92-CE
A MATHS
PAPER II

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1992

ADDITIONAL MATHEMATICS PAPER II

11.15 am-1.15 pm (2 hours)

This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions in Section B.

All working must be clearly shown.

Unless otherwise specified in a question, numerical answers must be given in exact value.

Section A (42 marks)

Answer **ALL** questions in this section.

1. Prove, by mathematical induction, that

$$1 \times 2 + 2 \times 5 + 3 \times 8 + \ldots + n(3n - 1) = n^2(n + 1)$$

for all positive integers n.

(5 marks)

2. In the expansion of $(1 + 3x)^2(1 + x)^n$, where n is a positive integer, the coefficient of x is 10.

(a) Find the value of n.

(b) Find the coefficient of x^2.

(5 marks)

3. A straight line with slope m passes through the point (4, 7).

(a) Write down the equation of the line.

(b) If the distance from the origin to the line is 1, find the two possible values of m.

(6 marks)

4. The slope of the tangent to a curve C at any point (x, y) on C is $x^2 - 2$. C passes through the point (3, 4).

(a) Find an equation of C.

(b) Find the coordinates of the point on C at which the slope of the tangent is -2.

(6 marks)

5. By using the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, find the general solution of the equation

$$\sin 2\theta\,(4\cos^2\theta - 3) - \sin\theta = 0.$$

(6 marks)

6.

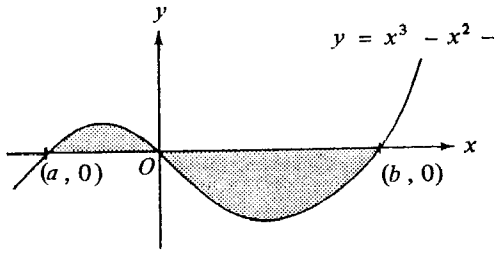

Figure 1

The curve $y = x^3 - x^2 - 2x$ cuts the x-axis at the origin and the points $(a, 0)$ and $(b, 0)$, as shown in Figure 1.

(a) Find the values of a and b.

(b) Find the total area of the shaded parts.

(6 marks)

7.

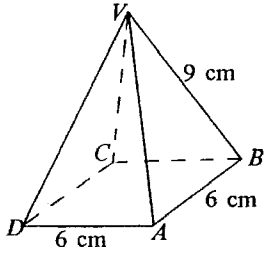

Figure 2

In Figure 2, $VABCD$ is a right pyramid with a square base of side 6 cm. $VB = 9$ cm. Find, correct to the nearest 0.1 degree,

(a) the angle between edge VB and the base $ABCD$,

(b) the angle between the planes VAB and VAD.

(8 marks)

Section B (48 marks)

Answer any **THREE** questions in this section.
Each question carries 16 marks.

8. (a) Let $y = \dfrac{\sin x}{2 + \cos x}$.

Show that $\dfrac{dy}{dx} = \dfrac{2}{2 + \cos x} - \dfrac{3}{(2 + \cos x)^2}$.

(4 marks)

(b) Using the substitution $t = \sqrt{3}\tan\theta$, evaluate

$$\int_0^1 \frac{dt}{t^2 + 3}.$$

(4 marks)

(c) Using the substitution $t = \tan\dfrac{x}{2}$ and the result of (b), evaluate

$$\int_0^{\frac{\pi}{2}} \frac{dx}{2 + \cos x}.$$

(4 marks)

(d) Using the results of (a) and (c), evaluate

$$\int_0^{\frac{\pi}{2}} \frac{dx}{(2 + \cos x)^2}.$$

(4 marks)

9. Given an ellipse $E : \frac{x^2}{25} + \frac{y^2}{16} = 1$.

Let the line $L : y = mx + c$ be a tangent to E.

(a) Show that $c^2 = 25m^2 + 16$.

(4 marks)

(b) Suppose L passes through the point (h, k). Using the result of (a), show that

$$(h^2 - 25)m^2 - 2hkm + (k^2 - 16) = 0.$$

(3 marks)

(c) Find equations of the two tangents from the point (7, 4) to E.

(5 marks)

(d) P is a variable point outside E and the two tangents from P to E are at right angles. Find an equation of the locus of P.

(4 marks)

10. Given a circle $C_1 : x^2 + y^2 - 2y - 4 = 0$.

(a) Find equations of the two circles centred at the point (8, 5) and touching C_1.

(7 marks)

(b) Find an equation of the line L_1 which touches C_1 at the point $(-1, 3)$.

(2 marks)

(c) F is the family of circles passing through the points of intersection of C_1 and the line $x - 2 = 0$.

(i) Write down an equation of F.

(ii) If L_1 in (b) also touches another circle C_2 of F, find an equation of C_2.

