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香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九一年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1991

附加數學卷一
ADDITIONAL MATHEMATICS PAPER I

評卷參考
MARKING SCHEME

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It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

Markers should therefore resist pleas from their students to have access to this document. Making it available to students would constitute misconduct on the part of the marker.

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員如向學生披露本評卷參考內容，即違背閱卷員守則。

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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method. <u>In general, a correct answer merits all the marks allocated to that part, provided that the method used is sound</u>.

2. In a question consisting of several parts each depending on the previous parts, marks should be awarded to steps or methods correctly deduced from previous erroneous answers. However, marks for the corresponding answer should <u>NOT</u> be awarded. In the marking scheme, 'M' marks are awarded for showing correct method use, and 'A' marks are awarded for the accuracy of the answers.

3. The symbol (pp-1) should be used to denote marks deducted for poor presentation (p.p.). Marks entered in the box should be the <u>net</u> total scored on that page. Note the following points :

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

4. Numerical answers should be given in exact value unless otherwise specified in the question. However answers not in exact values would be accepted this year provided that they are correct to at least 3 significant figures.

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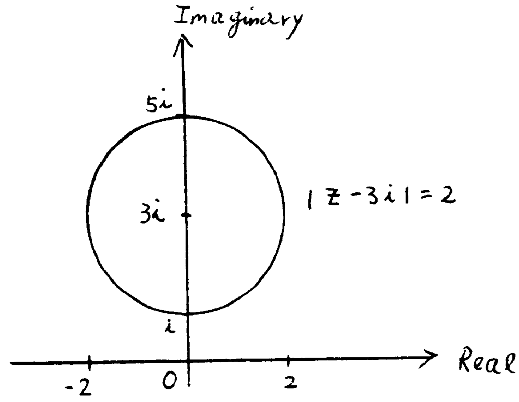
1.



circle	1A	Axes not labelled (pp-1)
correct centre	1A	
correct radius	1A	
$2 + 3i$	1A	(2,3) not accepted
	4	

2.

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{1}{h}\left(\dfrac{1}{1+x+h} - \dfrac{1}{1+x}\right)$	1A	
$= \dfrac{1}{h}\,\dfrac{-h}{(1+x+h)(1+x)}$	1A	
$= \dfrac{-1}{(1+x+h)(1+x)}$	1A	
$f'(x) = \lim_{h \to 0} \dfrac{-1}{(1+x+h)(1+x)}$	1M	
$= \dfrac{-1}{(1+x)^2}$	1A	
	5	

3.

$\lvert x - 2\rvert = \lvert x^2 - 4\rvert$		
$\lvert x - 2\rvert = \lvert x - 2\rvert\,\lvert x + 2\rvert$	2A	
$x = 2$ or $\lvert x + 2\rvert = 1$		
$x + 2 = \pm 1$		
$x = -1$ or -3		
$\therefore x = -1, -3$ or 2	1A+1A+1A	
	5	

Alternative solutions

(1) $x - 2 = x^2 - 4$ or $x - 2 = -(x^2 - 4)$	2A
$x - 2 = (x - 2)(x + 2)$; $x - 2 = -(x - 2)(x + 2)$	
$x = 2$ or $x + 2 = 1$; $x = 2$ or $x + 2 = -1$	
$x = -1$; $x = -3$	
$\therefore x = -1, -3$ or 2	1A+1A+1A

(2)	$(x - 2)^2 = (x^2 - 4)^2$ $(x - 2)^2 [(x + 2)^2 - 1] = 0$ $(x - 2)^2 (x + 3)(x + 1) = 0$ $x = 2, -1$ or -3	2A 1A+1A+1A	
(3)	3 cases : $x \geq 2$, $-2 < x < 2$, $x \leq -2$	1A	
	Case 1 : $x \geq 2$ $x - 2 = x^2 - 4$ $x = 2$ or -1 (rejected) $\therefore x = 2$		
	Case 2 : $-2 < x < 2$ $-(x - 2) = -(x^2 - 4)$ $x = -1$ or 2 (rejected) $\therefore x = -1$		
	Case 3 : $x \leq -2$ $-(x - 2) = x^2 - 4$	1A	Awarded only if the 3 equations are all correct
	$x = -3$ or 2 (rejected) $\therefore x = -3$ $\therefore x = -1, 2$ or -3	1A+1A+1A	

