91-CE
A MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1991

ADDITIONAL MATHEMATICS PAPER I

8.30 am-10.30 am (2 hours)
This paper must be answered in English

Answer ALL questions in Section A and any THREE questions in Section B.

All working must be clearly shown.

Unless otherwise specified in a question, numerical answers must be given in **exact value**.

SECTION A (42 marks)

Answer **ALL** questions in this section.

1. In an Argand diagram, sketch the locus of the point representing the complex number z which satisfies the equation

$$|z - 3i| = 2.$$

Let P be the point representing the complex number $4 + 3i$. Write down the complex number represented by the point on the locus which is nearest to P.

(4 marks)

2. Let $f(x) = \dfrac{1}{1 + x}$.

Find $f'(x)$ from first principles.

(5 marks)

3. Solve $|x - 2| = |x^2 - 4|$.

(5 marks)

4. Let $y = x + \sin 2x$, where $0 \le x \le \pi$.

Find (a) $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$,

(b) the maximum and minimum values of y.

(7 marks)

5.

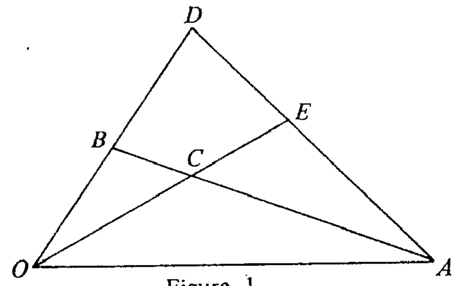

Figure 1

In Figure 1, OAD is a triangle and B is the mid-point of OD. The line OE cuts the line AB at C such that $AC : CB = 3 : 1$.

Let $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$.

(a) Express $\vec{OC}$ in terms of **a** and **b**.

(b) (i) Let $OC : CE = k : 1$. Express $\vec{OE}$ in terms of k, **a** and **b**.

(ii) Let $AE : ED = m : 1$. Express $\vec{OE}$ in terms of m, **a** and **b**.

Hence find k and m.

(7 marks)

6. Let C be the curve $y = \frac{1}{x} + x$, where $x \neq 0$. $P(1, 2)$ and $Q(\frac{1}{2}, \frac{5}{2})$ are two points on C.

(a) Find equations of the tangent and normal to C at P.

(b) Show that the tangent to C at Q passes through the point $A(0, 4)$.

(7 marks)

7. p, q and k are real numbers satisfying the following conditions :

$$\begin{cases} p + q + k = 2, \\ pq + qk + kp = 1. \end{cases}$$

(a) Express pq in terms of k.

(b) Find a quadratic equation, with coefficients in terms of k, whose roots are p and q.

Hence find the range of possible values of k.

(7 marks)

SECTION B (48 marks)

Answer any **THREE** questions in this section.
Each question carries 16 marks.

8. A, B and C are three points on a plane such that

$$\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j},$$

$$\overrightarrow{BC} = 7\mathbf{i} + \mathbf{j},$$

and $$\overrightarrow{OC} = x\mathbf{i} + y\mathbf{j},$$

where O is the origin.

(a) Find $\overrightarrow{CA}$, $\overrightarrow{OB}$ and $\overrightarrow{AB}$ in terms of x, y, $\mathbf{i}$ and $\mathbf{j}$.

(4 marks)

(b) Given $\overrightarrow{AB} \cdot \overrightarrow{BC} = 4\,\overrightarrow{BC} \cdot \overrightarrow{CA}$.

(i) Show that $y = 30 - 7x$.

(ii) If $|\overrightarrow{BC}| = \sqrt{5}\,|\overrightarrow{CA}|$ and x, y are positive,

(1) find x and y,

(2) show that CA is perpendicular to AB,

(3) show that O lies on AB.

(12 marks)

9. Let $f(x) = x^2 + 2x - 2$

and $g(x) = -2x^2 - 12x - 23$.

(a) Express $g(x)$ in the form $a(x+b)^2 + c$, where a, b and c are real constants.

Hence show that $g(x) < 0$ for all real values of x.

(3 marks)

(b) Let k_1 and k_2 $(k_1 > k_2)$ be the two values of k such that the equation $f(x) + kg(x) = 0$ has equal roots.

(i) Find k_1 and k_2.

(ii) Show that

$$f(x) + k_1 g(x) \le 0$$

and $f(x) + k_2 g(x) \ge 0$ for all real values of x.

(8 marks)

(c) Using (a) and (b), or otherwise, find the greatest and least values of $\frac{f(x)}{g(x)}$.

(5 marks)

10. (a) (i) Solve $t^2 + t + 1 = 0$, expressing the roots in the form $\cos\theta + i\sin\theta$, where $-\pi < \theta \le \pi$.

(ii) Hence find the roots of $z^6 + z^3 + 1 = 0$ in the form $\cos\theta + i\sin\theta$, where $-\pi < \theta \le \pi$.

(7 marks)

(b) (i) Show that

$$[z - \cos\theta - i\sin\theta][z - \cos(-\theta) - i\sin(-\theta)] = z^2 - 2z\cos\theta + 1.$$

(ii) Hence show that

$$z^6 + z^3 + 1 = (z^2 - 2z\cos\frac{2\pi}{9} + 1)(z^2 - 2z\cos\frac{4\pi}{9} + 1)(z^2 - 2z\cos\frac{8\pi}{9} + 1) \text{ .....}(*).$$

(5 marks)

(c) By substituting a suitable value of z into $(*)$, show that

$$\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9} = -\frac{1}{8}.$$

(4 marks)

11.

