香 港 考 試 局
HONG KONG EXAMINATIONS AUTHORITY

一 九 八 九 年 香 港 中 學 會 考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1989

附加數學 (卷二)
Additional Mathematics (Paper II)

評卷參考
Marking Scheme

請在學校任教之閱卷員特別留意

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

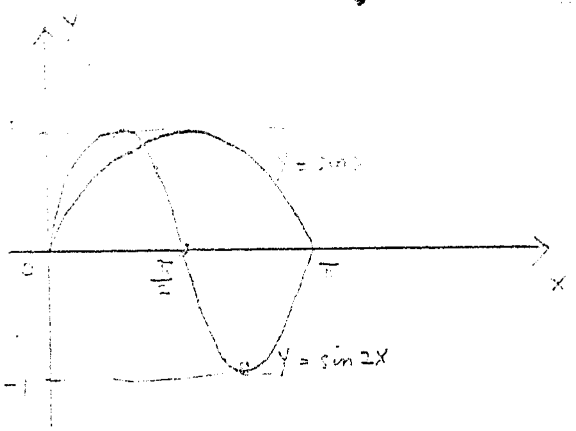
Special Notes for Teacher Markers

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

Markers should therefore resist pleas from their students to have access to this document. Making it available would constitute misconduct on the part of the marker and is, moreover in breach of the 1977 Hong Kong Examinations Authority Ordinance.

Solution	Marks	Remarks
1. $(1+x)^{10}(1-\frac{2}{x})^3$		
$= (1 + 10x + 45x^2 + 120x^3 + \ldots)(1 - \frac{6}{x} + \frac{12}{x^2} - \frac{8}{x^3})$	1A+1A	Accept nCr form, deduct 1 mark for missing ...
constant term		
$= 1(1) + 10(-6) + 45(12) + 120(-8)$	1M	一 要有 4 terms, 不 refer 到 上面.
$= 1 - 60 + 540 - 960$		
$= -479$	2A ― 5	
2. For $n = 1$, L.H.S. $= 1 \cdot 2 \cdot 3 = 6$		① n=1: ... = ... the statement is true for n=1 ✗
R.H.S. $= \frac{1(2)(3)(4)}{4} = 6$		
$\therefore$ the equality holds for $n = 1$.	1	
Assume the equality holds for some integer k .	1	② assume n=k is true
For $n = k+1$, L.H.S.		③ 首式亦可
$= (1)(2)(3) + (2)(3)(4) + \ldots + k(k+1)(k+2) + (k+1)(k+2)(k+3)$		
$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3)$	1	
$= \frac{1}{4}(k+1)(k+2)(k+3)(k+4)$	1	
$\therefore$ the equality holds for $n = k+1$.	[illegible]	
By the principle of Mathematical Induction, the equality holds for all +ve integers n.	1 ― 5	
3. $u = 2x^2 + 1$		
$du = 4x\,dx$	1A	
When $x = 0$, $u = 1$; $x = 2$, $u = 9$)	1A	
$\int_0^2 \frac{8x^3}{\sqrt{2x^2+1}}\,dx = \int_1^9 \frac{u-1}{\sqrt{u}}\,du$	1A	Integrand must be in terms of u.
$= \int_1^9 (\sqrt{u} - \frac{1}{\sqrt{u}})\,du$		
$= [\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}]_1^9$	1A	
$= \frac{40}{3}$ or 13.3	1A ― 5	

