89-CE
A MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1989

ADDITIONAL MATHEMATICS PAPER I

8.30 am-10.30 am (2 hours)
This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

SECTION A (42 marks)

Answer ALL questions in this section.

1. Let $y = x \sin 5x$.

   Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

   Hence find $\frac{d^2y}{dx^2} + 25y$. (4 marks)

2. Let $\vec{OA} = \mathbf{i} + 3\mathbf{j}$, $\vec{OB} = 4\mathbf{i} - \mathbf{j}$ and C be a point dividing AB internally in the ratio $k : 1$.

   (a) Express $\vec{OC}$ in terms of k , $\mathbf{i}$ and $\mathbf{j}$.

   (b) If OC is perpendicular to AB , find the value of k . (5 marks)

3. Find the coordinates of the two points on the curve $y = x^3$ at which the tangents to the curve have a slope of $\frac{3}{4}$.

   Hence find the equations of the two tangents to the curve $y = x^3$ which are parallel to the line $3x - 4y = 0$. (5 marks)

4. Let $\tan\theta = 2\tan x$ and $y = \tan(\theta - x)$ where $0 \leqslant x < \frac{\pi}{2}$.

   (a) Express y in terms of $\tan x$.

   (b) When $\frac{dy}{dx} = 0$, find the value of x . (5 marks)

5. Let $\frac{x^2+5x+1}{x^2-x+1} = r$ ....(*).

Express (*) in the form $ax^2+bx+c=0$.

Hence find the range of the values of r for real values of x .

(5 marks)

6. p and q are real numbers such that $(p+qi)^2 = 21-20i$.

Find the values of p and q .

Hence write down the two square roots of $21-20i$.

(6 marks)

7. Show that $\frac{1-\sin\theta+i\cos\theta}{1-\sin\theta-i\cos\theta} = i(\cos\theta+i\sin\theta)$.

Hence show that $\left(\frac{1-\sin\frac{7\pi}{36}+i\cos\frac{7\pi}{36}}{1-\sin\frac{7\pi}{36}-i\cos\frac{7\pi}{36}}\right)^6 = \frac{\sqrt{3}}{2}+\frac{1}{2}i$.

(6 marks)

8. Solve $(x-2)^2-5|x-2|-6=0$.

(6 marks)

SECTION B (48 marks)

Answer any THREE questions from this section.
Each question carries 16 marks.

9.

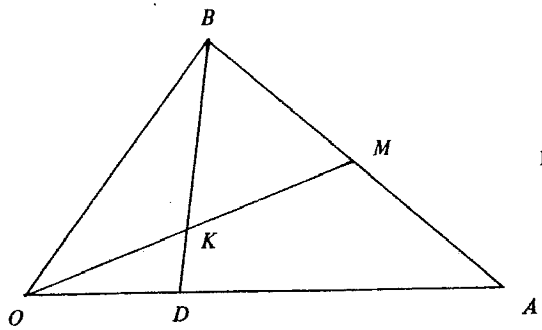

Figure 1

In Figure 1, M is the mid-point of AB and D is the point on OA such that $OD : DA = 1 : 2$. OM intersects BD at K . Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) (i) Express $\overrightarrow{OM}$ and $\overrightarrow{OD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Suppose $BK : KD = \lambda : 1-\lambda$.

Express $\overrightarrow{OK}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ .

Let $\overrightarrow{OK} = \mu\overrightarrow{OM}$. Find the values of λ and μ .

(7 marks)

(b) Suppose $\mathbf{a} = 12\mathbf{i}$ and $\mathbf{b} = 2\mathbf{i}+8\mathbf{j}$.

(i) Find $\overrightarrow{OM}$ and $\overrightarrow{DB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

(ii) Evaluate $\overrightarrow{OM}\cdot\overrightarrow{DB}$ and hence find $\angle BKM$.

(iii) Suppose P is the point on OB such that $OP : PB = 1 : 2$.

Find $\overrightarrow{AP}$ and $\overrightarrow{AK}$, and hence show that A , K , P are collinear.

(9 marks)

10. A solid right circular cylinder has length ℓ metres and base radius r metres. The sum of its length and the circumference of its cross-section is 2 metres.

(a) Find the maximum volume of the cylinder. (7 marks)

(b) Let the total surface area of the cylinder be S square metres.

(i) Express S in terms of r.

(ii) Find the value of r such that S is a maximum.

