90-CE
A MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1990

ADDITIONAL MATHEMATICS PAPER I

8.30 am-10.30 am (2 hours)
This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

SECTION A (42 marks)
Answer ALL questions in this section.

1. Let $f(x) = \sqrt{x^2 + k}\sin 2x$, where k is a constant.

   If $f'(0) = 1$, find the value of k.

   (5 marks)

2. Given $\overrightarrow{OA} = 5\mathbf{j}$, $\overrightarrow{OB} = -\mathbf{i} + 7\mathbf{j}$. P is a point such that $\overrightarrow{AP} = t\overrightarrow{AB}$.

   (a) Express $\overrightarrow{OP}$ in terms of t.

   (b) If OP is perpendicular to AB, find

   (i) the value of t,

   (ii) $\overrightarrow{OP}$.

   (6 marks)

3. (a) Express $\dfrac{1 + i\tan\theta}{1 - i\tan\theta}$ in polar form.

   (b) Hence, or otherwise, find the three cube roots of $\dfrac{1 + \sqrt{3}i}{1 - \sqrt{3}i}$.
   (Give your answers in polar form.)

   (6 marks)

4. α, β are the roots of the quadratic equation $x^2 - (k + 2)x + k = 0$.

   (a) Find $\alpha + \beta$ and $\alpha\beta$ in terms of k.

   (b) If $(\alpha + 1)(\beta + 2) = 4$, show that $\alpha = -2k$.
   Hence find the two values of k.

   (6 marks)

5. In the same Argand diagram, sketch the locus of the point representing the complex number z in each of the following cases :

(a) $|z - 3| = 1$;

(b) $|z| = |z - 2i|$.

Hence, or otherwise, find the complex number represented by the point of intersection of the two loci.

(6 marks)

6. Solve $(x+2)^2 - 8|x+2| + 15 \geqslant 0$.

(6 marks)

7. Given the curve $C : x^2 + 4xy + 5y^2 = 1$, find $\frac{dy}{dx}$.

Hence find the equations of the two tangents to C which are parallel to the line $y = -\frac{1}{2}x$.

(7 marks)

SECTION B (48 marks)
Answer any THREE questions from this section.
Each question carries 16 marks.

8.

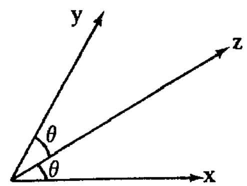

Figure 1(a)

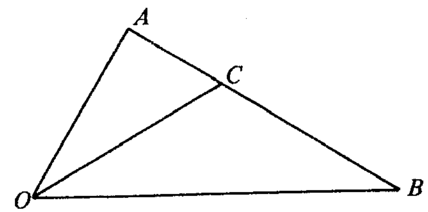

Figure 1(b)

(a) In Figure 1(a), $\mathbf{x}$ and $\mathbf{y}$ are unit vectors, each of which makes an angle θ with $\mathbf{z}$, where $0 < \theta < \frac{\pi}{2}$.

(i) Show that $\mathbf{x} \cdot \mathbf{z} = \mathbf{y} \cdot \mathbf{z}$.

(ii) Let $\mathbf{z} = m\mathbf{x} + n\mathbf{y}$.

By expressing $\mathbf{x} \cdot \mathbf{z}$ and $\mathbf{y} \cdot \mathbf{z}$ in terms of m, n and θ, show that $m = n$.

(8 marks)

(b) In Figure 1(b), $\overrightarrow{OA} = a\mathbf{u}$, $\overrightarrow{OB} = b\mathbf{v}$, where $\mathbf{u}$, $\mathbf{v}$ are unit vectors, $a > 0$ and $b > 0$. C is a point on AB such that $AC : CB = \lambda : 1$, where $\lambda > 0$.

(i) Express $\overrightarrow{OC}$ in terms of λ, a, b, $\mathbf{u}$ and $\mathbf{v}$.

(ii) If OC bisects $\angle AOB$, using the result of (a) (ii), show that $\lambda = \frac{a}{b}$.

(iii) Suppose $\overrightarrow{OA} = 3\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OB} = \frac{25}{3}\mathbf{i}$, and OC bisects $\angle AOB$. Using the result of (b) (ii), find $AC : CB$. Hence find $\overrightarrow{OC}$.