Solution	Marks	Remarks
(a) $\int \cos^2 2x \, dx$		
$= \int \frac{1}{2}(1 + \cos 4x)\, dx$	1A	
$= \frac{1}{2}x + \frac{1}{8}\sin 4x + c$	1A+1A	Deduct 1 mark for omitting c
(b) $\int \sin^2 2x \, dx$		
$= \int (1 - \cos^2 2x)dx$	1A	No mark if using $\int \frac{1}{2}(1 + \cos 4x)dx$
$= x - \int \cos^2 2x \, dx$		
$= \frac{1}{2}x - \frac{1}{8}\sin 4x + c'$	1A	
	5	
5 (a) $r = \sqrt{5^2 + (-12)^2} = 13$	1A	
$p = 7$	1A	
$\tan \alpha = \frac{12}{5}$		
$\alpha = 67.4° \ (67°23')$	1A	
$y = 13\sin(\theta - 67.4°) + 7$		
(b) Least value of $y = 13(-1) + 7$	1M	for putting $\sin(\theta - 67.4°) = -1$
$= -6$	1A	
	5	
6. $2\cos 2\theta + 5\sin\theta - 3 = 0$		
$2(1 - 2\sin^2\theta) + 5\sin\theta - 3 = 0$	1A	
$4\sin^2\theta - 5\sin\theta + 1 = 0$	1A	
$(4\sin\theta - 1)(\sin\theta - 1) = 0$		
$\sin\theta = \frac{1}{4}$ or 1	1A	
$\theta = 180k° + (-1)^k 14.5° \ (14°29')$	1A	$k\pi + (-1)^k(0.253)$
or $180k° + (-1)^k 90°$	1A	$k\pi + (-1)^k \frac{\pi}{2}$, $360k° + 90°$ use different units (pp-1)
where $k \in Z$		
	5	

Solution	Marks	Remarks
7. (a) Slope of $L_2 = \frac{17}{7}$		
$\left\lvert \dfrac{m - \frac{17}{7}}{1 + m(\frac{17}{7})} \right\rvert = \tan 45°$	1M	Accept formula with no absolute value sign or ± sign
$m = \frac{5}{12}$ or $-\frac{12}{5}$	1A+1A	
(b) For $m = \frac{5}{12}$, $L_1 : y = \frac{5}{12}x + c$		
$5x - 12y + 12c = 0$		
$\dfrac{5 - 12(2) + 12c}{\sqrt{5^2 + (-12)^2}} = \pm 5$	1M	Accept formula without ±
$-19 + 12c = \pm 65$		
$c = 7$ or $-\frac{23}{6}$	1A+1A 6	
8. (a) correct graph	1A	
		
$\sin 2x = \sin x$		
$2\sin x\cos x = \sin x$		
$\sin x = 0$ or $\cos x = \frac{1}{2}$	1A	
$x = 0, \pi$ or $\frac{\pi}{3}$	1A	not acceptable if in degree
(b) Required area =		
$\int_0^{\frac{\pi}{3}} (\sin 2x - \sin x)dx + \int_{\frac{\pi}{3}}^{\pi} (\sin x - \sin 2x)dx$	1M	for $\int_a^b (f_1(x) - f_2(x))dx$
$= [-\frac{1}{2}\cos 2x + \cos x]_0^{\frac{\pi}{3}} + [-\cos x + \frac{1}{2}\cos 2x]_{\frac{\pi}{3}}^{\pi}$		
$= (\frac{1}{4} + \frac{1}{2} + \frac{1}{2} - 1) + (1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{4})$		
$= \frac{1}{4} + 2\frac{1}{4}$		
$= 2\frac{1}{2}$	2A 6	