(iii) Suppose $0.15 \leqslant r \leqslant 0.25$.

Determine the range of the values of r for which S is

(1) increasing,

(2) decreasing.

Hence or otherwise, find the smallest value of S. (9 marks)

11. (a) Let α, β be the roots of the equation

$$x^2 + px + q = 0 \quad \text{.......................................}(*),$$

where p and q are real constants.

Find, in terms of p and q,

(i) $\alpha^2 + \beta^2$,

(ii) $\alpha^3 + \beta^3$,

(iii) $(\alpha^2 - \beta - 1)(\beta^2 - \alpha - 1)$. (6 marks)

(b) If the square of one root of (*) minus the other root equals 1, use (a), or otherwise, to show that

$$q^2 - 3(p-1)q + (p-1)^2(p+1) = 0 \quad \text{....................}(**).$$

(3 marks)

(c) Find the range of values of p such that the quadratic equation (**) in q has real roots. (4 marks)

(d) Suppose k is a real constant. If the square of one root of $4x^2 + 5x + k = 0$ minus the other root equals 1, use the result in (b), or otherwise, to find the value of k. (3 marks)

12.

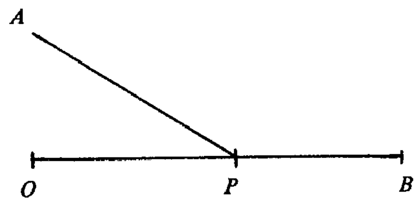

Figure 2

In Figure 2, B, due east of O, is the terminus of a railway OB of length b km, and A is a town a km north of O. A road AP is to be built connecting A to the railway at P so that goods can be transported from A to B via P. The cost of transporting 1 tonne of goods per km by road is \$ k $(k > 1)$ and \$ 1 per km by railway. Let $OP = x$ km, where $0 \leqslant x \leqslant b$, and let the cost of transporting 1 tonne of goods from A to B via P be \$ T.

(a) Suppose $b = 3a$.

(i) Find T in terms of x, a and k.

(ii) If $k = 2$, find, in terms of a, the minimum value of T.

(iii) Find the range of values of k for which $\dfrac{a}{\sqrt{k^2 - 1}} > 3a$.

Hence determine the range of values of k for which it would cost more to transport goods from A to B via P than directly from A to B without using railway. (12 marks)

(b) Let $k = 2$. Find, in terms of a, the minimum value of T for

(i) $b = 2a$,

(ii) $b = \dfrac{1}{2}a$.

(4 marks)

13. (a) In Figure 3, the points P, H and K represent respectively the complex number $z = x + iy$ and the real numbers h and k.

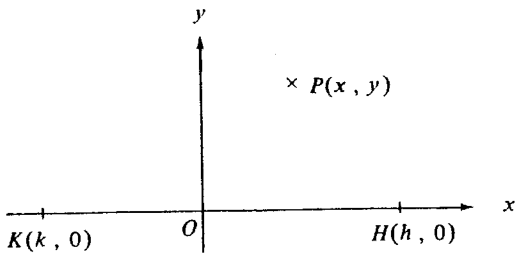

Figure 3

If P lies on the circle with HK as diameter, show that the real part of $\left(\frac{z-h}{z-k}\right)$ is 0.

(5 marks)

(b) z_1 and z_2 are the roots of $x^2 - 2x + 2 = 0$, where $-\pi < \arg z_2 < \arg z_1 < \pi$. α and β are the roots of $x^2 + 2tx - 4 = 0$, where t is a real number.

(i) Find z_1 and z_2.

(ii) Show that α and β are real and distinct and that they have opposite signs.

(iii) Show that $\frac{z_1 - \alpha}{z_1 - \beta} = \frac{(1-\alpha)(1-\beta) + 1 + (\alpha - \beta)i}{(1-\beta)^2 + 1}$ and obtain a similar expression for $\frac{z_2 - \alpha}{z_2 - \beta}$.

(iv) Suppose $\alpha > \beta$ and α, β, z_1, z_2 are represented respectively by the points A, B, C, D on the Argand plane. In addition, C and D lie on the circle with AB as diameter.

Show that $(1-\alpha)(1-\beta) + 1 = 0$, and hence find the value of t.

(11 marks)

END OF PAPER

89-CE
A MATHS
PAPER II

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1989

ADDITIONAL MATHEMATICS PAPER II

11.15 am-1.15 pm (2 hours)

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.