93-CE
A MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1993

ADDITIONAL MATHEMATICS PAPER I

8.30 am-10.30 am (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions in Section B.

All working must be clearly shown.

Unless otherwise specified in a question, the **exact values** of numerical answers must be given.

In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\overrightarrow{u}$ in their working.

Section A (42 marks)

Answer **ALL** questions in this section.

1. (a) Simplify $(\sqrt{2(x+\Delta x)} - \sqrt{2x})(\sqrt{2(x+\Delta x)} + \sqrt{2x})$.

   (b) Find $\frac{d}{dx}(\sqrt{2x})$ from first principles.

(5 marks)

2. (a) Express $\frac{50}{4+3i}$ in standard form.

   (b) By putting $z = a + bi$, where a, b are real numbers, solve the equation $5z + 3\bar{z} = \frac{50}{4+3i}$.

(5 marks)

3. α, β are the roots of the equation $x^2 + px + q = 0$ and $\alpha + 3$, $\beta + 3$ are the roots of the equation $x^2 + qx + p = 0$. Find the values of p and q.

(6 marks)

4. Express $\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3}$ in polar form.

   Hence find the three cube roots of $\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3}$, giving your answers in polar form.

(6 marks)

5. Solve $\left| -x^2 + 2x + 3 \right| \geq 5$ for real values of x.

(6 marks)

6. Given $\overrightarrow{OA} = 3\mathbf{i} - 2\mathbf{j}$, $\overrightarrow{OB} = \mathbf{i} + \mathbf{j}$. C is a point such that $\angle ABC$ is a right angle.

   (a) Find $\overrightarrow{AB}$.

   (b) Find $\overrightarrow{AB} \cdot \overrightarrow{AB}$ and $\overrightarrow{AB} \cdot \overrightarrow{BC}$.

   Hence find $\overrightarrow{AB} \cdot \overrightarrow{AC}$.

(7 marks)

7. Given the curve $C : x^2 - 2xy^2 + y^3 + 1 = 0$.

   (a) Find $\frac{dy}{dx}$.

   (b) Find the equation of the tangent to C at the point $(2, -1)$.

(7 marks)

Section B (48 marks)

Answer any **THREE** questions in this section.
Each question carries 16 marks.

8.

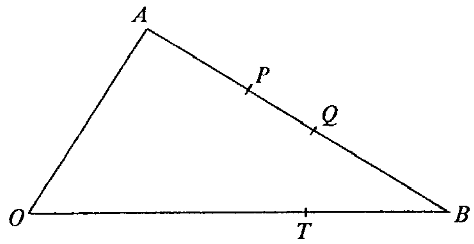

Figure 1

In Figure 1, OAB is a triangle. P , Q are two points on AB such that $AP : PB = PQ : QB = r : 1$, where $r > 0$. T is a point on OB such that $OT : TB = 1 : r$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in terms of r, $\mathbf{a}$ and $\mathbf{b}$.

(3 marks)

(b) Express $\overrightarrow{OT}$ in terms of r and $\mathbf{b}$.

Hence show that $\overrightarrow{TQ} = \dfrac{\mathbf{a} + (r^2 + r - 1)\mathbf{b}}{(r+1)^2}$.

(3 marks)

(c) Find the value(s) of r such that $\overrightarrow{OA}$ is parallel to $\overrightarrow{TQ}$.

(3 marks)

(d) Suppose $OA = 2$, $OB = 16$ and $\angle AOB = \dfrac{\pi}{3}$.

(i) Find $\mathbf{a}\cdot\mathbf{a}$ and $\mathbf{a}\cdot\mathbf{b}$.

(ii) Find the value(s) of r such that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{TQ}$.

(7 marks)

9.

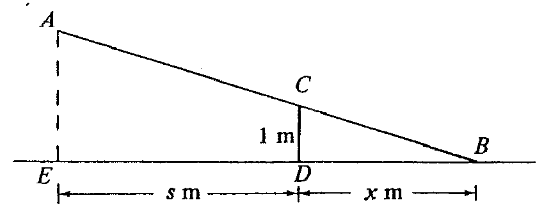

Figure 2

Figure 2 shows a straight rod AB of length 8 m resting on a vertical wall CD of height 1 m. The end B is free to slide along a horizontal rail such that AB is vertically above the rail. Let E be the projection of A on the rail, $DE = s$ m and $BD = x$ m, where $0 < x < 3\sqrt{7}$.

(a) Show that $s = \dfrac{8x}{\sqrt{1+x^2}} - x$.

(3 marks)

(b) Find the maximum value of s .

(7 marks)

(c) Let P m^2 be the area of the trapezium $CAED$.

(i) Show that $P = \dfrac{32x}{1+x^2} - \dfrac{x}{2}$.

(ii) Does P attain a maximum when s attains its maximum ? Explain your answer.