(8 marks)

9. Let $f(x) = x^2 + 4x + 1$. The curve $C_1 : y = f(x)$ cuts the x-axis at two points P and Q (See Figure 2).

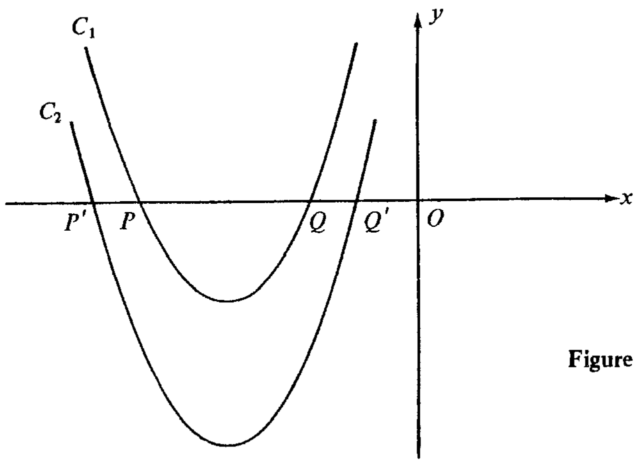

Figure 2

(a) (i) Write $f(x)$ in the form $(x + a)^2 + b$. Hence find the coordinates of the vertex of C_1.

(ii) Find the length of PQ. (Leave your answer in surd form.)

(5 marks)

(b) C_1 is shifted vertically downwards by m units to form the curve $C_2 : y = g(x)$. C_2 cuts the x-axis at two points P' and Q' (See Figure 2).

(i) Find the coordinates of the vertex of C_2 in terms of m. Hence, or otherwise, find $g(x)$.

(ii) Find the length of $P'Q'$ in terms of m.

(iii) If $P'Q' = 2PQ$, find the value of m.

(6 marks)

(c) C_1 is shifted horizontally towards the right by n units to form the curve $C_3 : y = h(x)$.

(i) Find the coordinates of the vertex of C_3 in terms of n. Hence find $h(x)$.

(ii) Find the two values of n such that C_3 passes through the origin.

(5 marks)

10. (a) C_1 is the curve $y = \dfrac{2x + 1}{x^2 + 2}$.

Find

(i) the range of values of x for which the slope of C_1 is negative;

(ii) the turning points of C_1; and for each point, state whether it is a maximum or a minimum point. (Testing for maximum/minimum is not required.)

(8 marks)

(b) In Figure 3, sketch the curve C_1 for $-3 \leqslant x \leqslant 3$.

(4 marks)

(c) C_2 is the curve $y = 1 - \dfrac{2x + 1}{x^2 + 2}$.

Using the result of (b) or otherwise, sketch the curve C_2 for $-3 \leqslant x \leqslant 3$ in Figure 3.

