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香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九一年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1991

附加數學卷二
ADDITIONAL MATHEMATICS PAPER II

評卷參考
MARKING SCHEME

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It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

Markers should therefore resist pleas from their students to have access to this document. Making it available to students would constitute misconduct on the part of the marker.

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員如向學生披露本評卷參考內容，即違背閱卷員守則。

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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method. <u>In general, a correct answer merits all the marks allocated to that part, provided that the method used is sound</u>.

2. In a question consisting of several parts each depending on the previous parts, marks should be awarded to steps or methods correctly deduced from previous erroneous answers. However, marks for the corresponding answer should <u>NOT</u> be awarded. In the marking scheme, 'M' marks are awarded for showing correct method use, and 'A' marks are awarded for the accuracy of the answers.

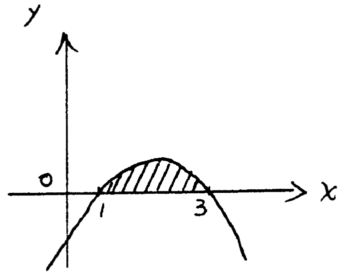
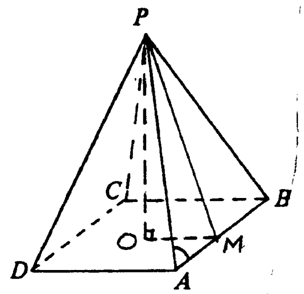
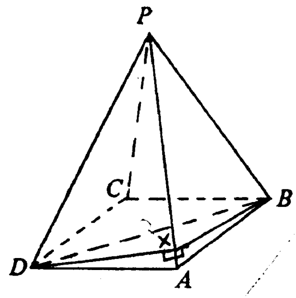
3. The symbol (pp-1) should be used to denote marks deducted for poor presentation (p.p.). Marks entered in the box should be the <u>net</u> total scored on that page. Note the following points :

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

4. Numerical answers should be given in exact value unless otherwise specified in the question. However answers not in exact values would be accepted this year provided that they are correct to at least 3 significant figures.

Solution	Marks	Remarks
1. (a) $(1 + x + ax^2)^8 = [1 + x(1 + ax)]^8$	1M	For grouping terms.
$= 1 + {}_8C_1x(1 + ax) + {}_8C_2x^2(1 + ax)^2 + {}_8C_3x^3(1 + ax)^3 + \ldots$		(pp-1) for omitting dots in all expressions
$\therefore k_1 = 8a + 28$	1A	Accept ${}_8C_1a + {}_8C_2$
$k_2 = 56a + 56$	1A	$2{}_8C_2 + {}_8C_3$
(b) $k_1 = 8a + 28 = 4$		
$a = -3$	1A	
$k_2 = 56(-3) + 56$		
$= -112$	1A	
	5	
2. $\int_0^{\pi/2} (\sin x + \cos x)^2 dx$		
$= \int_0^{\pi/2} (\sin^2 x + 2\sin x\cos x + \cos^2 x) dx$	1A	
$= \int_0^{\pi/2} (1 + 2\sin x\cos x) dx$ OR $= \int_0^{\pi/2} (1 + \sin 2x) dx$	1A	
$= [x + \sin^2 x]_0^{\pi/2}$ OR $= [x - \frac{1}{2}\cos 2x]_0^{\pi/2}$	1A+1A	
$= \frac{\pi}{2} + 1$	1A	Accept 2.57
	5	
3. $\cos 4\theta + \cos 2\theta = \cos\theta$		
$2\cos 3\theta\cos\theta = \cos\theta$	1A	
$\cos\theta = 0$ or $\cos 3\theta = \frac{1}{2}$	1A+1A	
$3\theta = 2n\pi \pm \frac{\pi}{3}$		
$\theta = 2n\pi \pm \frac{\pi}{2}$ (or $(2n + 1)\frac{\pi}{2}$) $\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$	1A+1A	$360n° \pm 90°$ (or $(2n + 1)90°$), $120n° \pm 20°$
(n being any integer.)		
	5	
4. $\left\|\frac{4 + 3k}{\sqrt{(2 - k)^2 + (1 + 2k)^2}}\right\| = 1$ (or $\frac{4 + 3k}{\sqrt{(2 - k)^2 + (1 + 2k)^2}} = \pm 1$)	1A	Omit absolute sign (pp-1)
$(4 + 3k)^2 = (2 - k)^2 + (1 + 2k)^2$		
$4k^2 + 24k + 11 = 0$		
$k = -\frac{1}{2}$ or $\frac{-11}{2}$	1A+1A	
Equations of lines : $x = 1$	1A	
$3x - 4y + 5 = 0$	1A	$y = \frac{3}{4}x + \frac{5}{4}$
	5	