Solution	Marks	Remarks
1. (a) $x = \tan\theta$		
$dx = \sec^2\theta\, d\theta$	1A	
$x = 0,\quad \theta = 0$		
$x = 1,\quad \theta = \frac{\pi}{4}$	1A	
$\int_0^1 \frac{dx}{1+x^2} = \int_0^{\frac{\pi}{4}} \frac{\sec^2\theta}{1+\tan^2\theta}\, d\theta$		
$= \int_0^{\frac{\pi}{4}} d\theta$	1A	
$= \frac{\pi}{4}$	1A / 4	
(b) $\frac{d}{dx}\left[\frac{x}{(1+x^2)^{n-1}}\right]$ $= \frac{(1+x^2)^{n-1} - 2x^2(n-1)(1+x^2)^{n-2}}{(1+x^2)^{2(n-1)}}$	2A	
$= \frac{1}{(1+x^2)^{n-1}} - 2(n-1)\frac{x^2}{(1+x^2)^n}$		
Integrating both sides with respect to x.	1M	
$\frac{x}{(1+x^2)^{n-1}} = \int\frac{dx}{(1+x^2)^{n-1}} - 2(n-1)\int\frac{x^2}{(1+x^2)^n}\,dx$		
$\int\frac{x^2}{(1+x^2)^n}\,dx = \frac{1}{2(n-1)}\left[\int\frac{dx}{(1+x^2)^{n-1}} - \frac{x}{(1+x^2)^{n-1}}\right]$	1 / 4	
(c) $\int\frac{dx}{(1+x^2)^n} = \int\frac{dx}{(1+x^2)^{n-1}} - \int\frac{x^2}{(1+x^2)^n}\,dx$	1A	
$= \int\frac{dx}{(1+x^2)^{n-1}} - \frac{1}{2(n-1)}\int\frac{dx}{(1+x^2)^{n-1}} + \frac{1}{2(n-1)}\cdot\frac{x}{(1+x^2)^{n-1}}$	1A	
$= \frac{2n-3}{2n-2}\int\frac{dx}{(1+x^2)^{n-1}} + \frac{1}{2(n-1)}\cdot\frac{x}{(1+x^2)^{n-1}}$	1 / 3	
(d) (i) $\int_0^1\frac{dx}{(1+x^2)^2}$		
$= \frac{1}{2}\int_0^1\frac{dx}{(1+x^2)} + \frac{1}{2}\left[\frac{x}{1+x^2}\right]_0^1$	1A+1A	
$= \frac{1}{8}(\pi+2)$ $\left(\frac{\pi}{8}+\frac{1}{4}\right)$	1A	(= 0.643)
(ii) $\int_0^1\frac{dx}{(1+x^2)^3}$		
$= \frac{3}{4}\int_0^1\frac{dx}{(1+x^2)^2} + \frac{1}{4}\left[\frac{x}{(1+x^2)^2}\right]_0^1$	1A	
$= \frac{1}{32}(3\pi+8)$	1A 1A / 5	(= 0.545)
$\left(\frac{3\pi}{32}+\frac{1}{4}\right)$		

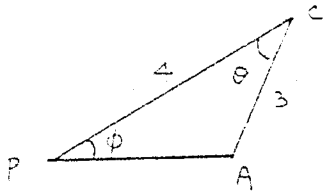
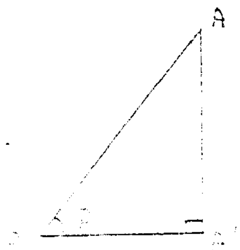
Solution	Marks	Remarks
9. (d)		
<u>Alt. Solution:</u>		
(i) $\displaystyle\int_0^1 \frac{dx}{(1+x^2)^2}$		
$\displaystyle = \int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$		
$\displaystyle = \int_0^{\frac{\pi}{4}} \frac{1}{2}(1+\cos 2\theta)\, d\theta$	2A	(integrable form)
$\displaystyle = \frac{1}{8}(\pi + 2)$	1A	
(ii) $\displaystyle\int_0^1 \frac{dx}{(1+x^2)^3}$		
$\displaystyle = \int_0^{\frac{\pi}{4}} \cos^4\theta\, d\theta$		
$\displaystyle = \int_0^{\frac{\pi}{4}} \frac{1}{4}[1 + 2\cos 2\theta + \frac{1}{2}(1+\cos 4\theta)]\, d\theta$	~~2A~~	or $\displaystyle\int_0^{\frac{\pi}{4}} (\frac{3}{8} + \frac{1}{2}\cos 2\theta - \frac{1}{8}\cos 4\theta)$
$\displaystyle = \frac{1}{32}(3\pi + 8)$	~~1A~~ 2A	

