

# RESTRICTED 內部文件

香港考試局  
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一九八八年香港中學會考  
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1988

附加數學 (卷二)  
Additional Mathematics (Paper II)

評卷參考  
Marking Scheme

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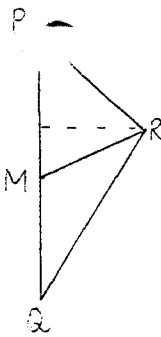
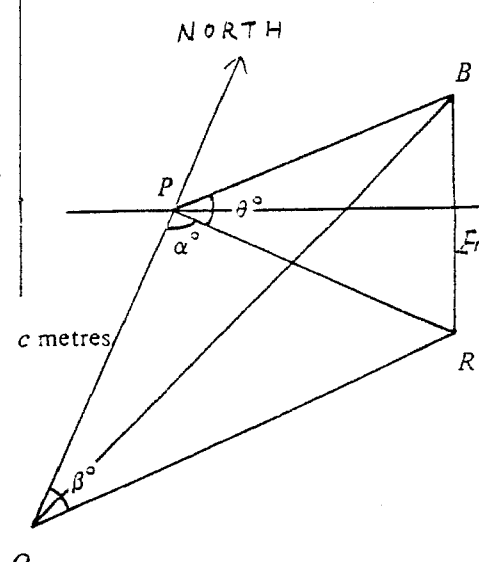


SOLUTIONS	MARKS	REMARKS
5. $n = 1, \quad \text{L.S.} = 1^2 = 1$		
$\text{R.S.} = \frac{1(2-1)(2+1)}{3} = 1$		
The equality holds for $n = 1.$	1	
Assume $1^2+3^2+\dots+(2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$		
for some positive integer $k.$	1	
$n = k + 1,$		<u>Alt. Solution:</u>
$\text{L.S.} = 1^2 + 3^2 + \dots + (2k-1)^2 + [(2(k+1) - 1)]^2$	1	$\text{L.S.} = \dots$
$= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2 \dots\dots\dots$	1	$= \frac{4k^3+12k^2+11k+3}{3}$
$= \frac{(2k+1)}{3} [2k^2 - k + 3(2k+1)]$		$\text{R.S.} = \frac{1}{3} (k+1)(2k+1)(2k+3)$
$= \frac{1}{3}(2k+1)(2k^2+5k+3)$		$= \frac{1}{3} (4k^3+12k^2+11k+3)$
$= \frac{1}{3}(2k+1)(2k+3)(k+1)$		$= \text{L.S.}$
$= \frac{1}{3}(k+1)(2k+1)(2k+3) \dots\dots\dots$	1	
Therefore equality holds for $n = k + 1.$		
By the Principle of Mathematical Induction, the equality holds for all positive integers $n.$	1	Award this mark only if a candidate has scored the first 5 marks.
	6	

6. Put $u = 9 - x^3$	1A	Put $v^2 = 9 - x^3$	1
$du = -3x^2 dx \dots\dots\dots$	1A	$2v dv = -3x^2 dx \dots\dots\dots$	1
When $x = 0, u = 9$ )		When $x = 0, v = 3$ )	
$x = 2, u = 1$ )	1A	$x = 2, v = 1$ )	
$\int_0^2 \frac{x^2 dx}{\sqrt{9-x^3}}$		$\int_0^2 \frac{x^2 dx}{\sqrt{9-x^3}} = \int_3^1 \frac{1}{3} \left(-\frac{2}{3}\right) dv$	
$= \int_9^1 \frac{1}{3} \frac{-du}{\sqrt{u}} \dots\dots\dots$	1A	$= \left[ \frac{2}{3} v \right]_1^3 \dots\dots\dots$	1
$= \frac{1}{3} \left[ \frac{\sqrt{u}}{\frac{1}{2}} \right]_9^1$		$= \frac{4}{3}$	
$= \frac{4}{3} \dots\dots\dots$	6	<u>Alt. Solution:</u>	
		Put $x^3 = 9 \sin^2 \theta$	1A
		$3x^2 dx = 18 \sin \theta \cos \theta d\theta$	1
		When $x = 0, \theta = 0$	
		$x = 2, \theta = 1.231$ }	1
		$\int_0^2 \frac{x^2 dx}{\sqrt{9-x^3}}$	
		$= \int_0^{1.231} 2 \sin \theta d\theta$	1
		$= [-2 \cos \theta]_0^{1.231}$	1



