A	
Put $x = -1$, $y = -1$ into C		
LHS $= y = -1$		
RHS $= \frac{1}{k+1}[2 + (k + 7)(-1) + 4] = -1 =$ LHS $\forall k \neq -1$	1	
Put $x = -2$, $y = -2$ into C		
LHS $= -2$		
RHS $= \frac{1}{k+1}[8 + (k + 7)(-2) + 4] = -2 =$ LHS $\forall k \neq -1$	1	
$\therefore C$ always passes through $(-1, -1)$ and $(-2, -2)$	6	
Alternative solution		
(c) $y = \frac{1}{k+1}[2x^2 + (k + 7)x + 4]$		
$(k + 1)y = 2x^2 + (k + 7)x + 4$		
$(y - 2x^2 - 7x - 4) + k(y - x) = 0$	1M+1A	
C always passes through the intersection points of the 2 curves $y - 2x^2 - 7x - 4 = 0$ and $y - x = 0$	2M	
$\begin{cases} y - 2x^2 - 7x - 4 = 0 \\ y - x = 0 \end{cases}$		
Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$	1A+1A	
(c) Let k_1, k_2 be two distinct values of k		
$\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 \ldots (1) \\ (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 \ldots (2) \end{cases}$	1M	
(1) − (2) : $(k_1 - k_2)y = (k_1 - k_2)x$	1M	
$y = x$ (since $k_1 \neq k_2$)	1A	
Subs. into (1) :		
$(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$	1M	
$2x^2 + 6x + 4 = 0$		
$x = -1$ or -2		
when $x = -1$, $y = -1$	1A	
$y = -2$, $y = -2$	1A	
$\therefore C$ always passes through 2 fixed points whose coordinates are $(-1, -1)$ and $(-2, -2)$		

p.12

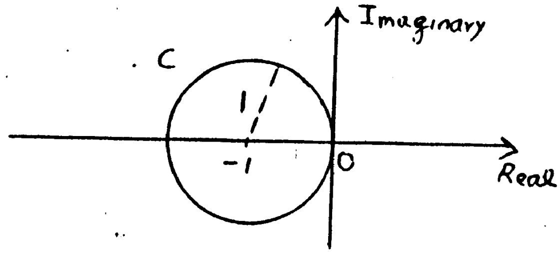
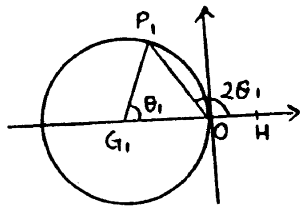
Solution	Marks	Remarks
11. (a) Put $x = 0$, $f(x) = -1$	1A	
$\therefore$ The y-intercept is -1.		
Put $f(x) = 0$, $\sqrt{3}\sin 3x - \cos 3x = 0$		
$\tan 3x = \frac{1}{\sqrt{3}}$	1A	
$x = \frac{\pi}{18}$ or $\frac{-5\pi}{18}$	1A+1A	No mark for degrees
$\therefore$ The x-intercepts are $\frac{\pi}{18}$ and $\frac{-5\pi}{18}$		
	4	
(b) $f'(x) = 3\sqrt{3}\cos 3x + 3\sin 3x$	1A	
$f''(x) = -9\sqrt{3}\sin 3x + 9\cos 3x$	1A	
	2	
(c) $f'(x) = 3\sqrt{3}\cos 3x + 3\sin 3x = 0$	1M	
$\tan 3x = -\sqrt{3}$		
$x = \frac{2\pi}{9}$ or $-\frac{\pi}{9}$	1A+1A	
$f''(-\frac{\pi}{9})$ $(= 18) > 0$ $\therefore$ it is a minimum	1M	
The minimum point is $(-\frac{\pi}{9}, -2)$	1A	
$f''(\frac{2\pi}{9})$ $(= -18) < 0$ $\therefore$ it is a maximum		
The maximum point is $(\frac{2\pi}{9}, 2)$	1A	
	6	

