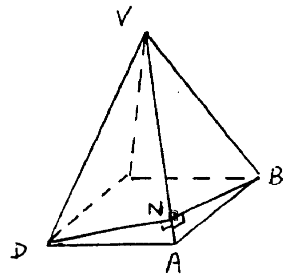
Solution	Marks	Remarks
1. For $n = 1$, L.H.S. $= 1 \times 2 = 2$		
R.H.S. $= 1^2(1 + 1) = 2$	1	
$\therefore$ the statement is true for $n = 1$		
Assume $1 \times 2 + 2 \times 5 + \ldots + k(3k - 1) = k^2(k + 1)$ (for some positive integer k)	1	
Then $1 \times 2 + 2 \times 5 + \ldots + k(3k - 1) + (k + 1)[3(k + 1) - 1]$		
$= k^2(k + 1) + (k + 1)[3(k + 1) - 1]$	1	
$= (k + 1)(k^2 + 3k + 2)$		
$= (k + 1)^2(k + 2)$	1	
$\therefore$ the statement is also true for $n = k + 1$ (if it is true for $n = k$)		
(By the principle of mathematical induction)		
$\therefore$ the statement is true for all +ve integers n.	1 / 5	
2. (a) Coefficient of $x = n + 6$	1A	
$n + 6 = 10$		
$n = 4$	1A	
(b) Coefficient of $x^2 = \dfrac{n(n-1)}{2} + 6n + 9$	2A	Accept nCr notation
$= \dfrac{4(4-1)}{2} + 6(4) + 9$		
$= 39$	1A / 5	
3. (a) $\dfrac{y-7}{x-4} = m$	1A	
$mx - y + (7 - 4m) = 0$		
(b) $\left\lvert \dfrac{7-4m}{\sqrt{m^2+1}} \right\rvert = 1$	1M+1A	Omit absolute sign (PP-1)
$(7 - 4m)^2 = m^2 + 1$		
$15m^2 - 56m + 48 = 0$	1A	
$m = \dfrac{4}{3}$ or $\dfrac{12}{5}$	2A / 6	1A + 1A

Solution	Marks	Remarks
4. (a) $\frac{dy}{dx} = x^2 - 2$	1A	or $y = \int (x^2 - 2)dx$
$y = \frac{1}{3}x^3 - 2x + c$	1A	
Put $x = 3$, $y = 4$	1M	
$c = 1$	1A	
$\therefore y = \frac{1}{3}x^3 - 2x + 1$		
(b) $x^2 - 2 = -2$	1A	
$x = 0$, $y = 1$	1A	
$\therefore$ The coordinates of the point is (0, 1)	6	
5. $\sin 2\theta(4\cos^2\theta - 3) - \sin\theta = 0$		
$2\sin\theta\cos\theta(4\cos^2\theta - 3) - \sin\theta = 0$	1A	For $\sin 2\theta = 2\sin\theta\cos\theta$
$2\sin\theta\cos 3\theta - \sin\theta = 0$	1M	For using the identity
$\sin\theta(2\cos 3\theta - 1) = 0$		
$\sin\theta = 0$ or $\cos 3\theta = \frac{1}{2}$	1A+1A	
$\theta = n\pi$ or $\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$ (n is an integer)	1A+1A	or $180n°$, $120n° \pm 20°$
	6	① P.P-1 for mixed units ② 120n P.P-1
6. (a) $x^3 - x^2 - 2x = 0$	1A	
$x = 0, -1, 2$		
$\therefore a = -1$, $b = 2$	1A	
(b) Area $= \int_{-1}^{0}(x^3 - x^2 - 2x)dx - \int_{0}^{2}(x^3 - x^2 - 2x)dx$	1M+1M	1M for $\int y\,dx$ 試用
		1M for $\int_a^0 - \int_0^b$ or $\int_0^a + \int_0^b$ or $\|\int_a^0\| + \|\int_0^b\|$
$= [\frac{x^4}{4} - \frac{x^3}{3} - x^2]_{-1}^{0} - [\frac{x^4}{4} - \frac{x^3}{3} - x^2]_{0}^{2}$	1A	For correct integration
$= \frac{5}{12} + \frac{8}{3}$		
$= \frac{37}{12}$	1A	
	6	