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4. (a) $\frac{dy}{dx} = 1 + 2\cos 2x$	1A	
$\frac{d^2y}{dx^2} = -4\sin 2x$	1A	
(b) $1 + 2\cos 2x = 0$	1M	
$\cos 2x = -\frac{1}{2}$		
$2x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ $\quad (0 \le x \le \pi)$		
$x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$	1A	Do not accept degrees, but carry forward
$\left.\frac{d^2y}{dx^2}\right\|_{x=\frac{\pi}{3}} = -2\sqrt{3} < 0 \quad \therefore$ max	1M	Accept $\left.\frac{d^2y}{dx^2}\right\|_{x=\frac{\pi}{3}} < 0 \quad \therefore$ max
$y_{max} = \frac{\pi}{3} + \sin\frac{2\pi}{3}$		
$= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$	1A	Accept 1.91 (awarded only if max. is checked)
$\left.\frac{d^2y}{dx^2}\right\|_{x=\frac{2\pi}{3}} = 2\sqrt{3} > 0 \quad \therefore$ min		
$y_{min} = \frac{2\pi}{3} + \sin\frac{4\pi}{3}$		
$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$	1A	Accept 1.23 (awarded only if min. is checked)
	7	

5. (a)	$\vec{OC} = \dfrac{\vec{a} + 3\vec{b}}{4}$	1A	$\frac{1}{4}\vec{a} + \frac{3}{4}\vec{b}$ Omit vector sign (pp-1)
(b) (i)	$\vec{OE} = \dfrac{k+1}{k}\vec{OC}$	1A	
	$= \dfrac{k+1}{4k}(\vec{a} + 3\vec{b})$	1A	
	$= \dfrac{k+1}{4k}\vec{a} + \dfrac{3(k+1)}{4k}\vec{b}$		
(ii)	$\vec{OD} = 2\vec{b}$	1A	
	$\vec{OE} = \dfrac{\vec{a} + 2m\vec{b}}{1+m}$	1A	
	$= \dfrac{1}{1+m}\vec{a} + \dfrac{2m}{1+m}\vec{b}$		
	$\therefore \begin{cases} \dfrac{k+1}{4k} = \dfrac{1}{1+m} \\ \dfrac{3(k+1)}{4k} = \dfrac{2m}{1+m} \end{cases}$	1M	
	Solving, $m = \dfrac{3}{2}$, $k = \dfrac{5}{3}$	1A	
		7	

6. (a)	$y' = -\frac{1}{x^2} + 1$	1A
	$y'\|_{x=1} = 0$	1A
	Equation of tangent at P : $y = 2$	1A
	Equation of normal at P : $x = 1$	1A
(b)	$y'\|_{x=\frac{1}{2}} = -3$	1A
	Equation of tangent at Q	
	$\dfrac{y - \frac{5}{2}}{x - \frac{1}{2}} = -3$	1A
	$y = -3x + 4$	
	Subs. $x = 0$, $y = 4$	
	$\therefore$ The tangent to C at Q passes through A.	1
		7

Alternative solution for (b)

$y'\|_{x=\frac{1}{2}} = -3$	1A
Slope of $AQ = \dfrac{4 - \frac{5}{2}}{0 - \frac{1}{2}}$	
$= -3$	1A
$=$ slope of tangent at Q	
$\therefore$ The tangent to C at Q passes through A.	1