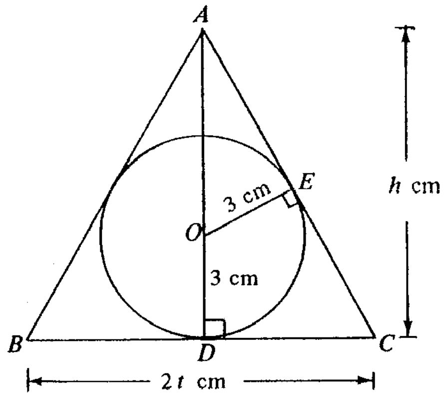

Figure 2(a)

ABC is a variable isosceles triangle with $AB = AC$ such that the radius of its inscribed circle is 3 cm. The height *AD* and the base *BC* of ΔABC are h cm and $2t$ cm respectively, where $h > 6$. (See Figure 2(a).) Let p cm be the perimeter of ΔABC.

(a) Show that $t^2 = \dfrac{9h}{h-6}$.

(3 marks)

(b) Show that $p = \dfrac{2h^{\frac{3}{2}}}{(h-6)^{\frac{1}{2}}}$.

(3 marks)

(c) Find

(i) the range of values of h for which $\dfrac{dp}{dh}$ is positive,

(ii) the minimum value of p .

(6 marks)

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If you attempt Question 11, fill in the details in the first three boxes above and tie this sheet into your answer book.

(d) (i) In Figure 2(b), sketch the graph of p against h for $h > 6$.

(ii) Hence write down the range of values of p for which two different isosceles triangles whose inscribed circles are of radii 3 cm can have the same perimeter p cm.

(4 marks)

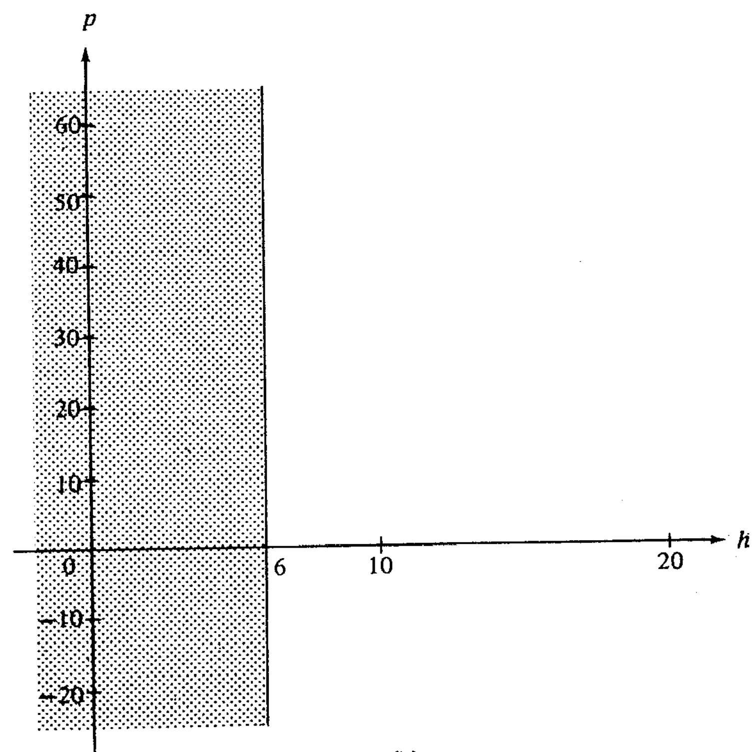

Figure 2(b)

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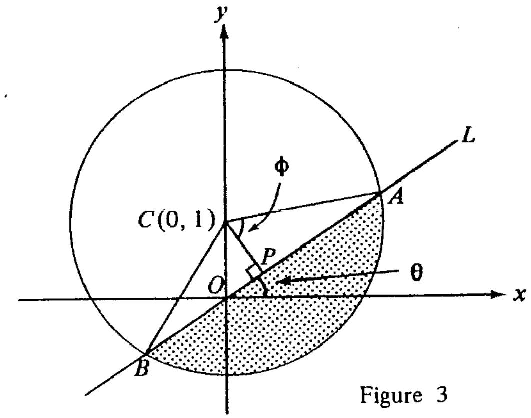

Figure 3

Figure 3 shows a circle of radius 2 centred at the point $C(0, 1)$. A variable straight line L with positive slope passes through the origin O and makes an angle θ with the positive x-axis. L intersects the circle at points A and B. Let S be the area of the shaded segment. P is the point on L such that CP is perpendicular to AB. Let $\angle PCA = \phi$.

(a) (i) Find the length of CP in terms of θ. Hence show that $\cos\theta = 2\cos\phi$.

(ii) Show that $S = 4\phi - 2\sin 2\phi$.

(5 marks)

(b) (i) Find $\dfrac{d\phi}{d\theta}$ in terms of θ and ϕ.

(ii) Hence find $\dfrac{dS}{d\theta}$ in terms of θ.

(7 marks)

(c) L rotates about O in the clockwise direction such that θ decreases steadily at a rate of $\dfrac{1}{30}$ radian per second. Find the rate of change of S with respect to time when $\theta = \dfrac{\pi}{3}$.

(4 marks)

END OF PAPER