5. (a) $\frac{dy}{dx} = 4 - 2x$		
$y = 4x - x^2 + c$	1A	
Subs. (1, 0)	1M	
$c = -3$	1A	
$\therefore y = -x^2 + 4x - 3$		
(b) $y = 0$ at $x = 1$ or 3	1A	
Area $= \int_1^3 (-x^2 + 4x - 3)\,dx$	1M	
$= [\frac{-x^3}{3} + 2x^2 - 3x]_1^3$	1A	
$= (-9 + 18 - 9) - (-\frac{1}{3} + 2 - 3)$		
$= \frac{4}{3}$	1A	
	7	
6. (a) Let M be the mid-point of AB, O be centre of $ABCD$		
$PM = 2\tan 60°$ — 1A		
$= 2\sqrt{3}$	1A	
$\cos\angle PMO = \frac{OM}{PM}$		
$= \frac{2}{2\sqrt{3}}$	1M	
$\angle PMO = 54.7°$	1A	
(b) Let X be the point on PA such that $DX \perp PA$, $BX \perp PA$		
$BX = 4\sin 60°$		
$= 2\sqrt{3}$	1A	
$OB = \frac{1}{2}\sqrt{4^2 + 4^2}$		
$= 2\sqrt{2}$	1A	
$\sin\frac{\angle BXD}{2} = \frac{OB}{BX}$		
$= \frac{2\sqrt{2}}{2\sqrt{3}}$	1M	
$\angle BXD = 109.5°$	1A	
	7	
Alternative solution for (b)		
$BX = DX = 2\sqrt{3}$	1A	
$BD = 4\sqrt{2}$	1A	
$\cos\angle BXD = \frac{BX^2 + DX^2 - BD^2}{2BX.DX}$		
$= \frac{(2\sqrt{3})^2 + (2\sqrt{3})^2 - (4\sqrt{2})^2}{2(2\sqrt{3})(2\sqrt{3})}$	1M	For cosine rule
$= -0.3333$		
$\angle BXD = 109.5°$	1A	

7. (a) For $n = 1$, L.H.S. $= 1^2 = 1$		
R.H.S. $= \frac{1}{6}(1)(2)(3) = 1$	1	
$\therefore$ the statement is true for $n = 1$		
Assume $1^2 + 2^2 + \ldots + k^2 = \frac{1}{6}k(k+1)(2k+1)$	1	
(for some +ve integer k)		
Then $1^2 + 2^2 + \ldots + k^2 + (k+1)^2$		
$= \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	1	
$= \frac{1}{6}(k+1)[k(2k+1) + 6(k+1)]$		
$= \frac{1}{6}(k+1)(k+2)(2k+3)$	1	
$\therefore$ the statement is also true for $n = k+1$ (if it is true for $n = k$)		
$\therefore$ (By the principle of mathematical induction) the statement is true for all +ve integers n	1	
(b) $1 \times 2 + 2 \times 3 + \ldots + n(n+1)$		
$= 1 \times (1+1) + 2 \times (2+1) + \ldots + n \times (n+1)$		
$= (1^2 + 2^2 + \ldots + n^2) + (1 + 2 + \ldots + n)$.	1A	
$= \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$	1A	
$= \frac{1}{3}n(n+1)(n+2)$	1A	$\frac{1}{3}(n^3 + 3n^2 + 2n)$
	8	