7. (a)	$pq = 1 - k(p + q)$	1A	can be omitted
	$= 1 - k(2 - k)$	1A	
	$= 1 - 2k + k^2$		
(b)	The equation is		
	$x^2 - (2 - k)x + (1 - 2k + k^2) = 0$	1A+1M	$x^2 + (k - 2)x + (k - 1)^2 = 0$
	$[-(2 - k)]^2 - 4(1 - 2k + k^2) \geq 0$	1M	do not accept > 0
	$4k - 3k^2 \geq 0$		
	$k(4 - 3k) \geq 0$	1A	or $k(3k - 4) \leq 0$
	$0 \leq k \leq \frac{4}{3}$	1A	
		7	

8. (a) $\vec{CA} = \vec{OA} - \vec{OC}$	1M	Omit vector sign (pp-1)
$= (3-x)\vec{i} - (y+1)\vec{j}$	1A	
$\vec{OB} = \vec{OC} - \vec{BC}$		
$= (x-7)\vec{i} + (y-1)\vec{j}$	1A	
$\vec{AB} = \vec{OB} - \vec{OA}$		
$= (x-10)\vec{i} + y\vec{j}$	1A	
	4	
(b) (i) $\vec{AB} \cdot \vec{BC} = 4\vec{BC} \cdot \vec{CA}$		
$7(x-10) + y = 4[7(3-x) - (y+1)]$	1M	
$5y = -35x + 150$		
$y = 30 - 7x$ ------ (1)	1	
(ii) (1) $\lvert\vec{BC}\rvert = \sqrt{5}\lvert\vec{CA}\rvert$		
$\sqrt{7^2+1^2} = \sqrt{5}\sqrt{(3-x)^2+(y+1)^2}$	1M	
$(3-x)^2 + (y+1)^2 = 10$		
$x^2 + y^2 - 6x + 2y = 0$ ------ (2)	~~1A~~	
Subs. (1) int (2)		
$x^2 + (30-7x)^2 - 6x + 2(30-7x) = 0$	1M	For substitution
$50x^2 - 440x + 960 = 0$	1A	
$x = 4$ or $\frac{24}{5}$	1A	
$x = 4,\ y = 2$		
$x = \frac{24}{5},\ y = \frac{-18}{5}$ rejected $\because y > 0$		
$\therefore\ x = 4,\ y = 2$	1A	
	7	

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(2) $\vec{CA} = -\vec{i} - 3\vec{j}$		
$\vec{AB} = -6\vec{i} + 2\vec{j}$		
$\vec{CA} \cdot \vec{AB} = -(-6) - 3(2) = 0$	1A	Omit dot sign (pp-1)
$\therefore CA \perp AB$	1	
Alternative solution for (b) (ii) (2)		
Slope of $CA = 3$		
Slope of $AB = -\frac{1}{3}$		
Slope of CA . Slope of $AB = -1$	1A	
$\therefore CA \perp AB$	1	
(3) $\vec{OA} = 3\vec{i} - \vec{j}$ OR $\vec{OA} = 3\vec{i} - \vec{j}$		
$\vec{OB} = -3\vec{i} + \vec{j}$ $\vec{AB} = -6\vec{i} + 2\vec{j}$	1A	
$\vec{OA} = -\vec{OB}$ $\vec{OA} = -\frac{1}{2}\vec{AB}$	1A	
$\therefore$ O lies on AB	1	
	12	
Alternative solution for (b) (ii) (3)		
(1) $\vec{OB} = -3\vec{i} + \vec{j}$	1A	
Slope of OB = $-\frac{1}{3}$ = slope of OA	1A	
$\therefore O$ lies on AB	1	
(2) Equation of AB : $x + 3y = 0$	1A	
$(0, 0)$ satisfy $x + 3y = 0$	1A	
$\therefore O$ lies on AB.	1	