Solution	Marks	Remarks
(a) $x^2+y^2-6x-8y+21+k(x^2+y^2-18x-14y+105)=0$	1A	$k=\frac{1}{2}$
or $k(x^2+y^2-6x-8y+21)+(x^2+y^2-18x-14y+105)=0$		$\rightarrow k=2$
or $(x^2+y^2-6x-8y+21)+k(2x+y-14)=0$		$\rightarrow k=-2$
etc.		$\rightarrow k=-\frac{1}{3}$
C_3 passes through (5, 6).		
$25+36-30-48+21+k(25+36-90-84+105)=0$	1M	
$k=\frac{1}{2}$	1A	
$C_3 : \frac{3}{2}x^2+\frac{3}{2}y^2-15x-15y+\frac{147}{2}=0$	1A	
$x^2+y^2-10x-10y+49=0$		
or $(x-5)^2+(y-5)^2=1$		
Centre is at (5, 5) and it lies on $y=x$	1	
	5	
(b) Let the equation of tangent be $y=mx$.	1A	**Alt. Solution:**
Sub. in equation of C_3 :		Distance from centre (5, 5) to the line = radius
$x^2+m^2x^2-10x-10mx+49=0$	1M	$\left\lvert\frac{5m-5}{\sqrt{m^2+1}}\right\rvert=1$ 1M+1M
$(1+m^2)x^2-10(1+m)x+49=0$		⋮
For tangency, $D=0$.		$12m^2-25m+12=0$
$100(1+m)^2-4(49)(1+m^2)=0$	1M	
$12m^2-25m+12=0$		
$(4m-3)(3m-4)=0$		
$m=\frac{3}{4}$ or $\frac{4}{3}$	2A	
Equations of tangents : $y=\frac{3}{4}x$, $y=\frac{4}{3}x$		
Length of tangents		
$=\sqrt{(\text{Dist. from }(0,0)\text{ to centre})^2-(\text{radius})^2}$	1M	
$=\sqrt{(5^2+5^2)-1^2}$		
$=7$	1A	
	7	

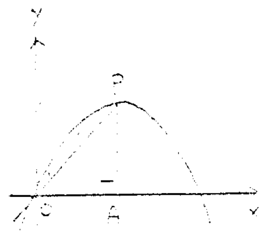
Solution	Marks	Remarks
10. (b) **Alt. Solution (1):**		
Sub. (0, 0) into equation of C_3.		
Length of tangent $= \sqrt{0^2 + 0^2 - 10(0) - 10(0) + 49}$	1M	
$= 7$	1A	
Alt. Solution (2):		
For points of contact:		
$x = \frac{10(m+1)}{2(m^2+1)} = \frac{5(m+1)}{m^2+1}$	1M	
$= \frac{28}{5}$ or $\frac{21}{5}$		
When $x = \frac{28}{5}$, $y = \frac{21}{5}$		
$x = \frac{21}{5}$, $y = \frac{28}{5}$		
Length of tangents $= \sqrt{(\frac{28}{5})^2 + (\frac{21}{5})^2}$		
$= 7$	1A	
(c) $x' = \frac{1}{3}a$, $y' = \frac{1}{3}b$	1A	
$a = 3x'$, $b = 3y'$	1M	for making a, b as subjects
P is a variable point on C_3.		
Therefore,		
$(3x')^2 + (3y')^2 - 10(3x') - 10(3y') + 49 = 0$	1M	
Equation of locus of M:		
$9x^2 + 9y^2 - 30x - 30y + 49 = 0$	1A	
	4	