Solution	Marks	Remarks
Alternative solution		
(a) $f(x) = 2\sin(3x - \frac{\pi}{6})$		or $f(x) = -2\cos(3x + \frac{\pi}{3})$
$f(0) = -1$	1A	
∴ The y-intercept is -1		
Put $f(x) = 0$, $\sin(3x - \frac{\pi}{6}) = 0$	1A	or $\cos(3x + \frac{\pi}{3}) = 0$
$x = \frac{\pi}{18}$ or $\frac{-5\pi}{18}$		
∴ The x-intercepts are $\frac{\pi}{18}$ or $\frac{-5\pi}{18}$	1A+1A	
	4	
(b) $f'(x) = 6\cos(3x - \frac{\pi}{6})$	1A	
$f''(x) = -18\sin(3x - \frac{\pi}{6})$	1A	
	2	
(c) $f'(x) = 6\cos(3x - \frac{\pi}{6}) = 0$	1M	
$x = \frac{2\pi}{9}$ or $-\frac{\pi}{9}$	1A+1A	
$f''(-\frac{\pi}{9})$ $(= 18) > 0$ ∴ it is a minimum	1M	
The minimum point is $(-\frac{\pi}{9}, -2)$	1A	
$f''(\frac{2\pi}{9})$ $(= -18) < 0$ ∴ it is a maximum		
The maximum point is $(\frac{2\pi}{9}, 2)$	1A	
OR		
$f(x) = 2\sin(3x - \frac{\pi}{6})$		
$f(x)$ is maximum when $\sin(3x - \frac{\pi}{6}) = 1$	1M	
$x = \frac{2\pi}{9}$	1A	
∴ The maximum point is $(\frac{2\pi}{9}, 2)$	1A	
$f(x)$ is minimum when $\sin(3x - \frac{\pi}{6}) = -1$	1M	
$x = -\frac{\pi}{9}$	1A	
∴ The minimum point is $(-\frac{\pi}{9}, -2)$	1A	
	6	

Solution	Marks	Remarks
(d)		



	2A	Shape
	1A	Labelled end points
	1A	Label the turning points and intercepts
	4	

Solution	Marks	Remarks
12.(a) $z + 1 = \cos\theta + i\sin\theta$	1A	
$\lvert z + 1 \rvert \; (= \sqrt{\cos^2\theta + \sin^2\theta}) = 1$	1	
	1A	For circle
	1A	For centre at $z = -1$
	1A	For radius = 1
	5	
(b) $\tan 2\theta_1 = \dfrac{\sin\theta_1}{\cos\theta_1 - 1}$	1A	
$\dfrac{2\sin\theta_1\cos\theta_1}{2\cos^2\theta - 1} = \dfrac{\sin\theta_1}{\cos\theta_1 - 1}$	1M	
$\sin\theta_1(2\cos\theta_1 - 1) = 0$		
$\cos\theta_1 = \frac{1}{2}$ or $\sin\theta = 0$ (rejected $\because 0 < \theta < \frac{\pi}{2}$)	1A+1A	
$\theta_1 = \frac{\pi}{3}$	1A	
Alternative solution		
Let G be the centre of C and H be a point on the positive real axis 		
$\angle OGP_1 = \theta_1$	1A	
$\angle HOP_1 = 2\theta_1$	1A	
Since $GP_1 = GO$, ΔGOP_1 is isosceles.		
$\angle GOP_1 = \dfrac{\pi - \theta_1}{2}$	1A	$\angle OP_1G = \pi - 2\theta_1$
$\dfrac{\pi - \theta_1}{2} + 2\theta_1 = \pi$	1A	$(\pi - 2\theta_1) \times 2 + \theta_1 = \pi$
$\theta_1 = \frac{\pi}{3}$	1A	
$z_1 = \cos\frac{\pi}{3} - 1 + i\sin\frac{\pi}{3}$	1M	
$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$	1A	
	7	

Solution	Marks	Remarks
(c) $z_2 = \cos(\frac{\pi}{3} + \pi) - 1 + i\sin(\frac{\pi}{3} + \pi)$	1M+1M+1A	1M for using C 1M for π
$= -\frac{3}{2} - \frac{\sqrt{3}}{2}i$	1A	
Alternative solutions		
P_1P_2 is a diameter of the circle C.	1A	
Let P_2 represent the complex no. $x + yi$		
$\frac{x - \frac{1}{2}}{2} = -1$, $\frac{y + \frac{\sqrt{3}}{2}}{2} = 0$	2M	
$x = -\frac{3}{2}$ $y = -\frac{\sqrt{3}}{2}$		
$\therefore P_2$ represents $-\frac{3}{2} - \frac{\sqrt{3}}{2}i$	1A	
$\lvert z_2 \rvert = \sqrt{3}$	1A	
$\angle GOP_2 = \frac{\pi}{6}$	1M	
$\text{Arg } z_2 = \frac{\pi}{6} - \pi$	1M	Accept $\pi + \frac{\pi}{6}$
$= \frac{-5\pi}{6}$		$\frac{7\pi}{6}$
$\therefore z_2 = \sqrt{3}(\cos\frac{-5\pi}{6} + i\sin\frac{-5\pi}{6})$		
$= -\frac{3}{2} - \frac{\sqrt{3}}{2}i$	1A	
	4	