Solution	Marks	Remarks
7. (a) $BD = \sqrt{6^2 + 6^2}$	1A	
$= \sqrt{72}$		
$\cos\angle VBD = \dfrac{\frac{1}{2}BD}{VB}$	1M	
$= \sqrt{18}/9$		
$\angle VBD = 61.9°$	1A	
(b) Let N be the point on VA such that $DN \perp VA$, $BN \perp VA$. The angle between the 2 planes is $\angle BND$.	1M	$\angle VAB = 70.5°$ $\angle AVB = 38.9°$
$\cos\angle VAB = \dfrac{1}{3}$		
$BN = 6\sin\angle VAB$ (or $9\sin\angle AVB$)	1M	
$= 4\sqrt{2}$	1A	Accept 5.7
$\sin\dfrac{\angle BND}{2} = \dfrac{\frac{1}{2}BD}{BN}$	1M	
$= \dfrac{\sqrt{18}}{4\sqrt{2}} = \dfrac{3}{4}$		
$\angle BND = 97.2°$	1A 8	

Alternative solution

Solution	Marks	Remarks
(b) Let N be the point on VA such that $DN \perp VA$, $BN \perp VA$. The angle between the 2 planes is $\angle BND$.	1M	
$BN = DN = 4\sqrt{2}$	1M+1A	Accept 5.7
$\cos\angle BND = \dfrac{BN^2 + DN^2 - BD^2}{2BN.DN}$	1M	
$= \dfrac{(4\sqrt{2})^2 + (4\sqrt{2})^2 - (\sqrt{72})^2}{2(4\sqrt{2})(4\sqrt{2})}$		
$= -0.125$		
$\angle BND = 97.2°$	1A	

Solution	Marks	Remarks
8. (a) $\frac{dy}{dx} = \frac{(2+\cos x)\cos x + \sin^2 x}{(2+\cos x)^2}$	1M+1A	1M for quotient rule or product rule
$= \frac{2\cos x + 1}{(2+\cos x)^2}$	1A	
$= \frac{(2\cos x + 4) - 3}{(2+\cos x)^2}$		
$= \frac{2}{2+\cos x} - \frac{3}{(2+\cos x)^2}$	1	
	4	
(b) $dt = \sqrt{3}\sec^2\theta d\theta$	1A	
$\int_0^1 \frac{dt}{t^2+3} = \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}}{3} d\theta$	1A+1A	1A for limits 1A for integrand
$= \frac{\sqrt{3}\pi}{18}$	1A	Accept $\frac{\pi}{6\sqrt{3}}$
	4	
(c) $dx = \frac{2dt}{1+t^2}$	1A	or $dt = \frac{1}{2}\sec^2\frac{x}{2}dx$
Since $\cos x = \frac{1-t^2}{1+t^2}$,	1A	
$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{2+\cos x} = \int_0^1 \frac{2}{t^2+3}dt$	1A	
$= \frac{\sqrt{3}\pi}{9}$	1A	Accept $\frac{\pi}{3\sqrt{3}}$
	4	
(d) $\frac{dy}{dx} = \frac{2}{2+\cos x} - \frac{3}{(2+\cos x)^2}$ Integrating with respect to x,		
$\int_0^{\frac{\pi}{2}} [\frac{2}{2+\cos x} - \frac{3}{(2+\cos x)^2}]dx = [\frac{\sin x}{2+\cos x}]_0^{\frac{\pi}{2}}$	1M+1A	1M for integrating both sides, (pp-1) for omitting limits
$= \frac{1}{2}$	1A	
$2\int_0^{\frac{\pi}{2}} \frac{dx}{2+\cos x} - 3\int_0^{\frac{\pi}{2}} \frac{dx}{(2+\cos x)^2} = \frac{1}{2}$		
$\int_0^{\frac{\pi}{2}} \frac{dx}{(2+\cos x)^2} = \frac{2\sqrt{3}\pi}{27} - \frac{1}{6}$	1A	Accept $\frac{2\pi}{9\sqrt{3}} - \frac{1}{6}$
	4	