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Solution	Marks	Remarks
8. (a) (i) $\tan x = k\tan y$		
$\sin x\cos y = k\cos x\sin y$	1A	
$\sin(x+y) = \sin x\cos y + \cos x\sin y$	1A	For addition formula
$= k\cos x\sin y + \cos x\sin y$		
$= (k+1)\cos x\sin y$	1	
(ii) $(k+1)\sin(x-y) = (k+1)(\sin x\cos y - \cos x\sin y)$	1A	For expanding $\sin(x-y)$
$= (k+1)(k\cos x\sin y - \cos x\sin y)$		
$= (k+1)(k-1)\cos x\sin y$		or $(k^2-1)\cos x\sin y$
$= (k-1)\sin(x+y)$	1M+1	1M for using result of (a) (i)
	6	
(b) (i) $\tan(\theta+10°) = k\tan(\theta-20°)$		
Using (a) (ii)		
$(k+1)\sin 30° = (k-1)\sin(2\theta-10°)$	1A	
$\sin(2\theta-10°) = \dfrac{k+1}{2(k-1)}$	1	
(ii) $\left\lvert\dfrac{k+1}{2(k-1)}\right\rvert \le 1$	2A	Do not accept < 1
$(k+1)^2 \le 4(k-1)^2$	1A	
$3k^2 - 10k + 3 \ge 0$		
$(3k-1)(k-3) \ge 0$	1A	
$k \ge 3$ or $k \le \dfrac{1}{3}$	1A	
	7	
Alternative solution		
$-1 \le \dfrac{k+1}{2(k-1)} \le 1$	1A+1A	
$\dfrac{k+1}{2(k-1)} \le 1$ and $\dfrac{k+1}{2(k-1)} \ge -1$		
$\dfrac{k+1}{2(k-1)} - 1 \le 0$ $\dfrac{k+1}{2(k-1)} + 1 \ge 0$		
$\dfrac{k-3}{k-1} \ge 0$ $\dfrac{3k-1}{k-1} \ge 0$		
$(k \ge 3$ or $k < 1)$ and $(k > 1$ or $k \le \dfrac{1}{3})$	1A+1A	Do not accept $k \le 1$
$\therefore k \ge 3$ or $k \le \dfrac{1}{3}$	1A	
(c) Subs. $k = -2$ into (b) (i)		
$\sin(2\theta-10°) = \dfrac{1}{6}$	1A	
$2\theta - 10° = 180n° + (-1)^n 9.6°$	1A	
$\theta = 90n° + 5° + (-1)^n 4.8°$	1A	
(n being any integer.)		
	3	

$\sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

			Marks	Remarks
9.	(a)	$x = 1 + s\cos\theta$	1A	
		$y = 2 + s\sin\theta$	1A	
			2	
	(b)	Subs. $x = 1 + s\cos\theta$, $y = 2 + s\sin\theta$ into C,		
		$(1 + s\cos\theta)^2 + (2 + s\sin\theta)^2 - 6(1 + s\cos\theta) - 10(2 + s\sin\theta) + 30 = 0$	1M	or Subs. $x = 1 + s_1\cos\theta$, $y = 2 + s_1\sin\theta$
		$s^2 - (4\cos\theta + 6\sin\theta)s + 9 = 0$	1	
		Since L and C intersects at H and K, so s_1 and s_2 are the roots of the above equation.	1	Similarly for subs. $x = 1 + s_2\cos\theta$, $y = 2 + s_2\sin\theta$
			3	
	(c)	$HK^2 = (s_2 - s_1)^2$	1A	
		$= (s_1 + s_2)^2 - 4s_1s_2$	1A	
		$= (4\cos\theta + 6\sin\theta)^2 - 36$	1A	
		$= 16\cos^2\theta + 48\sin\theta\cos\theta + 36\sin^2\theta - 36$		
		$= 48\sin\theta\cos\theta - 20\cos^2\theta$	1	
			4	
	(d)	$HK = 0$	1M	
		$48\sin\theta\cos\theta - 20\cos^2\theta = 0$		
		$\cos\theta = 0$ or $\tan\theta = \frac{5}{12}$	1A+1A	
		Equations of tangent :		
		$x = 1$	2A	
		and $\frac{y-2}{x-1} = \frac{5}{12}$	1M	point-slope form
		$5x - 12y + 19 = 0$	1A	$y = \frac{5}{12}x + \frac{19}{12}$
			7	

10. (a)	$\frac{x}{8} - \frac{2y}{9}\frac{dy}{dx} = 0$ or $\frac{xx_1}{16} - \frac{yy_1}{9} = 1$	1A	~~For LHS~~ only
	$\frac{dy}{dx} = \frac{9x}{16y}$; slope $= \frac{9x_1}{16y_1}$	1A	
	$= \frac{5}{4}$; $= \frac{5}{4}$	1M	
	$y = \frac{9x}{20}$		
	$\frac{x^2}{16} - \frac{1}{9}(\frac{9x}{20})^2 = 1$	1M	For substitution
	$x = \pm 5$	1A	
	The points are $(5, \frac{9}{4})$ and $(-5, -\frac{9}{4})$.	1A+1A	
		7	

