

13. (a) In Figure 3, the points P , H and K represent respectively the complex number $z = x + iy$ and the real numbers h and k .

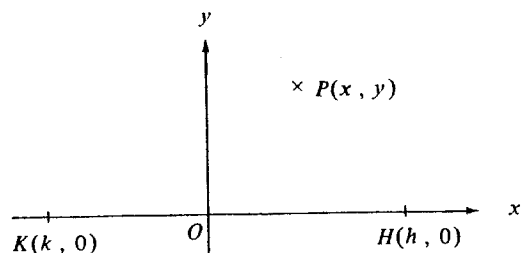


Figure 3

If P lies on the circle with HK as diameter, show that the real part of $\left(\frac{z-h}{z-k}\right)$ is 0. (5 marks)

- (b) z_1 and z_2 are the roots of $x^2 - 2x + 2 = 0$, where $-\pi < \arg z_2 < \arg z_1 < \pi$. α and β are the roots of $x^2 + 2tx - 4 = 0$, where t is a real number.
- Find z_1 and z_2 .
 - Show that α and β are real and distinct and that they have opposite signs.
 - Show that $\frac{z_1 - \alpha}{z_1 - \beta} = \frac{(1 - \alpha)(1 - \beta) + 1 + (\alpha - \beta)i}{(1 - \beta)^2 + 1}$ and obtain a similar expression for $\frac{z_2 - \alpha}{z_2 - \beta}$.
 - Suppose $\alpha > \beta$ and α, β, z_1, z_2 are represented respectively by the points A, B, C, D on the Argand plane. In addition, C and D lie on the circle with AB as diameter.

Show that $(1 - \alpha)(1 - \beta) + 1 = 0$, and hence find the value of t .

(11 marks)

END OF PAPER

89-CE
A MATHS
PAPER II

HONG KONG EXAMINATIONS AUTHORITY
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ADDITIONAL MATHEMATICS PAPER II

11.15 am-1.15 pm (2 hours)

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

SECTION A (42 marks)

Answer ALL questions in this section.

1. Find the constant term in the expansion of $(1+x)^{10} \left(1 - \frac{2}{x}\right)^3$.
(5 marks)

2. Prove, by mathematical induction, that

$$1 \times 2 \times 3 + 2 \times 3 \times 4 + \dots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

for all positive integers n .
(5 marks)

3. Using the substitution $u = 2x^2 + 1$, evaluate $\int_0^2 \frac{8x^3}{\sqrt{2x^2+1}} dx$.
(5 marks)

4. (a) Find $\int \cos^2 2x dx$.
(b) Using the result in (a), find $\int \sin^2 2x dx$.
(5 marks)

5. Let $y = 5 \sin \theta - 12 \cos \theta + 7$.

(a) Express y in the form $r \sin(\theta - \alpha) + p$ where r , α and p are constants and $0^\circ \leq \alpha < 90^\circ$.
(b) Using the result in (a), find the least value of y .
(5 marks)

6. Find the general solution of $2 \cos 2\theta + 5 \sin \theta - 3 = 0$.
(5 marks)

7. A straight line $L_1 : y = mx + c$, where m and c are constants, makes an angle of 45° with the line $L_2 : 17x - 7y + 14 = 0$.

- (a) Find the two values of m .
(b) If the distance from the point $(1, 2)$ to L_1 is 5, and m takes the greater of the two values obtained in (a), find the two values of c .
(6 marks)

8.

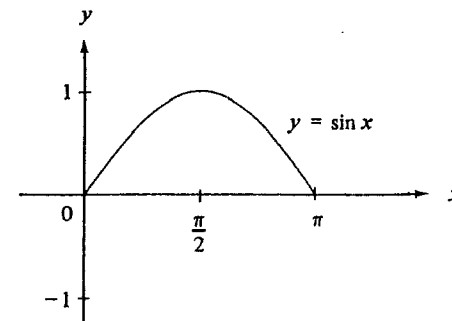


Figure 1

Figure 1 shows the graph of $y = \sin x$ for $0 \leq x \leq \pi$.

