Solution	Marks	Remarks
1. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$	1M	
$= (-3\vec{i} + 5\vec{j}) - (5\vec{i} - \vec{j})$		
$= -8\vec{i} + 6\vec{j}$	1A	Omit vector sign(pp-1)
$\lvert\overrightarrow{AB}\rvert = \sqrt{(-8)^2 + 6^2}$		
$= 10$	1A	
(b) $\overrightarrow{AP} = \frac{4}{10}\overrightarrow{AB}$	1M	$\overrightarrow{OP} = \frac{4\overrightarrow{OB} + 6\overrightarrow{OA}}{10}$
$= -\frac{16}{5}\vec{i} + \frac{12}{5}\vec{j}$	1A 5	$-3.2\vec{i} + 2.4\vec{j}$
2. (a) $\frac{\sqrt{3}+i}{\sqrt{3}-i} = \frac{2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})}{2(\cos\frac{-\pi}{6} + i\sin\frac{-\pi}{6})}$ (or $\frac{2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})}{2(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6})}$)	1A+1A	$\frac{2(\cos30° + i\sin30°)}{2(\cos(-30°) + i\sin(-30°))}$
$= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$	1A	$\cos60° + i\sin60°$
Alternative solution		
$\frac{\sqrt{3}+i}{\sqrt{3}-i} = \frac{(\sqrt{3}+i)(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)}$	1A	
$= \frac{1}{2} + \frac{\sqrt{3}}{2}i$	1A	
$= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$	1A	
(b) $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^{92} = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{92}$		
$= \cos\frac{92\pi}{3} + i\sin\frac{92\pi}{3}$	1M	$\cos5520° + i\sin5520°$
$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$	1A	$\cos120° + i\sin120°$ (can be omitted)
$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$	1A 6	(pp-1) for omitting degree sign.

Solution	Marks	Remarks
3. $\lvert x(x+5)\rvert > 6$		
$x(x+5) > 6$ or $x(x+5) < -6$	2A	Use "and" or ",,, and " (no mark)
$(x+6)(x-1) > 0$ or $(x+2)(x+3) < 0$	1A+1A	
$x > 1$ or $x < -6$ or $-3 < x < -2$	2A	"or" cannot be omitted.
$\therefore x < -6$ or $-3 < x < -2$ or $x > 1$	6	
Alternative solution		
(1) $x^2(x+5)^2 > 36$	1A	
$[x(x+5)-6][x(x+5)+6] > 0$	1M	
$(x+6)(x-1)(x+2)(x+3) > 0$	1A+1A	
$x < -6$ or $-3 < x < -2$ or $x > 1$	2A	
(2) Case 1 : $x \geq 0$		
$x(x+5) > 6$		
$(x+6)(x-1) > 0$		
$x > 1$ or $x < -6$		
Since $x \geq 0$, $\therefore x > 1$	1A	
Case 2 : $-5 < x < 0$		
$-x(x+5) > 6$		
$(x+2)(x+3) < 0$		
$-3 < x < -2$		
Since $-5 < x < 0$, $\therefore -3 < x < -2$	1A	
Case 3 : $x \leq -5$		
$x(x+5) > 6$		
$x > 1$ or $x < -6$		
Since $x \leq -5$, $\therefore x < -6$	1A	
	1M	For consider the 3 cases. (pp-1 for omitting some equality signs)
Combining the 3 cases,		
$x < -6$ or $-3 < x < -2$ or $x > 1$	2A	