$\vec{a} : \vec{b} = k : 1$

pp-1

9. (a) $g(x) = -2(x+3)^2 - 5$	1A+1A	1A for $-2(x+3)^2$ 1A for -5
$\because -2(x+3)^2 - 5 \le -5$ for all x		or $(x+3)^2 \ge 0$
$\therefore g(x) < 0$ for all x.	1	
	3	
(b) (i) $(x^2 + 2x - 2) + k(-2x^2 - 12x - 23) = 0$		
$(1-2k)x^2 + (2-12k)x - (2+23k) = 0$		
For equal roots		
$(2-12k)^2 + 4(1-2k)(2+23k) = 0$	1M	
$-40k^2 + 28k + 12 = 0$	1A	
$10k^2 - 7k - 3 = 0$		
$(10k+3)(k-1) = 0$		
$k = 1$ or $-\frac{3}{10}$		
$k_1 = 1$, $k_2 = -\frac{3}{10}$	1A+1A	Awarded only if the equation is correct
(ii) $f(x) + k_1 g(x)$		
$= (x^2 + 2x - 2) - (2x^2 + 12x + 23)$		
$= -x^2 - 10x - 25$		
$= -(x+5)^2$	1A	
$\therefore f(x) + g(x) \le 0$ for all x	1	
$f(x) + k_2 g(x)$		
$= (x^2 + 2x - 2) + \frac{3}{10}(2x^2 + 12x + 23)$		
$= \frac{8}{5}(x^2 + \frac{7}{2}x + \frac{49}{16})$		
$= \frac{8}{5}(x + \frac{7}{4})^2$	1A	$\frac{1}{10}(4x+7)^2$
$\therefore f(x) - \frac{3}{10}g(x) \ge 0$ for all x	1	
	8	

(c) $f(x) + g(x) \le 0$		
$f(x) \le -g(x)$	1A	
$\frac{f(x)}{g(x)} \ge -1$ for all x $(\because g(x) < 0)$	1M	accept omitting $g(x) < 0$
(and the equality holds when $x = -5$)		
$\therefore$ Least value $= -1$	1A	
$f(x) - \frac{3}{10} g(x) \ge 0$		
$f(x) \ge \frac{3}{10} g(x)$	1A	
$\frac{f(x)}{g(x)} \le \frac{3}{10}$ for all x $(\because g(x) < 0)$		accept omitting $g(x) < 0$
(and the equality holds when $x = -\frac{7}{4}$)		
$\therefore$ Greatest value $= \frac{3}{10}$	1A	(pp-1 if not specify the greatest and least values)
	5	

10. (a) (i) $t^2 + t + 1 = 0$		
$t = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$	1A	
$= \cos(\pm\frac{2\pi}{3}) + i\sin(\pm\frac{2\pi}{3})$	1A+1A	
(ii) Put $z^3 = t$	1M	
$z^3 = \cos(\pm\frac{2\pi}{3}) + i\sin(\pm\frac{2\pi}{3})$		
$z = \cos\frac{1}{3}(2k\pi \pm \frac{2\pi}{3}) + i\sin\frac{1}{3}(2k\pi \pm \frac{2\pi}{3})$	1A	
$K = -1,\ 0,\ 1$	2A	$k = 0, 1, 2$ (1A only)
OR $z = \cos\theta + i\sin\theta$ where $\theta = \pm\frac{2\pi}{9}, \pm\frac{4\pi}{9}, \pm\frac{8\pi}{9}$	1A+1A+1A	1 mark for every 2 answers.
	7	
(b) (i) $[z - \cos\theta - i\sin\theta][z - \cos(-\theta) - i\sin(-\theta)]$		
$= (z - \cos\theta - i\sin\theta)(z - \cos\theta + i\sin\theta)$	1A	For $\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$
$= (z - \cos\theta)^2 + \sin^2\theta$	1A	For an expression not involving i
$= z^2 - 2z\cos\theta + 1$	1	
(ii) $z^6 + z^3 + 1$		
$= [z - \cos\frac{2\pi}{9} - i\sin\frac{2\pi}{9}]\,[z - \cos(-\frac{2\pi}{9}) - i\sin(-\frac{2\pi}{9})]$ $[z - \cos\frac{4\pi}{9} - i\sin\frac{4\pi}{9}]\,[z - \cos(-\frac{4\pi}{9}) - i\sin(-\frac{4\pi}{9})]$ $[z - \cos\frac{8\pi}{9} - i\sin\frac{8\pi}{9}]\,[z - \cos(-\frac{8\pi}{9}) - i\sin(-\frac{8\pi}{9})]$		1M
$= (z^2 - 2z\cos\frac{2\pi}{9} + 1)\ (z^2 - 2z\cos\frac{4\pi}{9} + 1)\ (z^2 - 2z\cos\frac{8\pi}{9} + 1)\quad \ldots(*)$	1	
	5	
(c) Put $z = i$ in (*)	1A	or $z = -i$
$-i = 8i\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9}$	1A+1A	1A for L.H.S. 1A for R.H.S.
$\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9} = -\frac{1}{8}$	1	
	4	