SOLUTIONS	MARKS	REMARKS
8. (c) (i) From (b), $2n \int_0^{\frac{\pi}{2}} \cos^{2n} x dx - (2n-1) \int_0^{\frac{\pi}{2}} \cos^{2n-2} x dx$ $= [\sin x \cos^{2n-1} x]_0^{\frac{\pi}{2}} \dots\dots\dots$	1A	<u>OR</u> $[\sin x \cos^{2n-1} x + C]_0^{\frac{\pi}{2}}$
$2n \int_0^{\frac{\pi}{2}} \cos^{2n} x dx - (2n-1) \int_0^{\frac{\pi}{2}} \cos^{2n-2} x dx = 0$	1A	For R.S.
$\int_0^{\frac{\pi}{2}} \cos^{2n} x dx = \frac{2n-1}{2n} \int_0^{\frac{\pi}{2}} \cos^{2n-2} x dx$	1	
(ii) $\int_0^{\frac{\pi}{2}} \cos^6 x dx = \frac{5}{6} \int_0^{\frac{\pi}{2}} \cos^4 x dx$ $= \frac{5}{6} \cdot \frac{3}{4} \int_0^{\frac{\pi}{2}} \cos^2 x dx \dots\dots\dots$	1A	<u>Alt. Solution:</u>
$\int_0^{\frac{\pi}{2}} \cos^2 x dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} dx$ $= \frac{1}{2} [x]_0^{\frac{\pi}{2}}$ $= \frac{\pi}{4} \dots\dots\dots$	1A	$\int_0^{\frac{\pi}{2}} \cos^2 x dx = \int_0^{\frac{\pi}{2}} \frac{1}{2} (\cos 2x + 1) dx$ $= \frac{1}{2} \left[ \frac{1}{2} \sin 2x + x \right]_0^{\frac{\pi}{2}}$ $= \frac{\pi}{4} \dots\dots\dots$
Therefore, $\int_0^{\frac{\pi}{2}} \cos^6 x dx = \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{\pi}{4}$ $= \frac{5}{32} \pi \dots\dots\dots$	<u>1A</u> <u>7</u>	
(d) Put $v = \frac{\pi}{2} - x \dots\dots\dots$ $dv = -dx$ $x = 0, v = \frac{\pi}{2}; x = \frac{\pi}{2}, v = 0 \dots\dots\dots$	1A	
$\int_0^{\frac{\pi}{2}} \sin^6 x dx = \int_{\frac{\pi}{2}}^0 \sin^6 \left( \frac{\pi}{2} - v \right) (-dv)$ $= \int_0^{\frac{\pi}{2}} \cos^6 v dv \dots\dots\dots$ $= \frac{5}{32} \pi$	1A	NOTE: If a cand. claims $\int_0^{\frac{\pi}{2}} \sin^6 x dx = \int_0^{\frac{\pi}{2}} \cos^6 x dx$ $= \frac{5}{32} \pi \dots\dots\dots$
<u>Alt. Solution:</u> $\int_0^{\frac{\pi}{2}} \sin^6 x dx = \int_0^{\frac{\pi}{2}} (1 - \cos^2 x)^3 dx$ $= \int_0^{\frac{\pi}{2}} (1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x) dx$ $= [x]_0^{\frac{\pi}{2}} - \frac{3}{2} \cdot \frac{\pi}{2} + 3 \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} - \frac{5}{32} \pi$ $= \frac{5}{32} \pi \dots\dots\dots$	1A 1A 1M 1A	For using (c).