Solution	Marks	Remarks
9. (a) Substitute $y = mx + c$ into E,		
$16x^2 + 25(mx + c)^2 = 400$	1M	
$(25m^2 + 16)x^2 + 50mcx + 25c^2 - 400 = 0$	1A	
Since L is a tangent to E,		
$(50\ mc)^2 - 4(25m^2 + 16)\ (25c^2 - 400) = 0$	1M	
$(50mc)^2 - 4[(25mc)^2 - 400(25m^2) + 400c^2 - 400(16)] = 0$		
$c^2 = 25m^2 + 16$	1	
	4	
(b) Substitute (h, k) into L		
$c = k - mh$	1A	
Substitute into (a),		
$(k - mh)^2 = 25m^2 + 16$	1M	
$(h^2 - 25)m^2 - 2hkm + (k^2 - 16) = 0$	1	
	3	
(c) Put $h = 7$, $k = 4$		
$24m^2 - 56m + 0 = 0$	1M	
$m = 0$ or $\frac{7}{3}$	1A+1A	
$m = 0$: The equation of tangent is $y = 4$	1A	
$m = \frac{7}{3}$: The equation of tangent is $\frac{y-4}{x-7} = \frac{7}{3}$		
$7x - 3y - 37 = 0$	1A	$y = \frac{7}{3}x - \frac{37}{3}$
	5	
(d) Let $p(h, k)$ be a point on the locus		
$\frac{k^2 - 16}{h^2 - 25} = -1$	1M+2A	1M for $m_1 m_2 = -1$
$h^2 + k^2 - 41 = 0$	1A	
$\therefore$ The equation of the locus is $x^2 + y^2 - 41 = 0$.		
	4	

Alternative solution

(d) $(x^2 - 25)m^2 - 2xym + (y^2 - 16) = 0$		
$m = \frac{2xy \pm \sqrt{4(x^2 - 25)(y^2 - 16)}}{2(x^2 - 25)}$	1A	
$m_1 m_2 = -1$	1M	
$\frac{2xy + \sqrt{4(x^2 - 25)(y^2 - 16)}}{2(x^2 - 25)} \cdot \frac{2xy - \sqrt{4(x^2 - 25)(y^2 - 16)}}{2(x^2 - 25)} = -1$		
$(x^2 - 25)(x^2 + y^2 - 41) = 0$		
$x^2 - 25 = 0$ (rejected), $x^2 + y^2 - 41 = 0$	2A	

Solution	Marks	Remarks
10. (a) $x^2 + y^2 - 2y - 4 = 0$		
$x^2 + (y - 1)^2 = 5$		
C_1 is centred at (0, 1) with radius $\sqrt{5}$	1A	
Distance between centres		
$= \sqrt{(8-0)^2 + (5-1)^2} = 4\sqrt{5}$		
Radius of circle $= 4\sqrt{5} \pm \sqrt{5}$	1M+1M	1M for +, 1M for −
$= 3\sqrt{5}$ or $5\sqrt{5}$	1A+1A	
$\therefore$ Equations of circles are		
$(x-8)^2 + (y-5)^2 = 45$	1A	$x^2 + y^2 - 16x - 10y + 44 = 0$
$(x-8)^2 + (y-5)^2 = 125$	1A 7	$x^2 + y^2 - 16x - 10y - 36 = 0$
Alternative solution		
Let the equation of circle be		
$x^2 + y^2 - 16x - 10y + k = 0$	1A	
$C_1 : x^2 + y^2 - 2y - 4 = 0$		
$\therefore 16x + 8y - 4 - k = 0$	1A	
$\left(\frac{k + 4 - 8y}{16}\right)^2 + y^2 - 2y - 4 = 0$	1M	
$320y^2 - 16(36 + k)y + (k^2 + 8k - 1008) = 0$		
$256(36 + k)^2 - 4(320)(k^2 + 8k - 1008) = 0$	1M	
$k^2 - 8k - 1584 = 0$	1A	
$k = 44$ or -36	1A+1A	
Equations of circle are		
$x^2 + y^2 - 16x - 10y + 44 = 0$		
$x^2 + y^2 - 16x - 10y - 36 = 0$		
(b) $2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx} = 0$		
At $(-1, 3)$, $\frac{dy}{dx} = \frac{-2x}{(2y-2)} = \frac{1}{2}$	1A	
The equation is		
$\frac{y-3}{x+1} = \frac{1}{2}$		
$x - 2y + 7 = 0$	1A 2	
Alternative solutions		
(1) By using the formula $xx_1 + yy_1 - (y + y_1) - 4 = 0$		
The equation is $x(-1) + y(3) - (y + 3) - 4 = 0$	1A	
$x - 2y + 7 = 0$	1A	
(2) Slope of $L_1 = -1 \Big/ \frac{3-1}{-1-0} = \frac{1}{2}$	1A	
The equation is $x - 2y + 7 = 0$	1A	