Alternative solution

(a)	$y = \frac{5}{4}x + c$	1A	
	$9x^2 - 16(\frac{5}{4}x + c)^2 = 144$	1M	For substitution
	$2x^2 + 5cx + 2(c^2 + 9) = 0$		
	$25c^2 - 16c^2 - 144 = 0$	1M	
	$c = \pm 4$	1A	
	$2x^2 \pm 20x + 50 = 0$		
	$x = \pm 5$	1A	
	The points are $(5, \frac{9}{4})$ and $(-5, \frac{-9}{4})$.	1A+1A	

(b) $\frac{x^2}{16} - \frac{1}{9}(\frac{5}{4}x + c)^2 = 1$ — ~~1M~~ — For substitution

$2x^2 + 5cx + 2(c^2 + 9) = 0$

$x = \frac{x_1 + x_2}{2}$ — 1M

$x = -\frac{5c}{4}$ — 1A

$y = \frac{-9c}{16}$ — 1A

4

Alternative solution

(b) $x = \frac{-5c \pm \sqrt{25c^2 - 16(c^2 + 9)}}{4}$ — ~~1M~~

$x = \frac{1}{2}\left(\frac{-5c + \sqrt{25c^2 - 16(c^2 + 9)}}{4} + \frac{-5c - \sqrt{25c^2 - 16(c^2 + 9)}}{4}\right)$ — 1M — For $x = \frac{x_1 + x_2}{2}$

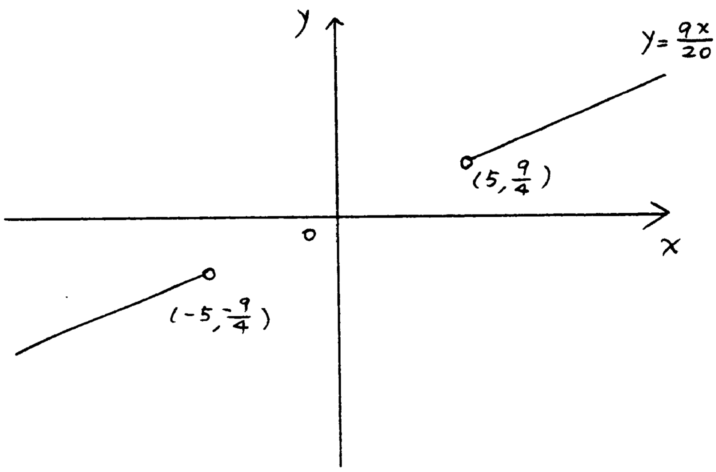
$= \frac{-5c}{4}$ — 1A

$y = \frac{-9c}{16}$ — 1A

(c) Eliminate c from $x = \frac{-5c}{4}$ and $y = \frac{-9c}{16}$, — 1M

Equation of locus : $y = \frac{9x}{20}$ $(x > 5 \text{ or } x < -5)$ — 1A — $(x > 5 \text{ or } x < -5)$ can be omitted.

(Note : The 2 limiting end-points can be included).



1A — straight line

2A — End points

5

11. (a) volume $= \int_0^4 \pi x^2 dy$	1A	
$= \int_0^4 \pi 4y dy$	1A	
$= [2\pi y^2]_0^4$		
$= 32\pi$	1A	
	3	
(b) (i) mass $= \int_0^4 \pi(16y - 3y^2) dy$	1A	
$= \pi[8y^2 - y^3]_0^4$		
$= 64\pi$	1A	
(ii) (1) $\int_0^h \pi x^2 dy = 16\pi$	1M	
$[2\pi y^2]_0^h = 16\pi$		
$2\pi h^2 = 16\pi$	1A	For $2\pi h^2$ only
$h = 2\sqrt{2}$	1A	Accept 2.83
(2) Mass of lower part $= \int_0^{2\sqrt{2}} \pi(16y - 3y^2) dy$	1M	
$= \pi[8y^2 - y^3]_0^{2\sqrt{2}}$		
$= (64 - 16\sqrt{2})\pi$	1A	Accept 41.4π, 130
Mass of upper part $= 64\pi - (64 - 16\sqrt{2})\pi$		
$= 16\sqrt{2}\pi$	1A	Accept 22.6π, 71.1
Ratio $= (64 - 16\sqrt{2})\pi : 16\sqrt{2}\pi$		
$= (2\sqrt{2} - 1) : 1$	1A	Accept 1.83 : 1, 1 : 0.546
	9	
(c) Volume of paint		
$= \pi\int_{-t}^4 4(y + t) dy - 32\pi$	1A+1M	1A for first term, 1M for difference
$= \pi[2(y + t)^2]_{-t}^4 - 32\pi$		
$= 2\pi(4 + t)^2 - 32\pi$		
$= 16\pi t + 2\pi t^2$	1A	
$\doteq 16\pi t$ ($\because$ t is small)	1	
	4	