(7 marks)

11. (a)

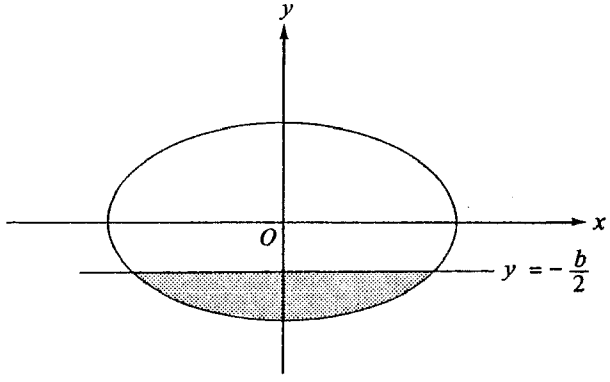

Figure 3 (a)

The shaded region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $y = -\frac{b}{2}$, as shown in Figure 3 (a), is revolved about the y-axis. Show that the volume of the solid of revolution is $\frac{5\pi a^2 b}{24}$.

(5 marks)

(b)

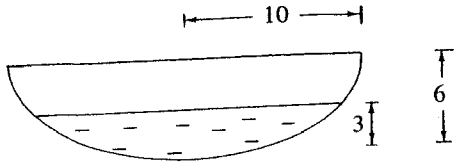

Figure 3 (b)

A bowl is generated by revolving the lower half of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the y-axis. The depth of the bowl is 6 units and the radius of its rim is 10 units. The bowl contains water to a depth of 3 units. (See Figure 3 (b).)

(i) Find the area of the water surface.

(ii) Using the result of (a), find the volume of water.

(iii) The water in the bowl is heated. At time t seconds after heating, the volume of water decreases at a rate of $\frac{\pi}{100}(25 + 2t)$ cubic units per second.

(1) Find the volume of water remaining in the bowl after t seconds.

(2) Calculate the time required to dry up the water in the bowl.

(11 marks)

12. (a) Using the identity $2\cos x \sin y = \sin(x + y) - \sin(x - y)$, show that

$2[\cos\theta + \cos(\theta + 2\alpha) + \cos(\theta + 4\alpha) + \cos(\theta + 6\alpha) + \cos(\theta + 8\alpha)]\sin\alpha$
$= \sin(\theta + 9\alpha) - \sin(\theta - \alpha)$.

Hence show that

$$\cos\theta + \cos(\theta + \frac{2\pi}{5}) + \cos(\theta + \frac{4\pi}{5}) + \cos(\theta + \frac{6\pi}{5}) + \cos(\theta + \frac{8\pi}{5}) = 0.$$

(7 marks)

(b)

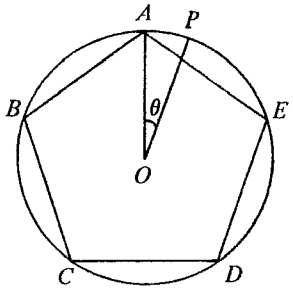

Figure 4

A, B, C, D and E are the vertices of a regular pentagon inscribed in a circle of radius r and centred at O. P is a point on the circumference of the circle such that $\angle POA = \theta$, as shown in Figure 4.

(i) By considering ΔOPD, show that
$PD^2 = 2r^2 - 2r^2\cos(\theta + \frac{6\pi}{5})$.

(ii) Show that $PA^2 + PB^2 + PC^2 + PD^2 + PE^2 = 10r^2$.

(iii) QP is a line perpendicular to the plane of the circle such that $QP = 2r$.
Find $QA^2 + QB^2 + QC^2 + QD^2 + QE^2$.

(9 marks)

END OF PAPER

1993

Additional Mathematics I

1. (a) $2\Delta x$

   (b) $\frac{1}{\sqrt{2x}}$

2. (a) $8 - 6i$

   (b) $1 - 3i$

3. $1, -5$

4. $\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$

   $\cos(\frac{2k\pi}{3} - \frac{\pi}{18}) + i\sin(\frac{2k\pi}{3} - \frac{\pi}{18})$, $k = -1, 0, 1$

5. $x \leq -2$ or $x \geq 4$

6. (a) $-2\mathbf{i} + 3\mathbf{j}$

   (b) 13, 0, 13

7. (a) $\frac{2x - 2y^2}{4xy - 3y^2}$

   (b) $2x + 11y + 7 = 0$

8. (a) $\frac{\overline{a} + r\overline{b}}{1 + r}$, $\frac{\overline{a} + (r^2 + 2r)\overline{b}}{(1 + r)^2}$

   (b) $\frac{1}{1 + r}\overline{b}$

   (c) $\frac{-1 + \sqrt{5}}{2}$

   (d) (i) 4, 16

   (ii) $\frac{1}{2}$

9. (b) $3\sqrt{3}$

10. (a) -13

    (b) $-7 - 4\sqrt{2} < k < -7 + 4\sqrt{2}$

    (c) $(-1, -1), (-2, -2)$

11. (a) $\frac{\pi}{18}, \frac{-5\pi}{18}; -1$

    (b) $3\sqrt{3}\cos 3x + 3\sin 3x$, $-9\sqrt{3}\sin 3x + 9\cos 3x$

    (c) $(-\frac{\pi}{9}, -2)$ is a minimum point.

    $(\frac{2\pi}{9}, 2)$ is a maximum point.