Solution	Marks	Remarks
(c) (i) $x^2 + y^2 - 2y - 4 + k(x - 2) = 0$ (k is a constant)	2A	Accept $(x - 2) + k(x^2 + \ldots) = 0$
(ii) Substitute L_1 into F,		
$(2y - 7)^2 + y^2 - 2y - 4 + k(2y - 7) - 2k = 0$	1M	
$5y^2 + (2k - 30)y + (45 - 9k) = 0$	1A	
$(2k - 30)^2 - 20(45 - 9k) = 0$	1M	
$4k^2 + 60k = 0$		
$k = -15$ ($k = 0$ (rejected))	1A	
$\therefore$ Equation of C_2 is $x^2 + y^2 - 15x - 2y + 26 = 0$	1A	
	7	
Alternative solutions for (ii)		
(1) Substitute $y = \frac{1}{2}(x + 7)$	1M	
$5x^2 + (4k + 10)x + (5 - 8k) = 0$	1A	
$(4k + 10)^2 - 20(5 - 8k) = 0$	1M	
$4k^2 + 60k = 0$		
(2) $x^2 + y^2 + kx - 2y = 2k + 4$		
$(x + \frac{k}{2})^2 + (y - 1)^2 = \frac{k^2}{4} + 2k + 5$		
centre is $(-\frac{k}{2}, 1)$, radius $= \sqrt{\frac{k^2}{4} + 2k + 5}$	1M	
If L_1 is tangent to a circle in F,		
$\left\lvert \frac{-\frac{k}{2} - 2 + 7}{\sqrt{5}} \right\rvert = \sqrt{\frac{k^2}{4} + 2k + 5}$	1M+1A	
$(5 - \frac{k}{2})^2 = 5(\frac{k^2}{4} + 2k + 5)$		
$4k^2 + 60k = 0$		

Solution	Marks	Remarks
11. (a) Volume $= \int_{-b}^{-\frac{b}{2}} \pi x^2 dy$	1A+1A	1A for $\int \pi x^2 dy$, 1A if others correct
$= \int_{-b}^{-\frac{b}{2}} \pi a^2 (1 - \frac{y^2}{b^2}) dy$	1M	
$= \pi a^2 [y - \frac{y^3}{3b^2}]_{-b}^{-\frac{b}{2}}$	1A	
$= \pi a^2 [\frac{-b}{2} + \frac{b}{24} + b - \frac{b}{3}]$		
$= \frac{5\pi a^2 b}{24}$	1	
	5	
(b) (i) Equation of ellipse is $\frac{x^2}{100} + \frac{y^2}{36} = 1$	1A	Accept $a = 10$, $b = 6$
Put $y = -3$	1A	
$x^2 = 75$		
$\therefore$ surface area $= \pi x^2$		
$= 75\pi$	1A	
(ii) Put $a = 10$, $b = 6$ into (a)	1A	
Volume $= \frac{5\pi(10)^2(6)}{24}$		
$= 125\pi$	1A	
(iii) (1) Let V be the volume of water remaining in the bowl.		
$\frac{dV}{dt} = -\frac{\pi}{100}(25 + 2t)$	1A	
$V = \frac{-\pi}{100}(25t + t^2) + c$	1A	
At $t = 0$, $V = 125\pi$ $\therefore c = 125\pi$	1M + 1A	1A
$\therefore V = 125\pi - \frac{\pi}{100}(25t + t^2)$	[redacted]	