(4 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

10. If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet into your answer book.

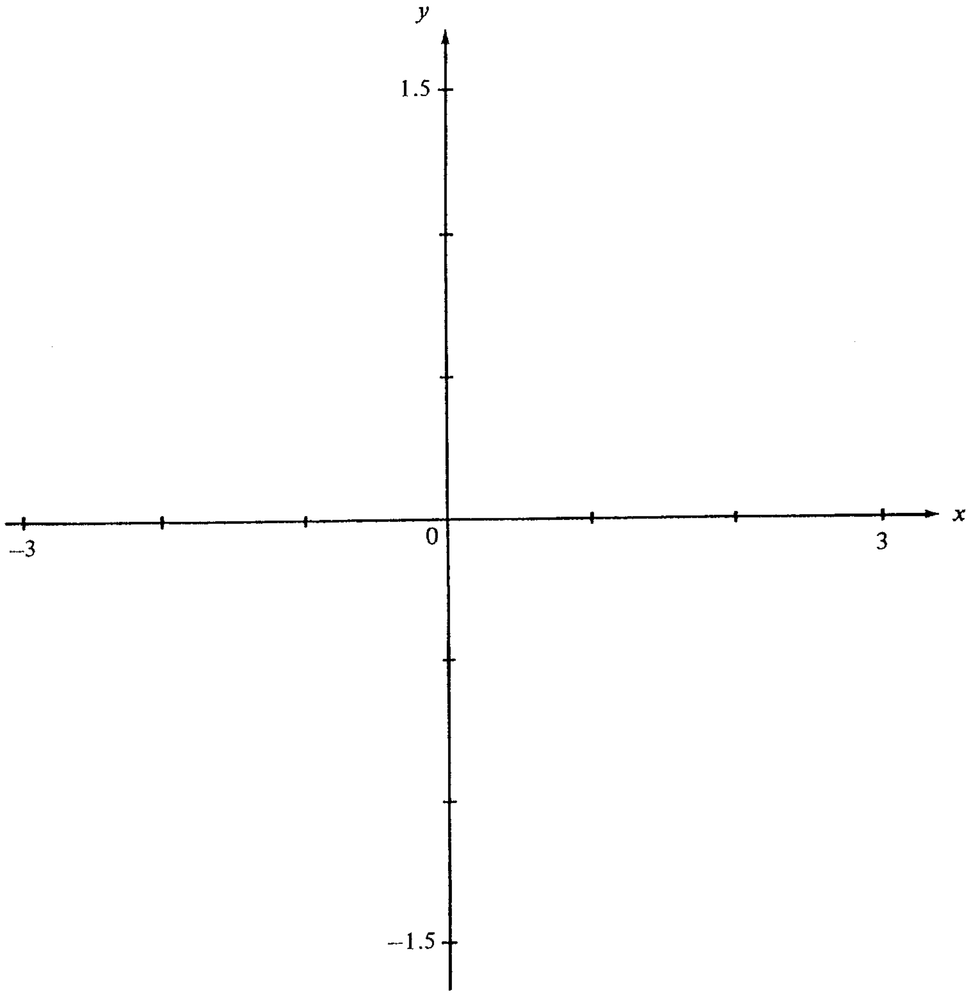

Figure 3

11.

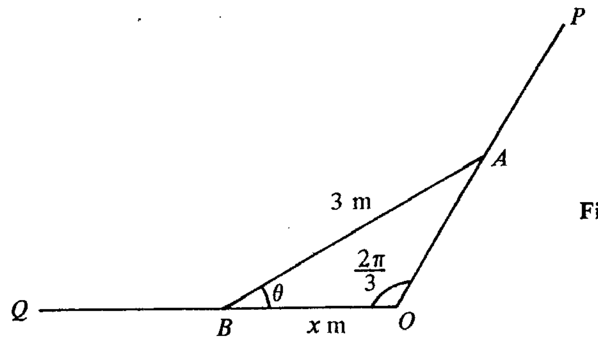

Figure 4

In Figure 4, POQ is a rail where OQ is horizontal and $\angle POQ = \frac{2\pi}{3}$. AB is a rod of length 3 m which is free to slide on the rail with end A on OP and end B on OQ. End A is initially at point O and end B is pushed towards O at a constant speed of $\frac{\sqrt{3}}{3}$ ms^{-1}. After t seconds, B is x metres from O and the rod makes an angle θ with the horizontal.

(a) Express x in terms of θ. (2 marks)

(b) Let S m^2 be the area of ΔAOB.

Show that $\frac{dS}{d\theta} = 3\sqrt{3}\sin\left(\frac{\pi}{3} - 2\theta\right)$.

Hence find the maximum value of S. (8 marks)

(c) (i) Show that $\frac{d\theta}{dt} = \frac{1}{6\cos\left(\frac{\pi}{3} - \theta\right)}$.

(ii) Find the range of the possible values of $\cos\left(\frac{\pi}{3} - \theta\right)$.

Hence determine the greatest and least values of $\frac{d\theta}{dt}$. (6 marks)

12. (a) Let $\bar{z}$ and $\text{Re}(z)$ denote the conjugate and the real part of a complex number z respectively.

Show that

(i) $z\bar{z}$ is real,

(ii) $z + \bar{z} = 2\text{Re}(z)$.

(2 marks)

(b)

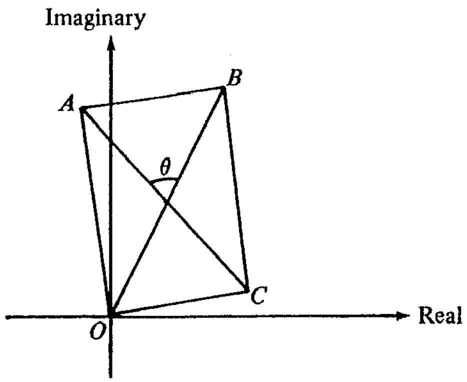

Figure 5

A, B and C are three points in the Argand diagram representing three distinct non-zero complex numbers p, q and r respectively, as shown in Figure 5. Let $p\bar{r} + \bar{p}r = 0$ and $OABC$ be a parallelogram.

(i) Show that $\text{Re}(p\bar{r}) = 0$ and $\text{Re}(\frac{p}{r}) = 0$.

(ii) Show that $OABC$ is a rectangle.

(iii) Let $\frac{p}{r} = 2i$.

Find $\frac{p-r}{p+r}$ in standard form.

Hence find the value of $\tan\theta$, where θ is the angle between the diagonals of $OABC$, as shown in Figure 5.

(14 marks)

END OF PAPER

90-CE
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PAPER II

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ADDITIONAL MATHEMATICS PAPER II

11.15 am-1.15 pm (2 hours)
This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.