- (a) Copy Figure 1 into your answer book and sketch the graph of $y = \sin 2x$ for $0 \leq x \leq \pi$ on the same figure.

Calculate the x -coordinates of the intersecting points of the two curves for $0 \leq x \leq \pi$.

- (b) Find the area bounded by the two curves for $0 \leq x \leq \pi$.
(6 marks)

SECTION B (48 marks)

Answer any **THREE** questions from this section.
Each question carries 16 marks.

9. Let n be an integer greater than 1.

(a) Using the substitution $x = \tan \theta$, evaluate $\int_0^1 \frac{dx}{1+x^2}$.
(4 marks)

(b) By differentiating $\frac{x}{(1+x^2)^{n-1}}$ with respect to x , show that

$$\int \frac{x^2}{(1+x^2)^n} dx = \frac{1}{2(n-1)} \left[\int \frac{dx}{(1+x^2)^{n-1}} - \frac{x}{(1+x^2)^{n-1}} \right].$$

(4 marks)

(c) Using the identity $\frac{1}{(1+x^2)^n} \equiv \frac{1}{(1+x^2)^{n-1}} - \frac{x^2}{(1+x^2)^n}$,
show that

$$\int \frac{dx}{(1+x^2)^n} = \frac{2n-3}{2n-2} \int \frac{dx}{(1+x^2)^{n-1}} + \frac{1}{2(n-1)} \cdot \frac{x}{(1+x^2)^{n-1}}.$$

(3 marks)

(d) Using the above results or otherwise, evaluate

(i) $\int_0^1 \frac{dx}{(1+x^2)^2}$,

(ii) $\int_0^1 \frac{dx}{(1+x^2)^3}$.

(5 marks)

10. Two circles $C_1 : x^2 + y^2 - 6x - 8y + 21 = 0$

and $C_2 : x^2 + y^2 - 18x - 14y + 105 = 0$

intersect at A and B . C_3 is another circle passing through A , B and the point $(5, 6)$.

- (a) Write down the equation of the family of circles passing through A and B .
Hence find the equation of C_3 and show that its centre lies on the line $y = x$.
(5 marks)
- (b) Find the length and the equations of the two tangents from the origin O to C_3 .
(7 marks)
- (c) P is the point (a, b) and $M(x', y')$ is the point dividing OP in the ratio $1 : 2$.
Express a and b in terms of x' and y' .
If P is a variable point on C_3 , find the equation of the locus of M .
(4 marks)

11. (x_0, y_0) is a point on the parabola $y^2 = 8x$ where $x_0 \neq 0$.

- (a) Find $\frac{dy}{dx}$.
Hence show that the equation of the tangent to the parabola at (x_0, y_0) is $y_0 y = 4x + 4x_0$.
(4 marks)
- (b) Using the result in (a), show that the equation of the tangent of slope m to the parabola is $y = mx + \frac{2}{m}$.
(4 marks)
- (c) Find the slopes of the two tangents from the point $(-4, -2)$ to the parabola.
(2 marks)
- (d) If the angle between the two tangents from the point $(-12, a)$ to the parabola is 45° , find a .
(6 marks)

12.

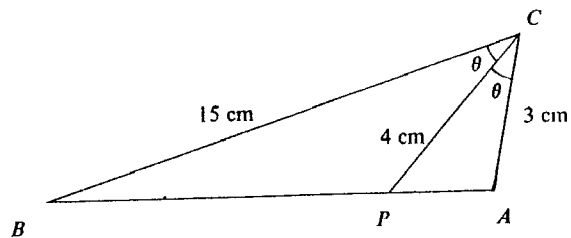


Figure 2(a)

In Figure 2(a), ABC is a triangular piece of paper. P is the point on AB such that CP bisects $\angle ACB$. $\angle ACP = \theta$, $AC = 3$ cm, $BC = 15$ cm and $CP = 4$ cm.