Solution	Marks	Remarks
(a) $y^2 = 8x$		$y = \sqrt{8x}$ without $\pm$ sign, max. 2 marks for part (a)
$2y \cdot \frac{dy}{dx} = 8$	1A	
$\frac{dy}{dx} = \frac{4}{y}$	1A	
Equation of tangent: $\frac{y - y_0}{x - x_0} = \frac{4}{y_0}$	1M	
$y_0 y - y_0^2 = 4x - 4x_0$		
$y_0 y = 4x + y_0^2 - 4x_0$		
$y_0 y = 4x + 8x_0 - 4x_0$ }	1	
$y_0 y = 4x + 4x_0$	4	
(b) Since $y_0 \neq 0$,		
$y = \frac{4}{y_0} x + \frac{4x_0}{y_0}$	1A	
Put $\frac{4}{y_0} = m$	1A	
$\frac{4x_0}{y_0} = \frac{4}{y_0} \cdot \frac{y_0^2}{8}$	1M	
$= \frac{y_0}{2} = \frac{1}{2} \cdot \frac{4}{m}$		
$= \frac{2}{m}$	1	
$\therefore$ Equation of tangent is $y = mx + \frac{2}{m}$	4	
(c) Equation of tangent: $y = mx + \frac{2}{m}$		
This passes through $(-4, -2)$.		
$-2 = -4m + \frac{2}{m}$	1A	
$2m^2 - m - 1 = 0$		
$m = 1$ or $-\frac{1}{2}$	1A 2	
(d) $a = -12m + \frac{2}{m}$		Alt. Solution:
$12m^2 + am - 2 = 0$	1A	$12m^2 + am - 2 = 0$
$m_1 m_2 = -\frac{1}{6}$ }	1A	$m = \frac{-a \pm \sqrt{a^2+96}}{24}$
$m_1 + m_2 = -\frac{a}{12}$		$m_1 - m_2 = \frac{\pm\sqrt{a^2+96}}{12}$
$(m_1 - m_2)^2 = (-\frac{a}{12})^2 - 4(-\frac{1}{6})$	1M	$\pm \frac{\sqrt{a^2+96}}{12(1 - 1/6)} = \tan 45°$ ... 1M+1A
$= \frac{a^2}{144} + \frac{2}{3}$		$a = \pm 2$
$\tan 45° = \pm \frac{\sqrt{\frac{a^2}{144} + \frac{2}{3}}}{1 + (-\frac{1}{6})}$	1M+1A	
$\frac{a^2}{144} + \frac{2}{3} = (\frac{5}{6})^2$		
$a = \pm 2$	1A 6	

Solution	Marks	Remarks
2. (a) $\triangle ABC = \frac{1}{2}(3)(15)\sin 2\theta$		
$\triangle APC = \frac{1}{2}(3)(4)\sin\theta$	1A+1A	all correct 2A one or two correct 1A
$\triangle BPC = \frac{1}{2}(4)(15)\sin\theta$		
$\frac{45}{2}\sin 2\theta = 6\sin\theta + 30\sin\theta$	1M	
$\frac{45}{2}(2)\cos\theta\sin\theta = 36\sin\theta$	1M	
$\cos\theta = \frac{36}{45}$		
$= \frac{4}{5}$	1 — 5	
(b) $AA' = 3\sin\theta = \frac{9}{5}$ (cm)	1A	
$BB' = 15\sin\theta = 9$ (cm)	1A	
$A'B' = 15\cos\theta - 3\cos\theta$		
$= 12\cos\theta$		
$= \frac{48}{5}$ (cm)	1A	
$AB^2 = (AA')^2 + (A'B)^2$	1M	or $(BB')^2 + (AB')^2$
$= (AA')^2 + (BB')^2 + (A'B')^2$	1M	
$= (\frac{9}{5})^2 + 9^2 + (\frac{48}{5})^2$		
$= \frac{882}{5}$		
$AB = \sqrt{\frac{882}{5}}$		deduct 1 mark for answers without units
$= 13.28$		
≈ 13.3 (cm)	1A — 6	
(c) $BP^2 = 15^2 + 4^2 - 2(15)(4)\cos\theta$		
$= 225 + 16 - 96$		
$= 145$	1A	
$AP^2 = 3^2 + 4^2 - 2(3)(4)\cos\theta$		
$= 9 + 16 - \frac{96}{5}$		
$= \frac{29}{5}$	1A	
$AB = 15 - 3$		
$= 12$	1A	
$\cos\angle APB = \dfrac{145 + \frac{29}{5} - 12^2}{2\sqrt{145}\sqrt{\frac{29}{5}}}$	1M	
$= 0.117$		
$\angle APB = 83.3°$	1A — 5	

Solution	Marks	Remarks
12. (c) **Alt. Solution (1):**		
		
$\angle APB = 180° - 2\phi$	1M	
$AP^2 = \dfrac{29}{5}$	1A	
$\dfrac{\sin\theta}{\sqrt{\frac{29}{5}}} = \dfrac{\sin\phi}{3}$	1M	
$\phi = 48.37°$	1A	
$\angle APB = (180° - 2\phi)$		
$= \underline{83.3°}$	1A	
Alt. Solution (2):		
		