Solution	Marks	Remarks
4. (a) $z_1 = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$	1A	Accept degree measures (can be omitted)
$= \sqrt{3} + i$	1A	
$z_3 = \cos(\frac{\pi}{2} + \frac{\pi}{6}) + i\sin(\frac{\pi}{2} + \frac{\pi}{6})$ | OR $z_3 = \frac{1}{2}iz_1$ |	1A	$z_3 = -\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ (can be omitted)
$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$	1A	
(b) $z_2 = z_1 + z_3$	1M	
$= (\sqrt{3} - \frac{1}{2}) + (\frac{\sqrt{3}}{2} + 1)i$	1A	
	6	
5. (a) Put $x = 0$,	1M	
$y = \pm 1$	1A	
$\therefore$ The points are (0, 1) and (0, -1)		
(b) Differentiate with respect to x,		$y^2 = \frac{2 + 3x}{2 - x}$
$(y^2 + 3) + (x - 2)(2y\frac{dy}{dx}) = 0$	1M (For product rule)	$2y\frac{dy}{dx} = \frac{3(2 - x) + (2 + 3x)}{(2 - x)^2}$
$\frac{dy}{dx} = \frac{y^2 + 3}{2y(2 - x)}$		$2y\frac{dy}{dx} = \frac{8}{(2 - x)^2}$
$\frac{dy}{dx}\Big\|_{(0,1)} = 1$	1M+1A	Subs. (0, 1), $\frac{dy}{dx} = 1$
$\frac{dy}{dx}\Big\|_{(0,-1)} = -1$	1A	Subs. (0, -1), $\frac{dy}{dx} = 1$
	6	

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Solution	Marks	Remarks
6. Consider 2 cases : (1) $\alpha = \beta$ (2) $\alpha = -\beta$	1M	
(1) $\alpha = \beta$		
Discriminant $= (2 - k)^2 + 4(k - 1) = 0$	1M+1A	1M for $\Delta = 0$
$k = 0$	1A	
(2) $\alpha = -\beta$		
Sum of roots $= -(k - 2) = 0$	1M	
$k = 2$	1A	
	6	
Alternative solutions		
$x^2 + (k - 2)x - (k - 1) = 0$		
$(x - 1)(x - (1 - k)) = 0$	1A	For factorisation
$x = 1$ or $1 - k$	1A+1A	
Since $\lvert\alpha\rvert = \lvert\beta\rvert$		
$\lvert 1 - k\rvert = 1$	1M	
$k = 0$ or 2	1A+1A	
$\alpha^2 = \beta^2$		
$(\alpha + \beta)(\alpha - \beta) = 0$	1M	
(1) $\alpha + \beta = 0$		
$-(k - 2) = 0$	1M	
$k = 2$	1A	
(2) $\alpha - \beta = 0$		
$(\alpha - \beta)^2 = 0$		
$(\alpha + \beta)^2 - 4\alpha\beta = 0$	1M	
$(2 - k)^2 + 4(k - 1) = 0$	1A	
$k = 0$	1A	

Solution	Marks	Remarks
7. (a) Let r cm be the radius of water surface when the depth of water is h cm.		
$\tan 30° = \frac{r}{h}$		
$r = \frac{h}{\sqrt{3}}$	1A	
$V = \frac{1}{3}\pi(\frac{h}{\sqrt{3}})^2 h$	1M	
$= \frac{\pi}{9}h^3$	1A	
(b) $\frac{dV}{dt} = \frac{\pi}{3}h^2\frac{dh}{dt}$	1M+1A	1M for chain rule
Put $\frac{dV}{dt} = -\pi$	1A	
At $h = 4$, $-\pi = \frac{\pi}{3}(4)^2\frac{dh}{dt}$		
$\frac{dh}{dt} = \frac{-3}{16}$	1A	
∴ The water is falling at a rate of $\frac{3}{16}$ cms^{-1}	7	