- (a) By considering the areas of $\triangle ABC$, $\triangle APC$ and $\triangle BPC$, show that $\cos \theta = \frac{4}{5}$. (5 marks)
- (b) $\triangle ABC$ is folded along CP so that the planes APC and BPC are perpendicular as shown in Figure 2(b). A' and B' are respectively the feet of the perpendiculars from A and B to CP and CP produced.

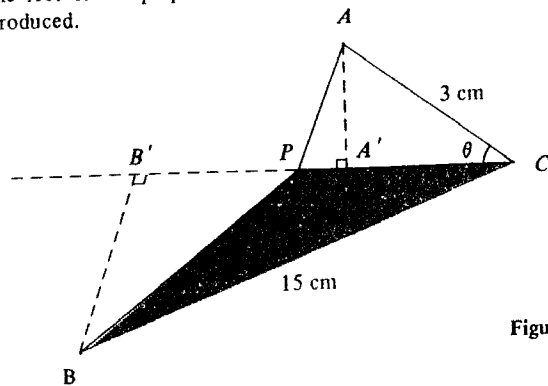


Figure 2(b)

- (i) Find AA' , BB' and $A'B'$.
- (ii) Find the distance between A and B . (6 marks)

12. (c) The paper is further folded along CP until CA lies along CB as shown in Figure 2(c). Find $\angle APB$. (5 marks)

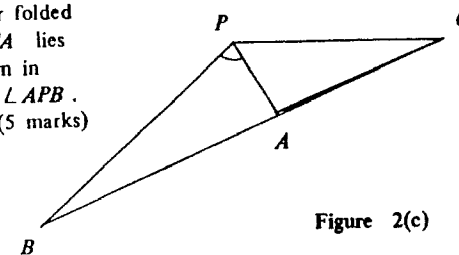


Figure 2(c)

13. The slope, at any point (x, y) , of a curve C is given by $\frac{dy}{dx} = -4x + k$, where k is a constant. The curve passes through the origin and the point $(5, 10)$.
- (a) Find the value of k and the equation of the curve C . (4 marks)
- (b) Find the area of the region bounded by the curve C and the x -axis. (3 marks)
- (c) In Figure 3, $P(a, b)$ is a point on the curve C . The area of the region bounded by the curve and the chord OP (the shaded region in the figure) is $\frac{1}{8}$ the area obtained in (b).

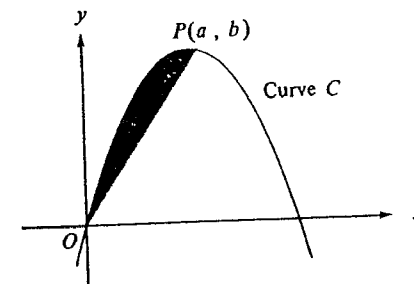


Figure 3

- (i) Find the value of a .
- (ii) If the shaded region is revolved about the x -axis, find the volume of the solid generated. (9 marks)

END OF PAPER



Additional Mathematics I

1. $\sin 5x + 5x \cos 5x$
 $10 \cos 5x - 25x \sin 5x$
 $10 \cos 5x$
2. (a) $\frac{1}{k+1} [(4k+1)i + (3-k)j]$
 (b) $\frac{9}{16}$
3. $(\frac{1}{2}, \frac{1}{8}), (-\frac{1}{2}, -\frac{1}{8})$
 $3x - 4y - 1 = 0$
 $3x - 4y + 1 = 0$
4. (a) $y = \frac{\tan x}{1 + 2 \tan^2 x}$
 (b) 0.615
5. $(r-1)x^2 - (r+5)x + (r-1) = 0$
 $7 > r > -1$
6. $p = 5, q = -2$
 $p = -5, q = 2$
 $5 - 2i, -5 + 2i$
8. -4, 8
9. (a) (i) $\vec{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$
 $\vec{OD} = \frac{1}{3}\mathbf{a}$
 (ii) $\vec{OK} = \frac{1}{3}\lambda\mathbf{a} + (1-\lambda)\mathbf{b}$
 $\lambda = \frac{3}{4}, \mu = \frac{1}{2}$
 (b) (i) $\vec{OM} = 7i + 4j$
 $\vec{DB} = -2i + 8j$
 (ii) 18, 74.3°