(6 marks)

10. C is the curve $y = \dfrac{1}{k+1}[2x^2 + (k+7)x + 4]$, where k is a real number not equal to -1.

(a) If C cuts the x-axis at two points P and Q and $PQ = 1$, find the value(s) of k.
(6 marks)

(b) Find the range of values of k such that C does not cut the x-axis.
(4 marks)

(c) Show that C always passes through two fixed points for all values of k not equal to -1. What are the coordinates of the two points ?
(6 marks)

11. Let $f(x) = \sqrt{3}\sin 3x - \cos 3x$, where $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$.

(a) Find the x- and y-intercepts of the curve $y = f(x)$.
(4 marks)

(b) Find $f'(x)$ and $f''(x)$.
(2 marks)

(c) Find the turning point(s) of the curve $y = f(x)$. For each point, test whether it is a maximum or a minimum point.
(6 marks)

(d) In Figure 3, sketch the curve $y = f(x)$.
(4 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

If you attempt Question 11, fill in the details in the first three boxes above and tie this sheet into your answer book.

11. (d) **(Continued)**

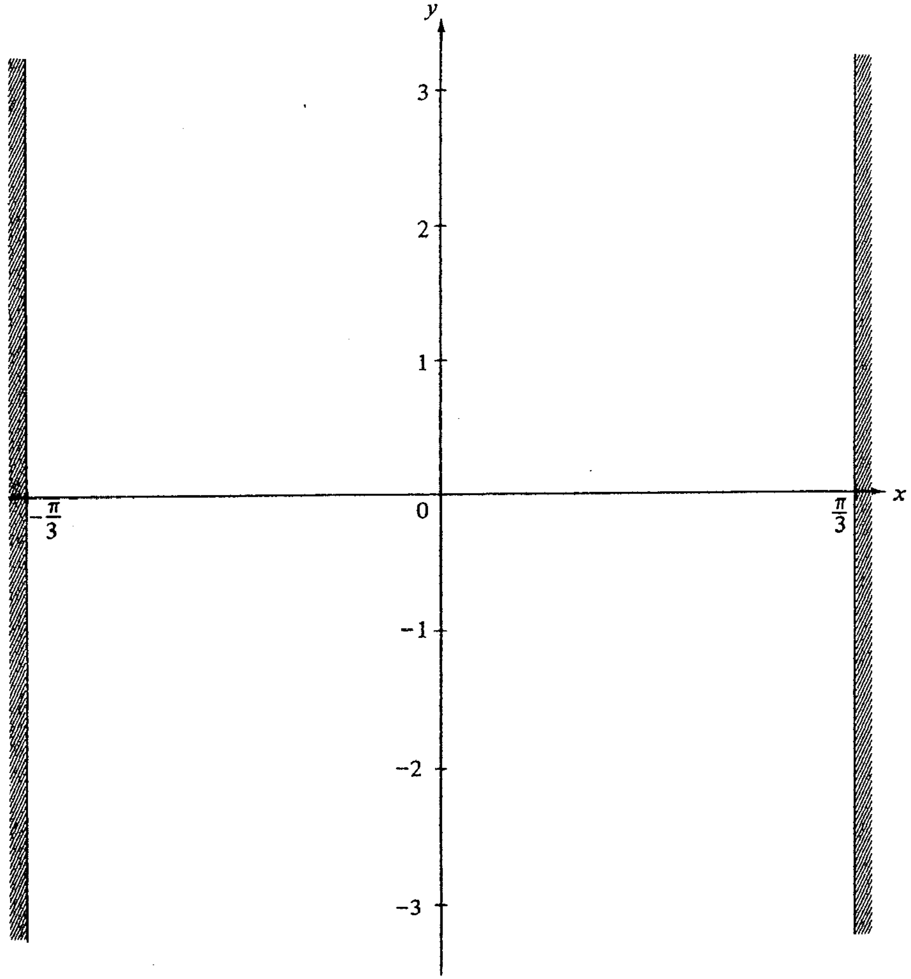

Figure 3

12. Let C be the locus of the point in an Argand diagram representing the complex number $z = (\cos\theta - 1) + i\sin\theta$, where $0 \le \theta < 2\pi$.

(a) Show that $|z + 1| = 1$.
Hence sketch C in an Argand diagram.
(5 marks)

(b) Let P_1 be the point on C representing the complex number $z_1 = (\cos\theta_1 - 1) + i\sin\theta_1$, such that $\arg z_1 = 2\theta_1$, and $0 < \theta_1 < \frac{\pi}{2}$.
Find the value of θ_1 and express z_1 in standard form.
(7 marks)

(c) Let P_2 be the point on C which is farthest away from the point P_1 in (b). Find the complex number represented by P_2 in standard form.
(4 marks)

END OF PAPER

93-CE
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PAPER II

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ADDITIONAL MATHEMATICS PAPER II

11.15 am-1.15 pm (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions in Section B.

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