Solution	Marks	Remarks
8. (a) $\vec{a}\cdot\vec{a} = \lvert\vec{a}\rvert^2$		Omit dot sign (pp-1)
$= 4$	1A	Omit vector sign (pp-1)
$\vec{a}\cdot\vec{b} = 2(3)\cos\frac{\pi}{3}$	1M	
$= 3$	1A	
	3	
(b) $OD = 2\cos\frac{\pi}{3} = 1$	1A	
$\vec{OD} = \frac{1}{3}\vec{b}$	1A	
	2	
(c) (i) $\vec{OH} = \dfrac{k\vec{a} + \frac{1}{3}\vec{b}}{k+1}$	1M+1A	
$\vec{OH}\cdot\vec{AB} = 0$	1M	
$\left(\dfrac{k\vec{a} + \frac{1}{3}\vec{b}}{k+1}\right)\cdot(\vec{b} - \vec{a}) = 0$		
$\dfrac{1}{k+1}(k\vec{a}\cdot\vec{b} - k\vec{a}\cdot\vec{a} + \frac{1}{3}\vec{b}\cdot\vec{b} - \frac{1}{3}\vec{b}\cdot\vec{a}) = 0$	1M	
$3k - 4k + \frac{1}{3}(9) - 1 = 0$		
$k = 2$	1A	
(ii) (1) $\vec{OC} = \dfrac{m\vec{a} + \vec{b}}{m+1}$	1A	
(2) $\vec{OC} = (n+1)\left(\dfrac{2\vec{a} + \frac{1}{3}\vec{b}}{3}\right)$	1M+1A	
(3) $\begin{cases} \dfrac{m}{m+1} = \dfrac{2(n+1)}{3} \\ \dfrac{1}{m+1} = \dfrac{n+1}{9} \end{cases}$	1M	
Solving, $m = 6$	1A	
$n = \frac{2}{7}$	1A	
	11	

Solution	Marks	Remarks
9. (a) Discriminant $\Delta = (p+1)^2 - 4(p-1)$	1A	
$= p^2 - 2p + 5$		
$= (p-1)^2 + 4 > 0$	1M+1	1M for knowing $\Delta > 0$ 1 for a correct proof.
$\therefore \alpha$, β are real and distinct.	3	[$\Delta' = (-2)^2 - 4(1)(5) = -16 < 0$ $\therefore$ real and distinct]
(b) $\begin{cases} \alpha + \beta = -(p+1) \\ \alpha\beta = (p-1) \end{cases}$	1A	
$(\alpha - 2)(\beta - 2) = \alpha\beta - 2(\alpha + \beta) + 4$		
$= (p-1) + 2(p+1) + 4$	1M	For complete substitution
$= 3p + 5$	1A 3	
(c) (i) Since $\beta < 2 < \alpha$,		
$\alpha - 2 > 0$, $\beta - 2 < 0$	1M	
From (b) $(\alpha - 2)(\beta - 2) = 3p + 5 < 0$		
$\therefore p < -\frac{5}{3}$	1	
(ii) $(\alpha - \beta)^2 < 24$		
$(\alpha + \beta)^2 - 4\alpha\beta < 24$	1A	
$(p+1)^2 - 4(p-1) < 24$		
$p^2 - 2p - 19 < 0$	1A	
$(p-1)^2 < 20$		or $(p-1+\sqrt{20})(p-1-\sqrt{20})<0$,
$1 - 2\sqrt{5} < p < 1 + 2\sqrt{5}$	1M+2A	or $1 - \sqrt{20} < p < 1 + \sqrt{20}$ ($-3.47 < P < 5.47$ – 1M only)
Combining with (i)		
$1 - 2\sqrt{5} < p < -\frac{5}{3}$	1A	or $1 - \sqrt{20} < p < -\frac{5}{3}$
The possible integral values of p are -2 or -3 .	1M+1A 10	
Alternative solution		
(c) (ii) $x^2 + (p+1)x + (p-1) = 0$		
$x = \dfrac{-(p+1) \pm \sqrt{p^2 - 2p + 5}}{2}$	1A	
$(\alpha - \beta)^2 < 24$		
$\left[\dfrac{-(p+1) + \sqrt{p^2 - 2p + 5}}{2} - \dfrac{-(p+1) - \sqrt{p^2 - 2p + 5}}{2}\right]^2 < 24$		
$p^2 - 2p - 19 < 0$	1A	