SOLUTIONS	MARKS	REMARKS
<p>9. (a) In <math>\triangle PQR</math>, <math>\frac{PR}{\sin \beta^\circ} = \frac{c}{\sin \angle PRQ}</math></p> <p><math>\sin \angle PRQ = \sin(180^\circ - \alpha^\circ - \beta^\circ)</math>  <math>= \sin(\alpha^\circ + \beta^\circ) \dots\dots\dots</math></p> <p>In <math>\triangle PBR</math>, <math>h = PR \tan \theta^\circ</math>  <math>= \frac{c \tan \theta^\circ \sin \beta^\circ}{\sin(\alpha^\circ + \beta^\circ)}</math></p>	<p>2A</p> <p>2A</p> <p>1M</p> <p><u>1</u></p> <p><u>6</u></p>	<p>Accept expressions with no degree measure</p>
<p>(b) (i) In <math>\triangle PQR</math>, <math>\frac{QR}{\sin \alpha^\circ} = \frac{c}{\sin \angle PRQ} \dots\dots\dots</math></p> <p><math>QR = \frac{c \sin 54^\circ}{\sin 80^\circ} (= 0.8215c)</math></p> <p><math>\tan \angle BQR = \frac{h}{QR} \dots\dots\dots</math></p> <p><math>= \frac{c \sin 46^\circ \tan 40^\circ}{\sin 100^\circ} \cdot \frac{\sin 80^\circ}{c \sin 54^\circ}</math></p> <p><math>\angle BQR = 36.7^\circ \dots\dots\dots</math></p>	<p>1A</p> <p>2M</p> <p>2A</p>	<p><math>h = 0.6129c</math></p>
<p>(ii) In <math>\triangle QMR</math>, <math>MR^2 = QM^2 + QR^2 - 2QM \cdot QR \cdot \cos 46^\circ</math></p> <p><math>MR = 0.5951c</math></p> <p><math>\tan \angle BMR = \frac{BR}{MR} \dots\dots\dots</math></p> <p><math>= \frac{c \tan 40^\circ \sin 46^\circ}{\sin 100^\circ} \cdot \frac{1}{0.5951c}</math></p> <p><math>\angle BMR = 45.8^\circ \dots\dots\dots</math></p>	<p>2M</p> <p>1M</p> <p>2A</p>	<p><u>Alt. Solution:</u></p> <p>In <math>\triangle PQR</math>, <math>PR = \frac{c \sin 46^\circ}{\sin 80^\circ}</math></p> <p><math>MR^2 = PM^2 + PR^2 - 2PM \cdot PR \cos 54^\circ</math></p>
<p>In <math>\triangle PMR</math>, <math>\frac{\sin \angle PMR}{PR} = \frac{\sin 54^\circ}{MR}</math></p> <p><math>\sin \angle PMR = \sin 54^\circ \cdot \frac{0.7304c}{0.5951c}</math></p> <p><math>\angle PMR = 83.2^\circ</math> or <math>96.8^\circ</math> (rejected)                  (Accept <math>\angle PMR = 83.2^\circ</math>)</p> <p>The bearing of B from M is N83.2°E.</p>	<p>1M</p> <p>2A</p> <p><u>1A</u></p> <p><u>14</u></p>	<p>for <math>\angle PMR</math></p>
		
<p><u>Alt. Solution:</u></p> <p>In <math>\triangle QMR</math>,</p> <p><math>\frac{\sin \angle QMR}{QR} = \frac{\sin 46^\circ}{MR} \dots\dots\dots</math> 1M</p> <p><math>\angle QMR = 96.8^\circ</math> or <math>83.2^\circ</math> 2A</p> <p>Rejecting <math>83.2^\circ</math>, the bearing of B from M is N83.2°E. 1A</p>		