$\angle APB = 180° - 2\phi$	1M	
$AA' = \dfrac{9}{5}$		
$CA' = 3\cos\theta = \dfrac{12}{5}$		
$PA' = 4 - \dfrac{12}{5} = \dfrac{8}{5}$	1A	
$\tan\phi = \dfrac{\frac{9}{5}}{\frac{8}{5}} = \dfrac{9}{8}$	1M	
$\phi = 48.37°$	1A	
$\angle APB = 180° - 2\phi$		
$= \underline{83.3°}$	1A	

Solution	Marks	Remarks
13. (a) $\frac{dy}{dx} = -4x + k$		
$y = \int(-4x + k)dx$		
$= -2x^2 + kx + c$	1A	No mark for part (a) if omitting c.
The curve C passes through (0, 0) and (5, 10).		
Sub. (0, 0) and (5, 10) into equation of C.	1M	
$c = 0$	1A	
$10 = -2(5)^2 + 5k$		
$k = 12$	1A	
$y = -2x^2 + 12x$		
	4	
(b) $y = 0$		
$x = 0$ or 6	1A	
Area $= \int_0^6 (-2x^2 + 12x)dx$	1M	for $\int_a^b ydx$
$= [-\frac{2}{3}x^3 + \frac{12x^2}{2}]_0^6$		
$= 72$	1A	
	3	
(c) (i) OP : $y = \frac{b}{a}x$	1A	
Area of shaded region		
$= \int_0^a [(-2x^2 + 12x) - \frac{b}{a}x]dx$	1M	for $\int_a^b (f_1(x)-f_2(x))dx$
$= \int_0^a (-2x^2 + 12x - \frac{-2a^2 + 12a}{a}x)dx$	1M	for substituting $b = -2a^2 + 12a$
$= [-\frac{2}{3}x^3 + 6x^2 + (2a - 12)\frac{x^2}{2}]_0^a$		
$= \frac{1}{3}a^3$	1A	
$\frac{1}{3}a^3 = \frac{1}{8}(72)$		
$a = 3$	1A	

Solution	Marks	Remarks
3. (c) (i) **Alt. Solution:**		
Area of $\triangle OAP = \frac{1}{2}ab$	1A	
$= \frac{1}{2}a(-2a^2 + 12a)$	1M	for substituting $b = -2a^2 + 12a$
$= 6a^2 - a^3$		
Area of shaded region + $\triangle OAP = \int_0^a (-2x^2 + 12x)dx$		
$= 6a^2 - \frac{2}{3}a^3$		
Area of shaded region $= (6a^2 - \frac{2}{3}a^3) - (6a^2 - a^3)$	1M	
$= \frac{1}{3}a^3$	1A	
$\frac{1}{3}a^3 = \frac{1}{8}(72)$		
$a = 3$	1A	
(ii) Vol. of solid generated by shaded region and $\triangle OAP$		
$= \int_0^3 \pi y^2 dx$		
$= \int_0^3 \pi(12x - 2x^2)^2 dx$	1M+1M	1M for $\int_a^b \pi y^2 dx$
$= 4\pi\left[\frac{x^5}{5} - \frac{12x^4}{4} + \frac{36x^3}{3}\right]_0^3$		
$= \frac{32\pi(3)^4}{5}$		
$= 518.4\pi$ (or 1628.60)		
Vol. of cone $= \frac{1}{3}\pi ab^2$		
$= \frac{1}{3}\pi(3)18^2$		
$= 324\pi$		
Required volume $= 518.4\pi - 324\pi$	1M	
$= 194.4\pi$ (or 610.73)	1A	
Alt. Solution:		
Required volume $= \int_0^3 \pi[(12x - 2x^2)^2 - (6x)^2]dx$	1M+1M +1A	1M for $\int_a^b \pi y^2 dx$, 1M for difference of volume
$= 4\pi\int_0^3 (27x^2 - 12x^3 + x^4)dx$		
$= 4\pi\left[\frac{27x^3}{3} - \frac{12x^4}{4} + \frac{x^5}{5}\right]_0^3$		
$= 194.4\pi$ (or 610.73)	1A	