Solution	Marks	Remarks
10. (a) $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$	1A	
$(\cos\theta + i\sin\theta)^3 = \cos^3\theta + 3\cos^2\theta(i\sin\theta) + 3\cos\theta(i\sin\theta)^2 + (i\sin\theta)^3$	1A	
Equating imaginary parts,		
$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$		
$= 3\sin\theta(1 - \sin^2\theta) - \sin^3\theta$		
$= 3\sin\theta - 4\sin^3\theta$	1 / 3	
(b) $16\sin^3\theta\cos^2\theta = 16[\frac{1}{2i}(z - \frac{1}{z})]^3 [\frac{1}{2}(z + \frac{1}{z})]^2$	1A+1A	
$= \frac{-1}{2i}(z^2 - \frac{1}{z^2})^2(z - \frac{1}{z})$		
$= \frac{-1}{2i}(z^4 - 2 + \frac{1}{z^4})(z - \frac{1}{z})$		
$= \frac{-1}{2i}(z^5 - 2z + \frac{1}{z^3} - z^3 + \frac{2}{z} - \frac{1}{z^5})$	1A	or $\frac{i}{2}$ (......)
$= \frac{-1}{2i}[(z^5 - \frac{1}{z^5}) - 2(z - \frac{1}{z}) -(z^3 - \frac{1}{z^3})]$	1M	For collecting terms
$= -\frac{1}{2i}(2i\sin 5\theta - 4i\sin\theta - 2i\sin 3\theta)$		
$= 2\sin\theta + \sin 3\theta - \sin 5\theta$	1 / 5	
(c) $\sin 5\theta + 9\sin 3\theta$		
$= (2\sin\theta + \sin 3\theta - 16\sin^3\theta\cos^2\theta) + 9\sin 3\theta$	1M	For using (b)
$= 2\sin\theta + 10(3\sin\theta - 4\sin^3\theta) - 16\sin^3\theta(1 - \sin^2\theta)$	1M	For using (a)
$= 16\sin^5\theta - 56\sin^3\theta + 32\sin\theta$	1A	
$\sin 5\theta + 9\sin 3\theta - 8\sin\theta = 0$		
$16\sin^5\theta - 56\sin^3\theta + 24\sin\theta = 0$	1A	
$8\sin\theta(2\sin^4\theta - 7\sin^2\theta + 3) = 0$		
$\sin\theta = 0$ or $\sin^2\theta = \frac{1}{2}$ or $\sin^2\theta = 3$	1A+1A	1A for $\sin\theta = 0$, 1A for others
$\sin\theta = 0$ or $\sin\theta = \pm\frac{\sqrt{2}}{2}$		
$\theta = 0, \pi$ or $\frac{\pi}{4}, \frac{3\pi}{4}$	1A+1A 8	1A for 0, π 1A for $\frac{\pi}{4}, \frac{3\pi}{4}$ no mark for degrees for extra −1 for each.

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Solution	Marks	Remarks
<u>Alternative solution</u>		
$\sin 5\theta + 9\sin 3\theta - 8\sin\theta = 0$		
$(\sin 5\theta + \sin 3\theta) + 8(\sin 3\theta - \sin\theta) = 0$		
$2\sin 4\theta\cos\theta + 16\sin\theta\cos 2\theta = 0$	1M	For sum to product
$8\sin\theta\cos^2\theta\cos 2\theta + 16\sin\theta\cos 2\theta = 0$		
$8\sin\theta\cos 2\theta(\cos^2\theta + 2) = 0$		
$\sin\theta = 0$ or $\cos 2\theta = 0$	1A+1A	
$\theta = 0, \pi$ or $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$	1A+1A	