12. (b) $\frac{\pi}{3}, -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

    (c) $-\frac{3}{2} - \frac{\sqrt{3}}{2}i$

1993

Additional Mathematics II

2. $2\cos(x + \frac{\pi}{6})$

   $2n\pi + \frac{\pi}{6}, 2n\pi - \frac{\pi}{2}$

3. (a) $4n, 8n^2 - 7n$

   (b) 5, 165

4. $y = 2x - 5$, $y = -\frac{x}{2} + 5$

5. (a) $(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$, $(\frac{5\pi}{4}, \frac{-\sqrt{2}}{2})$

   (b) $2\sqrt{2}$

6. (a) $y = x^3 - 3x^2 - x + 3$

   (b) $y = -x + 3$

7. (a) 75.5°, 11.6 cm

   (b) 62.2°

8. (b) (i) $\frac{6}{5}$

   (ii) $\frac{24}{13}$

9. (a) $(m - 1)\sin^{m-2}x\cos^{n+2}x - (n + 1)\sin^m x\cos^n x$

   (d) $\frac{3\pi}{512}$

10. (a) $y = \frac{s + t}{2}x - st$

    (b) $y = sx - s^2$

    (c) (ii) $\frac{\pi}{2}$

    (iv) $2y = x^2 + 2$

11. (a) $\frac{13}{4}, \frac{3}{4}$

    (d) $x^2 + (y - 3)^2 = 3^2$, $(x - \frac{24}{7})^2 + (y - \frac{3}{49})^2 = (\frac{3}{49})^2$

12. (b) (i) $4\pi^2$

    (ii) π minutes, $(\frac{\sqrt{17} - 3}{2})\pi$ minutes

1992

Additional Mathematics I

1. (a) $-8\mathbf{i} + 6\mathbf{j}$, 10

   (b) $-\frac{16}{5}\mathbf{i} + \frac{12}{5}\mathbf{j}$

2. (a) $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

   (b) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

3. $x < -6$ or $-3 < x < -2$ or $x > 1$

4. (a) $\sqrt{3} + i$, $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

   (b) $(\sqrt{3} - \frac{1}{2}) + (\frac{\sqrt{3}}{2} + 1)i$

5. (a) (0, 1), (0, -1)

   (b) 1, -1

6. 0, 2

7. (a) $V = \frac{\pi}{9}h^3$

   (b) $\frac{3}{16}$cm s^{-1}

8. (a) 4, 3

   (b) 1, $\frac{1}{3}\mathbf{b}$

   (c) (i) $\frac{k\mathbf{a} + \frac{1}{3}\mathbf{b}}{k+1}$, 2

   (ii) (1) $\frac{m\mathbf{a} + \mathbf{b}}{m+1}$

   (2) $(n+1)(\frac{2}{3}\mathbf{a} + \frac{1}{9}\mathbf{b})$

   (3) 6, $\frac{2}{7}$

9. (b) $3p + 5$

   (c) (ii) $1 - 2\sqrt{5} < p < -\frac{5}{3}$

   -2, -3

10. (c) $16\sin^5\theta - 56\sin^3\theta + 32\sin\theta$

    0, $\frac{\pi}{4}$, $\frac{3\pi}{4}$, π

11. (b) (ii) $0 < x \leq 4$, $4 \leq x < 5$

    (c) (ii) 51.0 cm^3

    (d) 50.6 cm^3

12. (a) (i) $\frac{\pi}{6}$, $\frac{5\pi}{6}$

    3

    (ii) $(\frac{\pi}{2}, -5)$

    (b) (ii) 5, 0

1992

Additional Mathematics II

2. (a) 4

   (b) 39

3. (a) $mx - y + (7 - 4m) = 0$

   (b) $\frac{4}{3}$, $\frac{12}{5}$

4. (a) $y = \frac{1}{3}x^3 - 2x + 1$

   (b) (0, 1)

5. $n\pi$, $\frac{2n\pi}{3} \pm \frac{\pi}{9}$

6. (a) -1, 2

   (b) $\frac{37}{12}$

7. (a) 61.9°

   (b) 97.2°

8. (b) $\frac{\sqrt{3}\pi}{18}$

   (c) $\frac{\sqrt{3}\pi}{9}$

   (d) $\frac{2\sqrt{3}\pi}{27} - \frac{1}{6}$

9. (c) $y = 4$, $7x - 3y - 37 = 0$

   (d) $x^2 + y^2 - 41 = 0$

10. (a) $(x-8)^2 + (y-5)^2 = 45$, $(x-8)^2 + (y-5)^2 = 125$

    (b) $x - 2y + 7 = 0$

    (c) (i) $x^2 + y^2 - 2y - 4 + k(x - 2) = 0$

    (ii) $x^2 + y^2 - 15x - 2y + 26 = 0$

11. (b) (i) 75π

    (ii) 125π

    (iii) (1) $V = 125\pi - \frac{\pi}{100}(25t + t^2)$

    (2) 100 s

12. (b) (iii) $30r^2$