SOLUTIONS	MARKS	REMARKS
10.(a) $x = a \sin \theta$		
$dx = a \cos \theta d\theta$ .....	1A	
When $x = 0$ , $\theta = 0$ ; $x = a$ , $\theta = \frac{\pi}{2}$	1A	
$\int_0^a \sqrt{a^2 - x^2} dx = \int_0^{\frac{\pi}{2}} a^2 \cos^2 \theta d\theta$ $= \frac{a^2}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2\theta) d\theta$ $= \frac{1}{2} a^2 \left[ \theta + \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{2}}$ $= \frac{\pi a^2}{4} \dots\dots\dots$	1A	
$\text{Area of ellipse} = 2 \int_{-a}^a y dx$ $= 4 \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx$ $= \frac{4b}{a} \int_0^a \sqrt{a^2 - x^2} dx$ $= \pi ab \dots\dots\dots$	1A	
	<u>1A</u>	
	<u>6</u>	
(b) (i) Volume of pebble = $\int_{-1}^1 \pi y^2 dx$	1A+1M	1A for limits
$= \int_{-1}^1 \pi \left(\frac{3}{4}\right)^2 (1 - x^2) dx$	1A	1M for $\int_a^b \pi y^2 dx$
$= \frac{9}{16} \pi \left[ x - \frac{x^3}{3} \right]_{-1}^1$		
$= \frac{3}{4} \pi \dots\dots\dots$	1A	
(ii) (1) $V = \int_{-b}^{-(b-h)} \pi x^2 dy$	1M+1A	1A for limits
$= \int_{-b}^{-b+h} \pi \cdot 4b^2 \left(1 - \frac{y^2}{b^2}\right) dy$	1A	1M for $\int_a^b \pi x^2 dy$
$= 4\pi b^2 \left[ y - \frac{y^3}{3b^2} \right]_{-b}^{-b+h}$		
$= 4\pi b^2 \left[ -b + h - \frac{(-b+h)^3}{3b^2} + b - \frac{b^3}{3b^2} \right]$		
$= 4\pi b^2 \left[ h - \frac{b}{3} + \frac{b^3 - 3b^2h + 3bh^2 - h^3}{3b^2} \right]$	1M	for expanding $(b-h)^3$
$= 4\pi b^2 \left[ \frac{3bh^2 - h^3}{3b^2} \right]$		
$= \frac{4\pi h^2}{3} (3b - h) \dots\dots\dots$	1	
$\frac{dV}{dh} = 8\pi bh - 4\pi h^2$	1A	
When $h = \frac{b}{2}$ , $\frac{dV}{dh} = 4\pi b^2 - \pi b^2$		
$= 3\pi b^2 \dots\dots\dots$	1A	
(2) $\delta V \approx \frac{dV}{dh} \cdot \delta h$	1M	
$\frac{3}{4}\pi \approx 3\pi (5)^2 \cdot \delta h \dots\dots\dots$	1M	For $\delta V = \text{vol. of pebble in (b)(i)}$
$\delta h \approx 0.01 \text{ (unit)}$	<u>1</u>	
	<u>14</u>	

SOLUTIONS	MARKS	REMARKS
11. (a) S lies on the perpendicular through K, slope of KS = -5	1A	
$\frac{y - 12}{x - 1} = -5$ .....	1A	
$5x + y - 17 = 0$		
S also lies on the perpendicular bisector of HK: Mid-point of HK is (-1, 9) ) Slope of HK = $\frac{12 - 6}{1 - (-3)} = \frac{3}{2}$ ) .....	1A	<u>Alt. Solution:</u> HS = KS $\sqrt{(x+3)^2 + (y-6)^2}$
$\frac{y - 9}{x + 1} = -\frac{2}{3}$	1M+1A	$= \sqrt{(x-1)^2 + (y-12)^2}$ 1M+
$-3y + 27 = 2x + 2$		$8x + 12y - 100 = 0$ ....
$2x + 3y - 25 = 0$		$2x + 3y - 25 = 0$
Solving the two equations, $x = 2, y = 7$ .....	1A	
S is the point (2, 7). Equation of C: $(x-2)^2 + (y-7)^2 = (2-1)^2 + (7-12)^2$	1M	
$(x-2)^2 + (y-7)^2 = 26$	1A	
$x^2 + y^2 - 4x - 14y + 27 = 0$	<u>8</u>	

Alt. Solution:

Let the equation of C be  
 $x^2 + y^2 + 2gx + 2fy + c = 0$  1M

This passes through (1, 12) and (-3, 6).  
 $1^2 + 12^2 + 2g + 24f + c = 0$  ..... 1A  
 $9 + 36 - 6g + 12f + c = 0$  1A

Differentiating the equation of C,  
 $2x + 2yy' + 2g + 2fy' = 0$  ..... 1M  
 $2 + 24(\frac{1}{5}) + 2g + 2f(\frac{1}{5}) = 0$  1A  
 $5g + f + 17 = 0$   
 Solving the three equations,  
 $g = -2$  )  
 $f = -7$  ) ..... 1A  
 $c = 27$  )