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Solution	Marks	Remarks
11. (a) Diagonal of base $= \sqrt{x^2 + x^2}$	1M	
$= \sqrt{2}x$ (cm)		
Height of pyramid $= \sqrt{(\frac{\sqrt{6}x}{2})^2 - (\frac{\sqrt{2}x}{2})^2}$		
$= x$ (cm)	1A	
$\therefore h = (10 - 2x) + x$		
$= 10 - x$	1 / 3	
(b) (i) $V = \frac{1}{3}x^2(x) + x^2(10 - 2x)$		
$= 10x^2 - \frac{5}{3}x^3$	1	
(ii) $\frac{dV}{dx} = 20x - 5x^2$	1A	
$\frac{dV}{dx} \ge 0$	1M	or $\frac{dV}{dx} > 0$
$20x - 5x^2 \ge 0$		
$5x(4 - x) \ge 0$		
$0 \le x \le 4$		
Since $0 < x < 5$, $\therefore 0 < x \le 4$	1A	or $0 < x < 4$
The range of values of x for which V is decreasing is $4 \le x < 5$.	1M+1A / 6	or $4 < x < 5$
(c) (i) Base side length $x \le 3.5$		
$h = 10 - x \le 7$	1A	
$\therefore 3 \le x \le 3.5$	1	
(ii) From (b) (ii), V is increasing on this interval		
$\therefore V$ is greatest when $x = 3.5$	1M	
Greatest volume $= 10(3.5)^2 - \frac{5}{3}(3.5)^3$		
$= 51.0$ (cm^3)	1A / 4	
(d) $x \le 4.7$ and $10 - x \le 5.5$		
$\therefore 4.5 \le x \le 4.7$	1A	
Since V is decreasing on this interval,		
$\therefore V$ is greatest when $x = 4.5$	1M	
Greatest volume $= 10(4.5)^2 - \frac{5}{3}(4.5)^3$		
$= 50.6$ (cm^3)	1A / 3	

Solution	Marks	Remarks
12. (a) (i) When $x = 0$, $y = 3$	1A	Accept (0, 3)
$\therefore$ The y-intercept is 3.		
When $y = 0$, $2\cos 2x - 4\sin x + 1 = 0$		
$2(1 - 2\sin^2 x) - 4\sin x + 1 = 0$	1A	
$4\sin^2 x + 4\sin x - 3 = 0$		
$\sin x = \frac{1}{2}$ or $\sin x = \frac{-3}{2}$ (rejected)	1A	
$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$	1A	Accept $(\frac{\pi}{6}, 0)$, $(\frac{5\pi}{6}, 0)$, no mark for degrees, (pp-1) if other correct roots are included.
$\therefore$ The x-intercepts are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$		
(ii) $\frac{dy}{dx} = -4\sin 2x - 4\cos x$	1A	
$-4\sin 2x - 4\cos x = 0$	1M	
$\cos x(2\sin x + 1) = 0$		
$\cos x = 0$ or $\sin x = -\frac{1}{2}$ (rejected)		
$x = \frac{\pi}{2}$	1A	(pp-1) if other correct roots are included
$\frac{d^2y}{dx^2} = -8\cos 2x + 4\sin x$		
$\left.\frac{d^2y}{dx^2}\right\|_{x = \frac{\pi}{2}} > 0$	1M	
$\therefore (\frac{\pi}{2}, -5)$ is a minimum point.	1A	Accept turning point