(ii) $\frac{dp}{dh} = 0$ when $h = 9$ — 1A

when $(6 <) h < 9$, $\frac{dp}{dh} < 0$

$h > 9$, $\frac{dp}{dh} > 0$

$\frac{d^2p}{dh^2} = \frac{54}{h^{1/2}(h-6)^{5/2}}$

$\frac{d^2p}{dh^2} > 0$ at $h = 9$

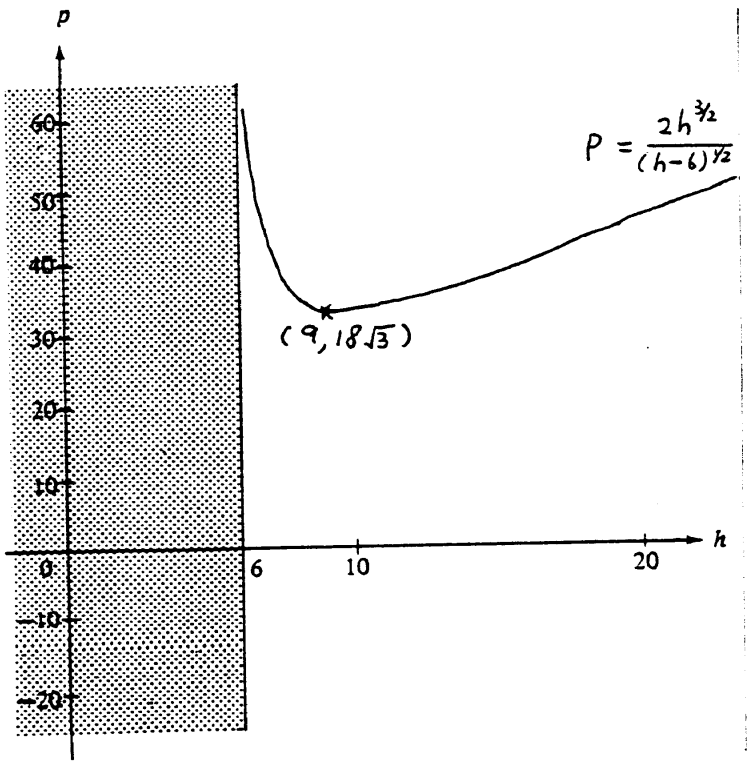
$\therefore p$ is minimum at $h = 9$ — 1M

$p_{min} = \frac{2 \cdot 9^{3/2}}{(9-6)^{1/2}}$

$= 18\sqrt{3}$ — 1A (6)

Accept 31.2 (awarded even if min. is not checked)

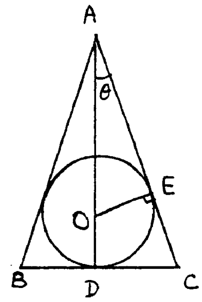
(d) (i)



1A Shape

1A Labelled minimum point

(ii) From the graph, $p > 18\sqrt{3}$ — 2A (4)