S is (2, 7). 1A

Equation of C is  $x^2 + y^2 - 4x - 14y + 27 = 0$  1A

Alt. Solution:  
 $\frac{12 + f}{1 + g} = -5$  1M+  
 $5g + f + 17 = 0$





SOLUTIONS	MARKS	REMARKS
12.(a) Equation of L: $\frac{y - 0}{x + 2} = m$ $y = m(x + 2)$ $y = mx + 2m$	1A	
Since A and B are the intersecting points of L and the parabola $y^2 = 8x$ , the coordinates of A and B satisfies the equations of L and the parabola, i.e. $y = mx + 2m$ and $y^2 = 8x$ .		
Eliminating y, $(mx + 2m)^2 = 8x$ ..... $m^2x^2 + (4m^2 - 8)x + 4m^2 = 0$	1M <hr/> <u>1</u> <hr/> <u>3</u>	
$\begin{matrix} x_1 + x_2 = \frac{8 - 4m^2}{m^2} \\ x_1 x_2 = 4 \end{matrix} \quad ) \dots\dots\dots$	1A	
$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$	1A	
$= \left( \frac{8 - 4m^2}{m^2} \right)^2 - 16$ .....	1M+1A	
$= 16 \left[ \frac{(2 - m^2)}{m^2} \right]^2 - 16$		
$= \frac{16(4 - 4m^2)}{m^4}$ .....	1A	
$= \frac{64(1 - m^2)}{m^4}$	<hr/> <u>5</u> <hr/>	
<p>Alt. Solution:</p> $x = \frac{-(4m^2 - 8) \pm \sqrt{(4m^2 - 8)^2 - 4(4m^2)(m^2)}}{2m^2} \quad 1A$ $(x_1 - x_2)^2 = \left[ \frac{2\sqrt{(4m^2 - 8)^2 - 16m^4}}{2m^2} \right]^2 \quad 2M+1A$ $= \frac{64(1 - m^2)}{m^4} \quad 1A$		

SOLUTIONS	MARKS	REMARKS
12. (c) $y_1 = mx_1 + 2m$ $y_2 = mx_2 + 2m$ ) .....	1A	Can be omitted.
$y_1 - y_2 = m(x_1 - x_2)$ .....	2A	
$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ $= (x_1 - x_2)^2 + m^2((x_1 - x_2)^2)$ .....	1M	
$= (1 + m^2)(x_1 - x_2)^2$ ) $= \frac{64(1 + m^2)(1 - m^2)}{m^4}$ ) .....	1	
	<u>5</u>	

Alt. Solution:

Eliminating  $x$  from  $y = mx + 2m$  and  $y^2 = 8x$ .

$my^2 - 8y + 16m = 0$  ..... 1A

$y_1 + y_2 = \frac{8}{m}$  )  
 $y_1 y_2 = 16$  ) .....

$(y_1 - y_2)^2 = (y_1 + y_2)^2 - 4y_1 y_2$   
 $= \left(\frac{8}{m}\right)^2 - 64$  ..... 1M  
 $= \frac{64(1 - m^2)}{m^2}$  ..... 1A

$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$   
 $= \frac{64(1 - m^2)}{m^4} + \frac{64(1 - m^2)}{m^2}$   
 $= \frac{64(1 - m^2)(1 + m^2)}{m^4}$  ..... 1

(a) From (c), $AB^2 = 0$ or from (a), $D = 0$ . $m^2 - 1 = 0$ $m = \pm 1$	1M	
	<u>1A+1A</u>	
	<u>3</u>	

(e) L: $y = \frac{\sqrt{3}}{3}x + \frac{2\sqrt{3}}{3}$ $x - \sqrt{3}y + 2 = 0$ Distance from C to L = $\left  \frac{2 - \sqrt{3}(0) + 2}{\sqrt{1+3}} \right $ $= 2$ .....	1M	Absolute value sign optional
	1A	
Length of AB = $\sqrt{\frac{64(1 + \frac{1}{3})(1 - \frac{1}{3})}{\frac{1}{9}}}$ $= 16\sqrt{2}$ .....	1A	
$\Delta ABC = \frac{1}{2}(2)(16\sqrt{2})$ $= 16\sqrt{2}$ .....	<u>1A</u>	
	<u>4</u>	