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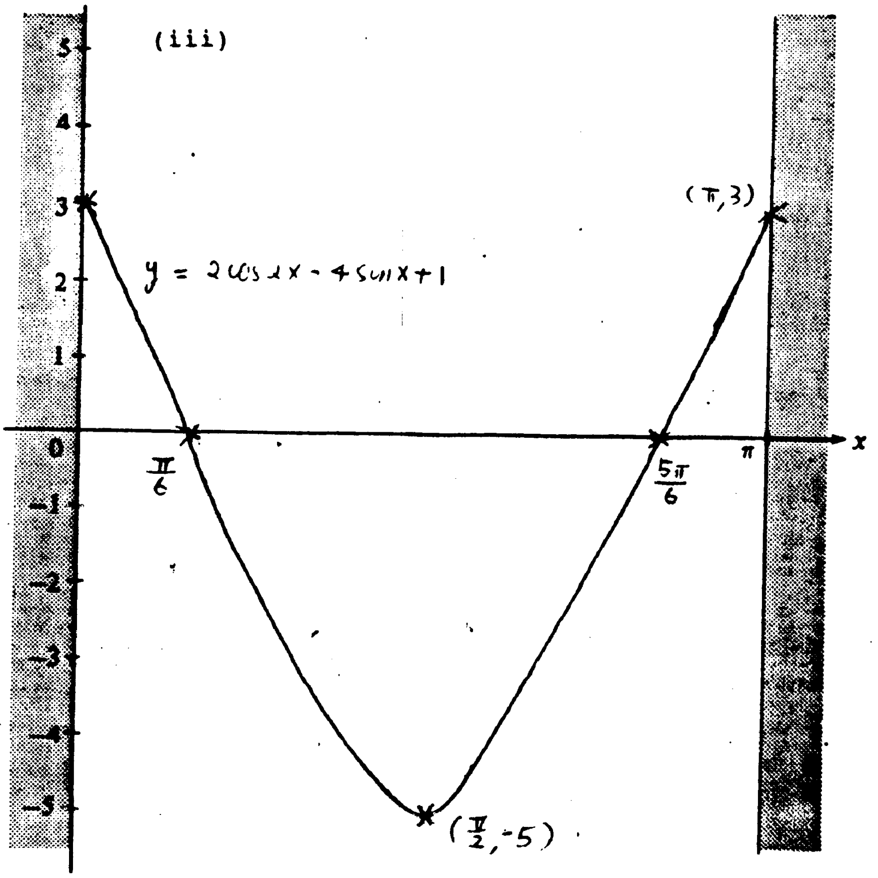
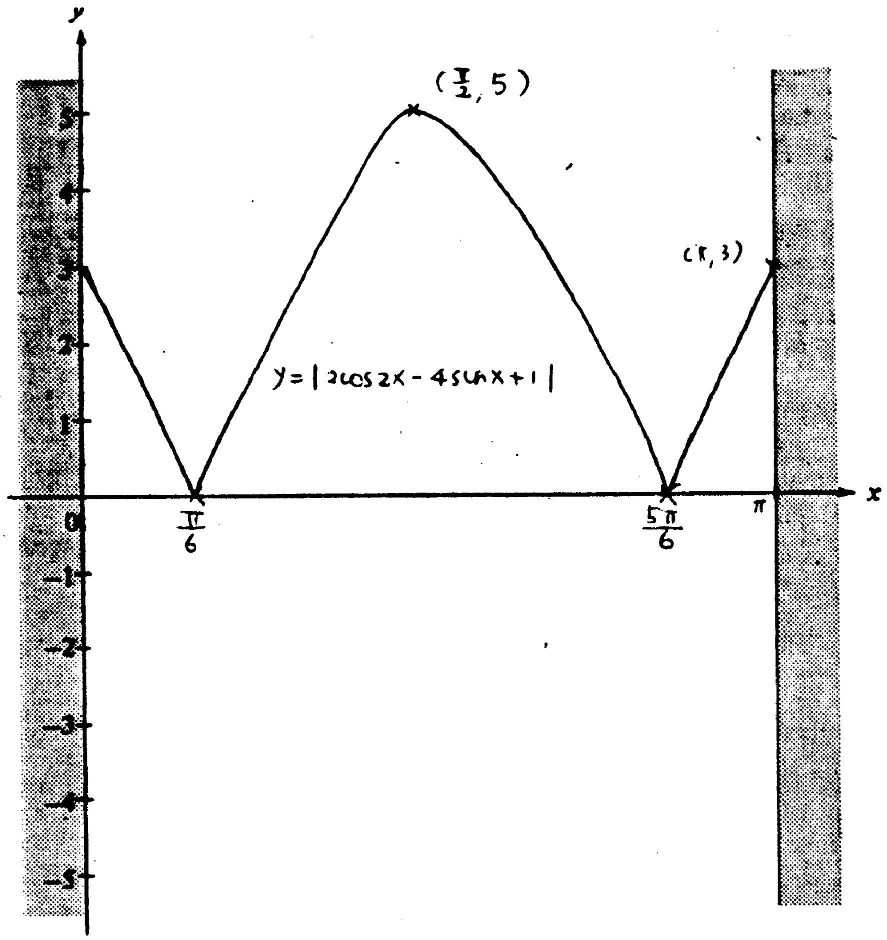
Solution	Marks	Remarks
(iii) 	1A 1A 1A 12	For V-shape labelled end points For labelling $(\frac{\pi}{6},0)$ $(\frac{5\pi}{6},0)$ and $(\frac{\pi}{2},-5)$
(b) (i) 	2M	For reflection Accept no labelling
(ii) Greatest value = 5 Least value = 0	1A 1A 4	