11. (a) $\Delta ACD \sim \Delta AOE$ (or $\frac{AE}{OE} = \frac{AD}{DC}$) — 1M

$\frac{\sqrt{(h-3)^2 - 3^2}}{3} = \frac{h}{t}$ — 1A

$t^2 = \frac{9h}{h-6}$ — 1 — 3

Alternative solution

$\Delta ACD \sim \Delta AOE$ (or $\frac{OE}{AO} = \frac{DC}{AC}$) — 1M

$\frac{t}{\sqrt{h^2 + t^2}} = \frac{3}{h-3}$ — 1A

$t^2(h^2 - 6h + 9) = 9(h^2 + t^2)$

$t^2 = \frac{9h}{h-6}$ — 1

(b) $p = 2t + 2\sqrt{h^2 + t^2}$ — 1A

$= 2\sqrt{\frac{9h}{h-6}} + 2\sqrt{h^2 + \frac{9h}{h-6}}$ — 1A

$= \frac{6h^{1/2} + 2h^{1/2}(h^2 - 6h + 9)^{1/2}}{\sqrt{h-6}}$

$= \frac{6h^{1/2} + 2(h-3)h^{1/2}}{(h-6)^{1/2}}$ ($\because h > 3$)

$= \frac{2h^{3/2}}{(h-6)^{1/2}}$ — 1 — 3

(c) (i) $\frac{dp}{dh} = \frac{(h-6)^{1/2}3h^{1/2} - h^{3/2}(h-6)^{-1/2}}{(h-6)}$ — 1M+1A — 1M for quotient rule

$= \frac{3h^{1/2}(h-6) - h^{3/2}}{(h-6)^{3/2}}$

$= \frac{2h^{1/2}(h-9)}{(h-6)^{3/2}}$

$\frac{dp}{dh} > 0$

$\frac{2h^{1/2}(h-9)}{(h-6)^{3/2}} > 0$

$h - 9 > 0$ ($\because h > 6$)
$h > 9$ — 1A

12. (a) (i) $\angle OCP = \theta$		
$CP = \cos\theta$	1A	
Also $CP = 2\cos\phi$	1A	
$\therefore \cos\theta = 2\cos\phi$	1	
(ii) S = area of sector CAB − area of ΔCAB	1M	
$= \frac{1}{2}(2)^2(2\phi) - \frac{1}{2}(2)^2\sin 2\phi$		
$= 4\phi - 2\sin 2\phi$	1	
	5	
(b) (i) $\cos\theta = 2\cos\phi$		
$-\sin\theta = -2\sin\phi \frac{d\phi}{d\theta}$	1A	
$\frac{d\phi}{d\theta} = \frac{\sin\theta}{2\sin\phi}$	1A	
(ii) $\frac{dS}{d\theta} = \frac{dS}{d\phi}\frac{d\phi}{d\theta}$	1M	
$= (4 - 4\cos 2\phi)\frac{d\phi}{d\theta}$	1A	1A for $\frac{ds}{d\phi}$
$= (4 - 4\cos 2\phi)\frac{\sin\theta}{2\sin\phi}$	1A	
$= 4\sin\theta\sin\phi$		
$= 4\sin\theta\sqrt{1 - \frac{1}{4}\cos^2\theta}$	2A	$2\sin\theta\sqrt{4 - \cos^2\theta}$
	7	
(c) $\frac{d\theta}{dt} = -\frac{1}{30}$	1A	
$\frac{dS}{dt} = \frac{dS}{d\theta}\cdot\frac{d\theta}{dt}$	1M	
$= 2\sin\theta\sqrt{4-\cos^2\theta}\frac{d\theta}{dt}$		
At $\theta = \frac{\pi}{3}$		
$\frac{ds}{dt} = 2\frac{\sqrt{3}}{2}\sqrt{4-(\frac{1}{2})^2}(-\frac{1}{30})$	1M	for substitution
$= \frac{-\sqrt{5}}{20}$ (per second)	1A	Accept −0.112
	4	