9. (b) (iii) $\vec{AP} = \frac{2}{3}(-17i + 4j)$
 $\vec{AK} = \frac{1}{2}(-17i + 4j)$
10. (a) $\frac{8}{27\pi} \text{ m}^3$
 (b) (i) $S = (2\pi - 4\pi^2)r^2 + 4\pi r$
 (ii) $r = \frac{1}{2\pi - 1}$
 (iii) (1) $0.15 < r < \frac{1}{2\pi - 1}$
 (2) $\frac{1}{2\pi - 1} < r < 0.25$
 Smallest value of $S = 1.07$
11. (a) (i) $p^2 - 2q$
 (ii) $-p(p^2 - 3q)$
 (iii) $q^2 - 3(p-1)q + (p-1)^2(p+1)$
 (c) $p < \frac{5}{4}$
 (d) $\frac{3}{2}$
12. (a) (i) $T = k\sqrt{a^2 + x^2} + (3a - x)$
 (ii) $(\sqrt{3} + 3)a$
 (iii) $1 < k < \frac{\sqrt{10}}{3}$
 $1 < k \leq \frac{\sqrt{10}}{3}$
 (b) (i) $(2 + \sqrt{3})a$
 (ii) $\sqrt{5}a$
13. (b) (i) $z_1 = 1 + i$
 $z_2 = 1 - i$
 (iii) $\frac{[(1-\alpha)(1-\beta) + 1] + (\beta-\alpha)t}{(1-\beta)^2 + 1}$
 (iv) $t = 1$

Additional Mathematics II

1. -479
3. $\frac{40}{3}$
4. (a) $\frac{1}{2}x + \frac{1}{8}\sin 4x + c$
 (b) $\frac{1}{2}x - \frac{1}{8}\sin 4x + c'$
5. (a) $y = 13 \sin(\theta - 67.4^\circ) + 7$
 (b) -6
6. $180k^\circ + (-1)^k 14.5^\circ$
 $180k^\circ + (-1)^k 90^\circ$
7. (a) $\frac{5}{12}, -\frac{12}{5}$
 (b) $7, -\frac{23}{6}$
8. (a) $0, \pi, \frac{\pi}{3}$
 (b) $2\frac{1}{2}$
9. (a) $\frac{\pi}{4}$
 (d) (i) $\frac{1}{8}(\pi + 2)$
 (ii) $\frac{1}{32}(3\pi + 8)$
10. (a) $x^2 + y^2 - 6x - 8y + 21 + k(x^2 + y^2 - 18x - 14y + 105) = 0$
 $x^2 + y^2 - 10x - 10y + 49 = 0$
 (b) 7
 $y = \frac{3}{4}x, y = \frac{4}{3}x$
 (c) $a = 3x', b = 3y'$
 $9x^2 + 9y^2 - 30x - 30y + 49 = 0$
11. (a) $\frac{dy}{dx} = \frac{4}{y}$
 (c) $1, -\frac{1}{2}$
 (d) ± 2
12. (b) (i) $\frac{9}{5} \text{ cm}, 9 \text{ cm}, \frac{48}{5} \text{ cm}$
 (ii) 13.3 cm
 (c) 83.3°
13. (a) 12
 $y = -2x^2 + 12x$
 (b) 72
 (c) (i) 3
 (ii) 194.4π