Solution	Marks	Remarks
Alternative solutions		
$V = 125\pi - \int_0^t \frac{\pi}{100}(25 + 2t)dt$	1M+1A	
$= 125\pi - \frac{\pi}{100}[25t + t^2]_0^t$	1A	
$= 125\pi - \frac{\pi}{100}(25t + t^2)$	1A	
Let V be the volume of water lost.		
$\frac{dV}{dt} = \frac{\pi}{100}(25 + 2t)$	1A	
$V = \frac{\pi}{100}(25t + t^2) + c$	1A	
At $t = 0$, $V = 0$ $\therefore c = 0$	1M	
Volume remaining		
$= 125\pi - \frac{\pi}{100}(25t + t^2)$	1A	
(2) $125\pi - \frac{\pi}{100}(25t + t^2) = 0$	1M	
$t^2 + 25t - 12500 = 0$		
$t = 100$ or -125 (rejected)		
$\therefore t = 100$ (seconds)	1A	
	11	

Solution	Marks	Remarks
12. (a) $2[\cos\theta + \cos(\theta + 2\alpha) + \ldots + \cos(\theta + 8\alpha)]\sin\alpha$		
$= 2\cos\theta\sin\alpha + 2\cos(\theta + 2\alpha)\sin\alpha + \ldots + 2\cos(\theta + 8\alpha)\sin\alpha$	1A	
$= \sin(\theta + \alpha) - \sin(\theta - \alpha) + \sin(\theta + 3\alpha) - \sin(\theta + \alpha) + \ldots + \sin(\theta + 9\alpha) - \sin(\theta + 7\alpha)$	1M	For using the identity.
$= \sin(\theta + 9\alpha) - \sin(\theta - \alpha)$	1	
Put $\alpha = \frac{\pi}{5}$	1A	
$2[\cos\theta + \cos(\theta + \frac{2\pi}{5}) + \ldots + \cos(\theta + \frac{8\pi}{5})]\sin\frac{\pi}{5}$		
$= \sin(\theta + \frac{9\pi}{5}) - \sin(\theta - \frac{\pi}{5})$	1A	
$= 2\sin\pi\cos(\theta + \frac{4\pi}{5})$		OR $= \sin(\theta + \frac{9\pi}{5}) - \sin(\theta + \frac{9\pi}{5})$
$= 0$	1	OR $= \sin(\theta - \frac{\pi}{5}) - \sin(\theta - \frac{\pi}{5})$
$\therefore \cos\theta + \cos(\theta + \frac{2\pi}{5}) + \ldots + \cos(\theta + \frac{8\pi}{5}) = 0$	1	OR $\because (\theta + \frac{9\pi}{5}) - (\theta - \frac{\pi}{5}) = 2\pi$
	7	
(b) (i) $PD^2 = r^2 + r^2 - 2r^2\cos(\frac{4\pi}{5} - \theta)$	1A	
$= 2r^2 - 2r^2\cos(\theta + \frac{6\pi}{5})$	1	
OR $PD^2 = [2r\sin\frac{1}{2}(\frac{4\pi}{5} - \theta)]^2$	1A	
$= 2r^2[1 - \cos(\frac{4\pi}{5} - \theta)]$		
$= 2r^2 - 2r^2\cos(\theta + \frac{6\pi}{5})$	1	

Solution	Marks	Remarks
(ii) $PA^2 = 2r^2 - 2r^2\cos\theta$	1A	Any one of PA^2, PB^2 or P
$PB^2 = 2r^2 - 2r^2\cos(\theta + \frac{2\pi}{5})$		
$PC^2 = 2r^2 - 2r^2\cos(\theta + \frac{4\pi}{5})$		
$PE^2 = 2r^2 - 2r^2\cos(\theta + \frac{8\pi}{5})$	1A	or $2r^2 - 2r^2\cos(\frac{2\pi}{5} - \theta)$
$\therefore PA^2 + PB^2 + PC^2 + PD^2 + PE^2$		
$= 10r^2 - 2r^2[\cos\theta + \cos(\theta + \frac{2\pi}{5})$		
$+ \dots + \cos(\theta + \frac{8\pi}{5})]$	1A	
$= 10r^2$	1	
(iii) $QA^2 = QP^2 + PA^2$ $(\because \angle QPA = \frac{\pi}{2})$	1A	
$= 4r^2 + PA^2$		
Similarly, $QB^2 = 4r^2 + PB^2, \dots, QE^2 = 4r^2 + PE^2$		
$\therefore QA^2 + QB^2 + QC^2 + QD^2 + QE^2$		
$= 20r^2 + (PA^2 + PB^2 + PC^2 + PD^2 + PE^2)$	1A	
$= 20r^2 + 10r^2$		
$= 30r^2$	1A	
	9	