12. (a) $y = (1+x)^{m+1}(1-x)^n$

$\frac{dy}{dx} = (m+1)(1+x)^m(1-x)^n - n(1+x)^{m+1}(1-x)^{n-1}$ — 1A+1A

$\therefore (m+1)\int (1+x)^m(1-x)^n dx$

$= (1+x)^{m+1}(1-x)^n + n\int (1+x)^{m+1}(1-x)^{n-1}dx$ — 1

3

(b) From (a),

$(m+1)\int_{-1}^{1}(1+x)^m(1-x)^n dx$

$= [(1+x)^{m+1}(1-x)^n]_{-1}^{1} + n\int_{-1}^{1}(1+x)^{m+1}(1-x)^{n-1}dx$ — 1A

$= n\int_{-1}^{1}(1+x)^{m+1}(1-x)^{n-1}dx$ — 1A

$\therefore \int_{-1}^{1}(1+x)^m(1-x)^n dx = \frac{n}{m+1}\int_{-1}^{1}(1+x)^{m+1}(1-x)^{n-1}dx$ — 1

3

(c) $\int_{-1}^{1}(1+x)^8 dx = [\frac{1}{9}(1+x)^9]_{-1}^{1}$ — 1A — Expansion not accepted

$= \frac{512}{9}$ — 1A — Accept $\frac{2^9}{9}$, 56.9

2

(d) $x = \tan\theta$

$\cos^2\theta = \frac{1}{1+x^2}$ — 1A — Accept $\cos\theta = \frac{1}{\sqrt{1+x^2}}$

$\cos 2\theta = \frac{1-x^2}{1+x^2}$ — 1A

$dx = \sec^2\theta d\theta$ — 1A

$d\theta = \frac{dx}{\sec^2\theta} = \frac{dx}{1+x^2}$ — 1A

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\cos^2 2\theta(1+\tan\theta)^4}{\cos^6\theta}d\theta$

$= \int_{-1}^{1}\frac{(\frac{1-x^2}{1+x^2})^2(1+x)^4}{(\frac{1}{1+x^2})^3}\frac{dx}{1+x^2}$ — ~~(pp-1) for not~~ changing the limits of integration

$= \int_{-1}^{1}(1-x^2)^2(1+x)^4 dx$

$= \int_{-1}^{1}(1+x)^6(1-x)^2 dx$ — 1

Solution	Marks	Remarks
Alternative solution		
$x = \tan\theta$		
$dx = \sec^2\theta d\theta$	1A	
$\int_{-1}^{1}(1+x)^6(1-x)^2dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}(1+\tan\theta)^6(1-\tan\theta)^2\sec^2\theta d\theta$	1A	(pp-1) for not changing the limits of integration
$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{(1+\tan\theta)^4}{\cos^2\theta}(1-\tan^2\theta)^2d\theta$	1A	
$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{(1+\tan\theta)^4}{\cos^2\theta}\frac{(\cos^2\theta-\sin^2\theta)^2}{\cos^4\theta}d\theta$	1A	
$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\cos^2 2\theta(1+\tan\theta)^4}{\cos^6\theta}d\theta$	1	
$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\cos^2 2\theta(1+\tan\theta)^4}{\cos^6\theta}d\theta$		
$= \int_{-1}^{1}(1+x)^6(1-x)^2dx$		
$= \frac{2}{7}\int_{-1}^{1}(1+x)^7(1-x)dx$	1A	
$= \frac{2}{7}\cdot\frac{1}{8}\int_{-1}^{1}(1+x)^8dx$	1A	
$= \frac{2}{7}\cdot\frac{1}{8}\cdot\frac{512}{9}$		
$= \frac{128}{63}$	1A	Accept $\frac{2^7}{63}$, 2.03
	8	